Int. J. Math. And Appl., 7(1)(2019), 203–210
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Soft Precompactness via Soft Ideals

P. Gomathi Sundari[1], R. Muthu Vijayalakshmi[1] and N. Rajesh[1,*]

1 Department of Mathematics, Rajah Serfoji Government College, Thanjavur, Tamil Nadu, India.

Abstract: In this paper, we introduce and study the concept of some covering properties via soft ideal preopen sets.

MSC: 54D10.

Keywords: Soft topological space, soft pre-$\tilde{I}$-compact space, soft p-closed space.

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1. Introduction

The concept of soft sets was first introduced by Molodtsov [13] in 1999 as a general mathematical tool for dealing with uncertain objects. In [13, 14], Molodtsov successfully applied the soft theory in several directions,such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. After presentation of the operations of soft sets [11], the properties and applications of soft set theory have been studied increasingly [3, 7, 14, 16]. In recent years, many interesting applications of soft set theory have been expanded by embedding the ideas of fuzzy sets [1, 2, 4, 9–12, 14, 15, 21]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations [5]. Recently, in 2011, Shabir and Naz [18] initiated the study of soft topological spaces. They defined soft topology on the collection τ of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces and established their several properties. In this paper, we introduce and study the concept of some covering properties via soft ideal preopen sets.

2. Preliminaries

In this section, we present the basic definitions and results of soft set theory which will be needed in the sequel.

Definition 2.1 ([13])**.** *Let X be an initial universe and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F,A) denoted by F_A is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parametrized family of subsets of the universe X. For a particular $e \in A, F(e)$ may be considered the set of approximate elements of the soft set (F, A) and if $e \neq A$, then $F(e) = \phi$, that is, $F_A = \{F(e) : e \in A \subseteq E, F : A \to P(X)\}$. The family of all these soft sets over X denoted by $SS(X)_A$.*

* E-mail: nrajesh_topology@yahoo.co.in

Definition 2.2 ([11]). *Let $F_A, G_B \in SS(X)_E$. Then F_A is soft subset of G_B, denoted by $F_A \tilde{\subseteq} G_B$, if*

(1). $A \subseteq B$

(2). $F(e) \subseteq G(e)$ for all $e \in A$.

In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of $F_A, G_B \tilde{\supseteq} F_A$.

Definition 2.3 ([11]). *Two soft subset F_A and G_B over a common universe set X are said to be soft equal if F_A is a soft subset of G_B and G_B is a soft subset of F_A.*

Definition 2.4 ([3]). *The complement of a soft set (F, A), denoted by $(F, A)'$, is defined by $(F, A)' = (F', A), F' : A \to P(X)$ is a mapping given by $F'(e) = X - F(e)$ for all $e \in A$ and F' is called the soft complement function of F. Clearly $(F')'$ is the same as F and $((F, A)')' = (F, A)$.*

Definition 2.5 ([18]). *The difference between two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E) - (G, E)$ is the soft set (H,E) where for all $e \in E, H(e) = F(e) - G(e)$.*

Definition 2.6 ([18]). *Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E) whenever $x \in F(e)$ for all $e \in E$.*

Definition 2.7 ([11]). *A soft set (F, A) over X is said to be a NULL soft set denoted by ϕ or ϕ_A if for all $e \in A, F(e) = \phi$ (null set).*

Definition 2.8 ([11]). *A soft set (F, A) over X is said to be an absolute soft set denoted by $\tilde{A}$ or X_A if for all $e \in A, F(e) = X$. Clearly we have $X_A^{'} = \phi_A$ and $\phi_A' = X_A$.*

Definition 2.9 ([11]). *The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B \\ G(e), & \text{if } e \in B - A \\ F(e) \cup G(e), & \text{if } e \in A \cap B \end{cases}$$

Definition 2.10 ([11]). *The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and for all $e \in C, H(e) = F(e) \cap G(e)$. Note that, in order to efficiently discuss, we consider only soft sets (F, E) over a universe X in which all the parameter set E are same. We denote the family of these soft sets by $SS(X)_E$.*

Definition 2.11 ([22]). *Let I be an arbitrary indexed set and $L = \{(F_i, E), i \in I\}$ be a subfamily of $SS(X)_E$.*

(1). The union of L is the soft set (H, E), where $H(e) = \cup_{i \in I} F_i(e)$ for each $e \in E$. We write $\cup_{i \in I}(F_i, E) = (H, E)$.

(2). The intersection of L is the soft set (M, E), where $M(e) = \cap_{i \in I} F_i(e)$ for each $e \in E$. We write $\cap_{i \in I}(F_i, E) = (M, E)$.

Definition 2.12 ([18]). *Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if*

(1). $\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X, \forall e \in E$,

(2). the union of any number of soft sets in τ belongs to τ,

(3). the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.13 ([17]). *Let (X, τ, E) be a soft topological space. A soft set (F, A) over X is said to be closed soft set in X, if its relative complement $(F, A)'$ is an open soft set.*

Definition 2.14 ([17]). *Let(X, τ, E) be a soft topological space. The members of t are said to be open soft sets in X.*

Definition 2.15 ([18]). *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft closure of (F, E), denoted by $\mathrm{Cl}(F, E)$ is the intersection of all closed soft super sets of (F, E). Clearly $\mathrm{Cl}(F, E)$ is the smallest closed soft set over X which contains (F, E), that is, $\mathrm{Cl}(F, E) = \tilde{\cap}\{(H, E) : (H, E)$ is closed soft set and $(F, E)\tilde{\subseteq}(H, E)\})$.*

Definition 2.16 ([22]). *Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft interior of (G,E), denoted by $\mathrm{Int}(G, E)$ is the union of all open soft subsets of (G,E). Clearly $\mathrm{Int}(G, E)$ is the largest open soft set over X which contained in (G, E), that is, $\mathrm{Int}(G, E) = \tilde{\cup}\{(H, E) : (H, E)$ is an open soft set and $(H, E)\tilde{\subseteq}(G, E)\})$.*

Definition 2.17 ([22]). *The soft set $(F, E) \in SS(X)_E$ is called a soft point in X_E if there exists $x \in X$ and $e \in E$ such that $F(e) = \{x\}$ and $F(e') = \phi$ for each $e' \in E - \{e\}$ and the soft point (F, E) is denoted by x_e.*

Definition 2.18 ([19]). *The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it.*

Definition 2.19 ([22]). *The soft point x_e is said to be belonging to the soft set (G, A), denoted by $x_e\tilde{\in}(G, A)$, if for the element $e \in A, F(e) \subseteq G(e)$.*

Definition 2.20 ([22]). *A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood of the soft point $x_e\tilde{\in}X_E$ if there exists an open soft set (H, E) such that $x_e\tilde{\in}(H, E)\tilde{\subseteq}(G, E)$. A soft set (G, E) in a soft topological space (X, τ, E) is called a soft neighborhood of the soft (F, E) if there exists an open soft set (H, E) such that $(F, E)\tilde{\in}(H, E)\tilde{\subseteq}(G, E)$. The neighborhood system of a soft point x_e, denoted by $N_\tau(x_e)$, is the family of all its neighborhoods.*

Theorem 2.21 ([20]). *Let (X, τ, E) be a soft topological space. For any soft point x_e, $x_e\tilde{\in}\,\mathrm{Cl}(F, E)$ if and only if each soft neighborhood of x_e intersects (F, E).*

Definition 2.22 ([18]). *Let (X, τ, E) be a soft topological space and Y be a non null subset of X. Then $\tilde{Y}$ denotes the soft set (Y, E) over X for which $Y(e) = Y$ for all $e \in E$.*

Definition 2.23 ([18]). *Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$ and Y be a nonnull subset of X. Then the sub soft set of (F,E) over Y denoted by (F_Y, E), is defined as follows: $F_Y(e) = Y \cap F(e)$ for all $e \in E$. In other words $(F_Y, E) = \tilde{Y}\tilde{\cap}(F, E)$.*

Definition 2.24 ([18]). *Let (X, τ, E) be a soft topological space and Y be a non null subset of X. Then $\tau_Y = \{(F_Y, E) : (F, E) \in \tau\}$ is said to be the soft relative topology on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).*

Theorem 2.25 ([18]). *Let (Y, τ_Y, E) be a soft subspace of a soft topological space (X, τ, E) and $(F, E) \in SS(X)_E$. Then*

(1). If (F, E) is an open soft set in Y and $\tilde{Y} \in \tau$, then $(F, E) \in \tau$.

(2). (F, E) is an open soft set in Y if and only if $(F, E) = \tilde{Y}\tilde{\cap}(G, E)$ for some $(G, E) \in \tau$.

(3). (F, E) is a closed soft set in Y if and only if $(F, E) = \tilde{Y}\tilde{\cap}(H, E)$ for some (H, E) is τ-closed soft set.

Definition 2.26 ([6]). *Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with a fixed set of parameters E, then $\tilde{I}\tilde{\subseteq}SS(X)_E$ is called a soft ideal on X with a fixed set E if*

(1). $(F,E)\in\tilde{I}$ and $(G,E)\in\tilde{I}\Rightarrow(F,E)\tilde{\cup}(G,E)\in\tilde{I}$,

(2). $(F,E)\in\tilde{I}$ and $(G,E)\tilde{\subseteq}(F,E)\Rightarrow(G,E)\in\tilde{I}$

Example 2.27 ([6]). *Let X be a universe set. Then each of the following families is a soft ideal over X with the same set of parameters E,*

(1). $\tilde{I}=\{\phi\}$,

(2). $\tilde{I}=SS(X)_E=\{(F,E):(F,E)$; is a soft set over X with the fixed set of parameters $E\}$,

(3). $\tilde{I}_f=\{(F,E)\in SS(X)_E:(F,E)$ is finite$\}$, called soft ideal of finite soft sets,

(4). $\tilde{I}_c=\{(F,E)\in SS(X)_E:(F,E)$ is countable$\}$, called soft ideal of countable soft sets,

(5). $\tilde{I}_{(F,E)}=\{(G,E)\in SS(X)_E:(G,E)\tilde{\subseteq}(F,E)\}$.

(6). $\tilde{I}_n=\{(G,E)\in SS(X)_E:\mathrm{Int}(\mathrm{Cl}(G,E))=\phi\}$, called soft ideal of nowhere dense soft sets in (X,τ,E).

Definition 2.28 ([22]). *A family ψ of soft sets is called a soft cover of a soft set (F,E) if $(F,E)\tilde{\subseteq}\tilde{\cup}\{(F_i,E):(F_i,E)\in\psi,i\in I\}$. It is an open soft cover if each member of ψ is an open soft set. A soft subcover of ψ is a subfamily of ψ which is also a soft cover of (F,E).*

Definition 2.29 ([22]). *A family ψ of soft sets is said to be have the finite intersection property if the soft intersection of the members of each finite subfamily of ψ is not null soft set.*

Definition 2.30 ([22]). *A soft topological space (X,τ,E) is called soft compact space if each open soft cover of $\tilde{X}$ has a finite soft subcover.*

Definition 2.31 ([20]). *A soft topological space (X,τ,E) is called soft lindelöf if each open soft cover of $\tilde{X}$ has a soft countable subcover.*

Definition 2.32 ([8]). *A soft topological space (X,τ,E) is called soft semi compact space if each soft cover by preopen soft sets of $\tilde{X}$ has a finite soft subcover.*

3. Soft pre-$\tilde{I}$-compact Spaces

Definition 3.1. *A soft subset (F,E) of a space $(X,\tau,E,\tilde{I})$ is said to be soft $\tilde{I}$-compact if every soft cover $\{(G_\alpha,E):\alpha\in\Lambda\}$ of (F,E) by open soft sets, there exists a finite subset Λ_0 of Λ such that $(F,E)\backslash\tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha,E)\in\tilde{I}$. The space $(X,\tau,E,\tilde{I})$ is said to be soft $\tilde{I}$-compact if $\tilde{X}$ is soft $\tilde{I}$-compact as a soft subset.*

Proposition 3.2. *(1). Every soft compact topological space (X,τ,E) is soft $\tilde{I}$-compact for any soft ideal $\tilde{I}$ on $\tilde{X}$.*

(2). If $\tilde{I}=\{\tilde{\phi}\}$, then (X,τ,E) is soft compact $\Leftrightarrow$ it is soft $\tilde{I}$-compact.

(3). If the $\star$-soft topology $(X,\tau^\star,E)$ is soft $\tilde{I}$-compact, then the soft topology (X,τ,E) is soft $\tilde{I}$-compact.

Proof. The proof follows from the definition. □

Theorem 3.3. *Let (X, τ, E) be a soft topological space, $\tilde{I}$ be a soft ideal on X with the same set of parameters E and $(X, \tau^\star, E)$ be its $\star$-soft topological space. Then $(X, \tau^\star, E)$ is soft $\tilde{I}$-compact if and only if (X, τ, E) is soft $\tilde{I}$-compact.*

Proof. Let (X, τ, E) be a soft $\tilde{I}$-compact and $\{(G_\alpha, E) : \alpha \in \Lambda\}$ be $\tau^\star$-open soft cover of $\tilde{X}$. Then $(G_\alpha, E) = (V_\alpha, E) - (I_\alpha, E) \forall \alpha \in \Lambda$, where $(V_\alpha, E) \in \tau$ and $(I_\alpha, E) \in \tilde{I}$. It follows that $\{(V_\alpha, E) : \alpha \in \Lambda\}$ is a τ-open soft cover of $\tilde{X}$. Thus by soft $\tilde{I}$-compactness of (X, τ, E), $\exists \Lambda_o \subseteq \Lambda$ finite such that $\tilde{X} - \tilde{\cup}_{\alpha\in\Lambda_0}(V_\alpha, E) \in \tilde{I}$. So $\tilde{X} - \tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha, E) = \tilde{X} - \tilde{\cup}_{\alpha\in\Lambda_0} [(V_\alpha, E) - (I_\alpha, E)] \tilde{\subseteq} [\tilde{X} - \tilde{\cup}_{\alpha\in\Lambda_0}(V_\alpha, E)] \tilde{\cup} [\tilde{\cup}_{\alpha\in\Lambda_0}(I_\alpha, E)] \in \tilde{I}$, where $(I_\alpha, E) \in \tilde{I} \forall \alpha \in \Lambda$. Hence $(X, \tau^\star, E)$ is soft $\tilde{I}$-compact. The necessary of the theorem follows from Proposition 3.2 (3). □

Theorem 3.4. *Let (X, τ, E) be a soft topological space, $\tilde{I}$ be a soft ideal on X with the same set of parameters E and $(X, \tau^\star, E)$ be its $\star$-soft topological space. Then we have the following:*

(1). If $(X, \tau^\star, E)$ is soft compact, then (X, τ, E) is soft compact.

(2). If $(X, \tau^\star, E)$ is soft compact, then (X, τ, E) is soft $\tilde{I}$-compact.

(3). If $(X, \tau^\star, E)$ is soft $\tilde{I}$-compact, then (X, τ, E) is soft $\tilde{I}$-compact.

Proof. The proof follows from Proposition 3.2 and Theorem 3.3. □

Definition 3.5. *A family ψ of preopen soft sets is called a preopen soft cover of a soft set (F, E) if $(F, E) \tilde{\subseteq} \tilde{\cup}\{(F_i, E) : (F_i, E) \in \psi, i \in I\}$. A preopen soft subcover of ψ is a subfamily of ψ which is also a preopen soft cover of (F, E).*

Definition 3.6. *A soft subset (F, E) of the space $(X, \tau, E, \tilde{I})$ is said to be soft pre-$\tilde{I}$-compact if for every preopen cover $\{(G_\alpha, E) : \alpha \in \Lambda\}$ of (F, E) there exists a finite subset Λ_o of Λ such that $(F, E) - \tilde{\cup}_{\alpha\in\Lambda_o}(G_\alpha, E) \in \tilde{I}$. The space $(X, \tau, E, \tilde{I})$ is said to be soft pre-$\tilde{I}$-compact if $\tilde{X}$ is soft pre-$\tilde{I}$-compact as a soft subset.*

Proposition 3.7. *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal with the same set of parameters E and $(X, \tau^\star, E)$ its $\star$-soft topology. Then*

(1). If $(X, \tau, E, \tilde{I})$ is a soft pre-$\tilde{I}$-compact, then it is soft $\tilde{I}$-compact.

(2). If $(X, \tau^\star, E)$ is a soft pre-$\tilde{I}$-compact, then (X, τ, E) is also soft precompact.

Proof. The proof follows from the respective definitions. □

Theorem 3.8. *A soft topological space (X, τ, E) is soft precompact if, and only if $(X, \tau, E, \tilde{I}_f)$ is soft pre-$\tilde{I}_f$-compact, where $\tilde{I}_f$ is the soft ideal of finite soft subsets of $\tilde{X}$.*

Proof. Let (X, τ, E) be a soft precompact topological space and let $\{(G_\alpha, E) : \alpha \in \Lambda\}$ be a preopen soft cover of $\tilde{X}$. Then there exists a finite subset Λ_o of Λ such that $\tilde{X} = \tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha, E)$. It follows that $X - \{(G_\alpha, E) : \alpha \in \Lambda\} = \phi \in \tilde{I}_f$. Hence $(X, \tau, E, \tilde{I}_f)$ is soft pre-$\tilde{I}_f$-compact. Conversely, let $(X, \tau, E, \tilde{I}_f)$ be a soft pre-$\tilde{I}_f$-compact and let $\{(G_\alpha, E) : \alpha \in \Lambda\}$ be a preopen soft cover of $\tilde{X}$. Then there exists a finite subset Λ_o of Λ such that $\tilde{X} - \tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha, E) \in \tilde{I}_f$. Thus $\tilde{X} = \tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha, E)$. Hence (X, τ, E) is soft precompact. □

Theorem 3.9. *Let $(X, \tau, E, \tilde{I})$ be a soft pre-$\tilde{I}$-compact space and $\tilde{J}$ be a soft ideal on X with the same set of parameters E such that $\tilde{I} \subseteq \tilde{J}$. Then $(X, \tau, E, \tilde{J})$ is a soft pre-$\tilde{J}$-compact.*

Proof. The proof follows from the respective definitions. □

Definition 3.10. *A soft subset (F,E) of the soft topological space (X,τ,E) is said to be soft p-closed if each preopen soft cover of (F,E) has a finite soft subcover whose soft closure covers (F,E). The space (X,τ,E) is said to be soft p-closed if $\tilde{X}$ is soft p-closed as a soft subset.*

Definition 3.11. *A soft subset (F,E) of the soft topological space (X,τ,E) is said to be soft preclosed if each preopen soft cover of (F,E) has a finite preopen soft subcover whose soft preclosure covers (F,E). The space (X,τ,E) is said to be preclosed soft if $\tilde{X}$ is a preclosed soft as a soft subset.*

Theorem 3.12. *If the space $(X,\tau,E,\tilde{I}_f)$ is soft pre-$\tilde{I}_f$-compact, then (X,τ,E) is soft p-closed (resp. soft preclosed).*

Proof. We give the proof for the case of soft p-closed. Let $\{(G_\alpha,E):\alpha\in\Lambda\}$ be a preopen soft cover of $\tilde{X}$. Then there exists a finite subset Λ_o of Λ such that $\tilde{X}-\tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha,E)\in\tilde{I}_f$. This means that $\tilde{X}-\tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha,E)\subseteq\tilde{\cup}_{\alpha\in\Lambda_0}cl(G_\alpha,E)$. Hence $\tilde{X}=\tilde{\cup}_{\alpha\in\Lambda_0}\operatorname{Cl}(G_\alpha,E)$. Therefore (X,τ,E) is soft p-closed soft. The rest of the proof is similar. □

Theorem 3.13. *Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal. If $\tilde{I}_n\tilde{\subseteq}\tilde{I}$ and (X,τ,E) is soft p-closed, then (X,τ,E) is soft pre-$\tilde{I}$-compact.*

Proof. Let $\{(G_\alpha,E):\alpha\in\Lambda\}$ be a preopen soft cover of $\tilde{X}$. Then there exists a finite subset Λ_o of Λ such that $\tilde{X}=\tilde{\cup}_{\alpha\in\Lambda_0}\operatorname{Cl}(G_\alpha,E)\,\tilde{\subseteq}\tilde{\cup}_{\alpha\in\Lambda_0}\operatorname{Cl}(\operatorname{Int}(G_\alpha,E))$, where $(G_\alpha,E)\in SPO(X)$. It follows that $\tilde{X}=\tilde{\cup}_{\alpha\in\Lambda_0}\operatorname{Cl}(\operatorname{Int}(G_\alpha,E))$. Hence $\tilde{X}-\tilde{\cup}_{\alpha\in\Lambda_0}\operatorname{Cl}(\operatorname{Int}(G_\alpha,E))\,\tilde{\subseteq}\,\tilde{X}\,\tilde{\cap}\,\operatorname{Int}(\operatorname{Cl}(\tilde{\cup}_{\alpha\in\Lambda_o}(G_\alpha,E))')\,\tilde{\subseteq}\,\operatorname{Int}(\operatorname{Cl}(\tilde{X}-\tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha,E)))=\tilde{\phi}$. Thus $\tilde{X}-\tilde{\cup}_{\alpha\in\Lambda_0}\operatorname{Cl}(G_\alpha,E)\in\tilde{I}_n\subseteq\tilde{I}$. Therefore, (X,τ,E) is soft pre-$\tilde{I}$-compact. □

Theorem 3.14. *Let (X,τ,E) be a soft topological space. Then (X,τ,E) is soft pre-$\tilde{I}_n$-compact if, and only if (X,τ,E) is soft p-closed.*

Proof. Let $\{(G_\alpha,E):\alpha\in\Lambda\}$ be a preopen soft cover of $\tilde{X}$. Then there exists a finite subset Λ_o of Λ such that $\tilde{X}-\tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha,E)\in\tilde{I}_n$. Then $\tilde{\phi}=\operatorname{Int}(\operatorname{Cl}(\tilde{X}-\tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha,E)))=\operatorname{Int}(\operatorname{Cl}(\tilde{X}\tilde{\cap}[\tilde{\cap}_{\alpha\in\Lambda_0}(G_\alpha,E)')])\tilde{\supseteq}\operatorname{Int}(\tilde{X}\cap\tilde{[}\tilde{\cap}_{\alpha\in\Lambda_0}(G_\alpha,E)')])=\tilde{X}\tilde{\cap}[\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Int}(\operatorname{Cl}(G_\alpha,E)')]=\tilde{X}\tilde{\cap}[\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Int}(\operatorname{Cl}(G_\alpha,E)')]=\tilde{X}\tilde{\cap}(\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Cl}(\operatorname{Int}(G_\alpha,E)))$, where $(G,E)'\in SPC(X)$. Hence $\tilde{X}=\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Cl}(\operatorname{Int}((G_\alpha,E)))\tilde{\subseteq}\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Cl}((G_\alpha,E))$. Then $\tilde{X}=\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Cl}((G_\alpha,E))$. Thus (X,τ,E) is soft p-closed. Conversely, Let $\{(G_\alpha,E):\alpha\in\Lambda\}$ be a preopen soft cover of $\tilde{X}$. Then there exists a finite subset Λ_o of Λ such that $\tilde{X}=\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Cl}((G_\alpha,E))$. So $\tilde{X}-\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Cl}((G_\alpha,E))=\tilde{\phi}$. Then $\tilde{X}-\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Cl}(\operatorname{Int}(G_\alpha,E))=\tilde{X}\tilde{\cap}[\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Int}(\operatorname{Cl}(G_\alpha,E)'))]\tilde{\subseteq}\tilde{X}\tilde{\cap}[\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Int}(G_\alpha,E)')=\tilde{\phi}$. Then $\tilde{\phi}=\operatorname{Int}(\operatorname{Cl}(\tilde{X}-\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Cl}(\operatorname{Int}(G_\alpha,E))))=\operatorname{Int}(\operatorname{Cl}(\tilde{X}\tilde{\cap}[\tilde{\cap}_{\alpha\in\Lambda_0}\operatorname{Int}(\operatorname{Cl}(G_\alpha,E)')]))\tilde{\supseteq}\operatorname{Int}(\operatorname{Cl}(\tilde{X}-\tilde{\cap}_{\alpha\in\Lambda_0}(G_\alpha,E)))$. It follows that $\tilde{X}-\tilde{\cap}_{\alpha\in\Lambda_0}(G_\alpha,E)\in\tilde{I}_n$. Thus (X,τ,E) is soft pre-$\tilde{I}_n$-compact, which completes the proof. □

Definition 3.15. *A soft subset (F,E) of the topological space (X,τ,E) is soft pre-Lindelöf if each preopen soft cover of $\tilde{X}$ has a soft countable subcover.*

Theorem 3.16. *If the space $(X,\tau,E,\tilde{I}_c)$ is soft pre$\tilde{I}_c$-compact, then the space (X,τ,E) is soft prelindelöf, where $\tilde{I}_c$ is the soft ideal of countable soft subsets of $\tilde{X}$.*

Proof. Let $\{(G_\alpha,E):\alpha\in\Lambda\}$ be a preopen soft cover of $\tilde{X}$. Then there exists a finite subset Λ_o of Λ such that $\tilde{X}-\tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha,E)\in\tilde{I}_c$. Since $\tilde{X}=\tilde{\cup}[\tilde{X}-\tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha,E)]=\tilde{\cap}[\tilde{X}\tilde{\cup}(\tilde{\cap}_{\alpha\in\Lambda_0}(G_\alpha,E))']=\tilde{X}$. This means that $\tilde{X}-\tilde{\cup}_{\alpha\in\Lambda_0}(G_\alpha,E)$ is a soft countable subcover of $\tilde{X}$ Therefore (X,τ,E) is soft pre-lindelöf. □

Theorem 3.17. *Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal. Then the following are equivalent:*

(1). (X,τ,E) is soft pre-$\tilde{I}$-compact.

(2). For every family $\{(F_j, E) : j \in J\}$ of preclosed soft subsets of $\tilde{X}$ for which $\tilde{\cap}\{(F_j, E) : j \in J\} = \tilde{\phi}$, there exists finite $J_o \tilde{\subset} J$ such that $\tilde{\cap}\{(F_j, E) : j \in J_o\} \in \tilde{I}$.

Proof. (1) $\Rightarrow$ (2) : Let $\{(F_j, E) : j \in J\}$ be a family of preclosed soft sets of $\tilde{X}$ such that $\tilde{\cap}\{(F_j, E) : j \in J_o\} = \tilde{\phi}$. Then $\{\tilde{X} - (F_j, E) : j \in J\}$ is a family of preopen soft sets of $\tilde{X}$ such that $\tilde{X} = \tilde{\cup}\{\tilde{X} - (F_j, E) : j \in J\}$. By (1),there exists finite $J_o \in J$ such that $\tilde{X} - (\tilde{\cup}\{\tilde{X} - (F_j, E) : j \in J_o\}) \in \tilde{I}$, that is, $\tilde{X} - (\tilde{X}\tilde{\cup}(\tilde{\cap}_{j\in J_o}(F_j, E)')) = \tilde{X}\tilde{\cap}(\tilde{\cap}_{j\in J_o}(F_j, E))$ $= \tilde{X} - (\tilde{X} - \cap_{j\in J_o}(F_j, E)) = \tilde{\cap}_{j\in J_o}(F_j, E) \in \tilde{I}$.

(2) $\Rightarrow$ (1): Suppose that $\{(G_j, E) : j \in J\}$ be a family of preopen soft cover of $\tilde{X}$. Then $\{\tilde{X} - (G_j, E) : j \in J\}$ is a family of preclosed soft sets of $\tilde{X}$ with $\tilde{\cap}_{j\in J}(\tilde{X} - (G_j, E)) = \tilde{\phi}$ By (2), there exists finite $J_o \tilde{\subset} J$ such that $\tilde{\cap}_{j\in J}(\tilde{X} - (G_j, E)) \in \tilde{I}$. Thus $\tilde{\cap}_{j\in J_o}(\tilde{X} - (G_j, E)) = \tilde{X} - \tilde{\cup}_{j\in J_o}(G_j, E) \in \tilde{I}$. Therefore, (X, τ, E) is soft pre-$\tilde{I}$-compact. □

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