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Graph Algorithms and Abstraction for Number Theoretic Concepts

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Abstract: In this paper, we discuss graph theoretic representation of the number theoretic concepts such as relatively prime numbers, Greatest Common Divisor (GCD), Geometric mean (GM) and Fibonacci numbers using graph labeling. We also propose a new generalization of Geometric mean.

Keywords: Graph, Greatest Common Divisor, Geometric mean.

1. Introduction

Unlike the abstract algebraic treatment and rigid analytical treatment, graph theory gives more intuitive combinatorial and simple pedagogical appeal. In this paper, we discuss number theoretic concepts such as GCD of a given set of numbers, Geometric mean of a given set of numbers and Fibonacci numbers using graph labeling. We also propose a new generalization of Geometric mean. We define a new type of labeled graph called the divisor graph of a set of given numbers, whose significance is that the numbers in one component are all relatively prime numbers to the numbers of any other component, if any. The GCD of a set of numbers equals the product of the prime factors common to all the numbers. Geometric mean (GM) of a given set of numbers, $a_1.a_2\dots a_n$ is given by $\sqrt[n]{a_1a_2\dots .a_n}$. For Number theoretic terms and definitions not explicitly defined here, one can refer Childs [1]. For graph theoretic terms and definitions not explicitly defined here, one can refer Harary [2].

2. Main Results

GCD of a set of numbers is the greatest common divisor of all the numbers of that set of numbers. The GCD of a set of numbers is also equal the product of the prime factors common to all the numbers. Two distinct numbers are called relatively prime numbers, if they have no common divisors other than one. We first discuss a new type of labeled graph called divisor graph, to find relatively prime numbers among a set of given numbers.

2.1. Divisor Graphs

A *Divisor graph* of a set S of positive integers is an edge-labeled graph G*, whose vertices are the elements in S and two vertices u, v are adjacent in G* if u and v have a common divisor and that common divisor is the label of the edge $\{u, v\}$.

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The edge $\{u, v\}$ will be a multiple edge, if u, v have more than one common divisor.

The significance of the divisor graph of a set of given numbers is that the numbers in a component of the divisor graph are all relatively prime numbers to all the numbers in any other component, if any. Isolated vertices in Divisor graphs correspond to the Prime numbers.

2.2. GCD of a Set of Numbers

The GCD of a set of numbers equals the product of the prime factors common to all the numbers. It is generally calculated by repeatedly taking the GCD of pairs of numbers in some order.

For an array $a[n]$ of n elements, the algorithm is as follows:

(1). $GCD = a[0]$.

(2). For $i = 1$ to $n - 1$; $GCD = GCD\{GCD, a[i]\}$.

Now, we present the graph algorithm to compute the GCD is as follows:

(1). Let S be a given set of n positive integers. Let G* be the Divisor graph of the elements of S.

(2). If G^* is not a complete graph, then the numbers in S have no GCD.

(3). If G^* is a complete graph, then all its edges will have a common label and this common edge label is the GCD of the given set of numbers.

Example 2.1. *Let S=$\{2, 4, 6, 8, 10\}$. Then G^* will be a complete graph with 5 vertices. Also all the edges will have a common label, 2 which is the GCD of numbers in S. Note that the edge $\{4, 8\}$ has a multiple edge of multiplicity 2 and have labels 2 and 4.*

2.3. Generalized GM of a Set of Numbers

Geometric mean (GM) of a given set of numbers, $a_1.a_2 \ldots a_n$ is given by $= \sqrt[n]{a_1a_2 \ldots a_n}$. GM can be calculated repeatedly by taking the n$^{\text{th}}$ root of pairs of numbers, since $(\sqrt[n]{a_1a_2})(\sqrt[n]{a_3a_4}) \cdots = \sqrt[n]{a_1a_2 \ldots a_n}$. For an array $a[n]$ of n elements, it can be calculated by an algorithm as follows:

(1). $GM = \{a[0] * a[1]\}^{1/n}$.

(2). For $i = 2$ to $n - 1$; $GM = GM^*\{a[i]\}^{1/n}$.

The Generalized GM of a set of numbers is defined as follows:

(1). For a given set S of n positive integers, consider a graph G with n vertices and assign the elements of S as its vertex labels.

(2). For any edge $\{a_i, a_j\}$, assign the label $\sqrt[n]{a_ia_j}$.

(3). The product of all the edge labels is defined as the generalized GM of the given set of numbers.

The generalized GM of the given numbers assigned as vertex labels of a graph G is very much depends on the nature of the graph. Thus we can have a notion of generalized geometric means of a set of numbers. The generalized GM of the given numbers with respect to a graph G will be equal to the usual GM of the given numbers, if the graph G is a 1-regular graph, If the graph G is a 2-regular graph, like a cycle graph, then the generalized GM of the given numbers will be twice the usual GM of the given numbers. The following theorem is the complete information of the importance of graph structure on generalized GM of the given numbers.

Theorem 2.2. *For a graph G with the vertex degrees, $\{d_1, d_2 \ldots d_n\}$, the generalized GM and the usual GM are related by the following equation:*

$$GGM = \left({a_1}^{d_1} {a_2}^{d_2} \ldots {a_n}^{d_n}\right) GM$$

In a similar way, we can define the generalized Arithmetic mean and the generalized Harmonic mean of a set of given numbers. In the statistical analysis, various classes of data may have inter-relationships other than class frequencies, which is not usually taken into account by statisticians. The graph representation of the classes accounts for the inter-relationships among the classes so that it gives a better measure of central tendency.

2.4. Fibonacci Graphs

Construct a graph G recursively as follows: Let a_1, a_2 be two vertices with labels 1, 1. Let a_3 be a new vertex which is adjacent to both a_1, a_2 and has the label, $1 + 1 = 2$. (That is, the sum of the labels of a_1, a_2). Let a_4 be a new vertex which is adjacent to both a_2, a_3 and has the label $2 + 1 = 3$ and so on. The resulting graph is defined as the Fibonacci graph and denoted by F^*. In this section, we list some properties of Fibonacci graph.

Theorem 2.3. *The labels of the Fibonacci graph with n vertices are the first n Fibonacci numbers.*

Theorem 2.4. *The Fibonacci graph is nearly a cubic graph. It has exactly two vertices have degree 2 (the first and last vertices).*

Theorem 2.5. *Fibonacci graphs are planar.*

References

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