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Mathematical Approach for Twenty-Two Microtones: Frequency Ratios in Hindustani Classical Music & their Implementation in 22 Shruti Harmonium

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Abstract: This article shows the calculations of ratio's of 22 shrutis of Indian classical music relation between the Shrutis and its implementation in 22 shruti harmonium.

Keywords: Twenty-Two Microtones, Hindustani Classical Music, Shruti Harmonium.

1. Introduction

The Sanskrit word, shruti, has been used in various ways, even in the context of North Indian classical music, also described as Raaga Sangeet. For our purposes, shrutis are musical notes used in Indian raagas. (A raaga is a melodic mode or frame. Its description is much more than just a scale, and includes not only allowed lines of ascent, descent, signature phrases, but also much more refined information on how the notes are approached, their relative emphasis, etc. Musicians are supposed to develop and create music, by exploring moods and emotions within the language/grammar of a particular raaga.) Long-forgotten understanding and awareness of this shruti system is evident from ancient texts such as the aforementioned Natya Shastra, which mentions that there are 22 shrutis is based on calculation of ratios for the shruties based on relationship with 7 natural shuddha swara's sa,Re,Ga,Ma,Pa,Dha,Ni whise is gift of god through the nature d22 Shrutis, frequency ratios,

2. State in Art

A lot of people have tried to solve this problem. One scholar from Dombivili named Ranade has written a book "Shruti-Rahasya' and given the Shruti-values as obtined by his research and he has also quoted the work of other prominent people including Pundits Achrekar, Mulay, Clemants, Onkarnath, Brihaspati and of course, Bharatmuni (Table 1).

> "Indian classical music and most other music, especially the concept of ragas and taals rhythmic modes or metres, is based on the concept of samvaad, meaning dialog, accord or intercommunication between the parts of the whole."

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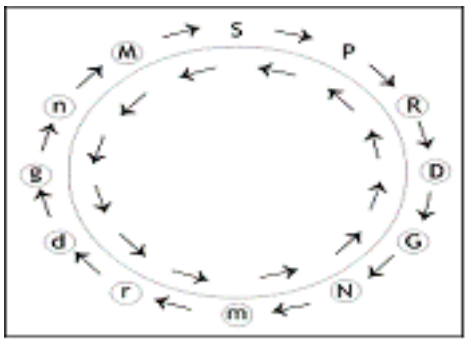
Shruti	Achrekar	Maloy	Clements	Gokarnath	Srivastava	Ranade
1	0	0	0	0	0	0
2	5.46875	4.4701	5	5.3497	4.1668	5.3453
3	6.66	6.66	6.66	6.66	9.6666	6.7864
4	11.11	11.11	11.11	11.11	11.11	10.9816
5	12.5	17	12.5	16.3408	17.6166	12.5
6	18.51851	18.51851	18.51851	18.51851	18.51851	18.5166
7	20	20	20	20	20.1166	20.1333
8	25	25	25	25	25	24.8666
9	28.4137	31.6872	26.5625	31.6872	31.675	28.3825
10	33.33	33.33	33.33	33.33	33.33	33.33
11	40.625	35	35	35	35	40.4658
12	42.22	40.625	40.625	40.625	40.625	42.3825
13	45.145	45.145	42.22	45.145	45.145	47.9794
14	50	50	50	50	50	50
15	56.2031	56.25	57.5	56.9102	56.25	58.0361
16	60	60	60	60	64.5833	60.1816
17	66.66	66.66	66.66	66.66	66.66	66.4779
18	68.75	75.5625	68.75	75.5625	75.5333	68.75
19	77.77	77.77	77.5	77.77	77.77	77.77
20	80	80	80	80	80	80.2008
21	87.5	87.5	87.5	87.5	87.5	87.2875
22	89.6296	97.7777	89.84375	97.5308	97	89.64375

Table 1.

There are also other values including some on the internet, regarding the frequencies of 22 Shrutis. Unfortunately, all these researchers in music have given different values of all the Shrutis and therefore in the end, the reader is left in considerable confusion. Plus, in the absence of the underlying mathematical principle, it was not surprising that the end points were different. Therefore it was decided to explore the logic and the mathematics behind the construction of 22 Shrutis. The indian stalwarts and other performers were singing or playing these 22 Shrutis, many of them, very well. There was however, no method of measuring the positions of the frquencies of the Shrutis they played of sang. The 20 Shrutis of "Chala Swaras and 2 "Achala" swaras were thus recovered beyond doubt.

3. Shruti-Nirman Chakra

The clockwise cycle is shadja:Pancham (S:P) and the anticlockwise cycle is Shadja:Madhyam (S:M).



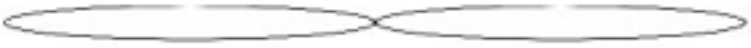
Thus, we get 24 naturally arising notes or Shrutis at the natural ratios of S:P and S:M. Out of these, S and P have been considered "Achala" or immovable or fixed by our ancestors. Subtracting these 2 from 24, we get 22 Shrutis. The exact frequency positions of the 22 Shrutis were however obscure for all these years.

4. Harmonics in Indian Classical Music

The most natural example of sound production in the nature is of a vibrating single string. We all know that a string tightly stretched across 2 ends produces a sound on plucking.

(1). "1st Harmonic" or "Shadja" or "Fundamental Tone" : When a string is made to vibrate by plucking, to start with it vibrates in it's full length; and the makes a certain sound. This is called as "Shadja" or the "Fundamental Tone", or "1st Harmonic". For the purpose of understanding, we shall take it's frequency as 100 hz. Ratio for Shadja 1/1

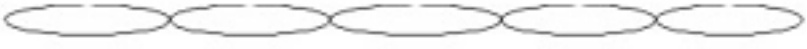
(2). "2nd Harmonic" or "Tara Shadja" : Immediately thereafter, as the energy put in the string for plucking reduces, the string starts vibrating in 2 parts. This produces a sound of 200 hz called as the 2nd Harmonic. This is "Tara Shadja" as we know. Ratio will be 2/1



(3). "3rd Harmonic" or "Pancham" : Immediately thereafter, as the energy put in the string for plucking reduces further, the string starts vibrating in 3 parts. This produces a sound of 300 hz called as 3rd Harmonic. This is however perceived by our brain as of 150 hz (our brain has the spectrum of perception of 100 hz to 200 hz) or "Pancham" (5th) as we know. Ratio will be 3/2

(4). "4th Harmonic" or "Ati-Tara Shadja" : Immediately thereafter, as the energy put in the string for plucking reduces further, the string starts vibrating in 4 parts. This produces a sound of 400 hz called as 4th Harmonic. This is perceived by our brain as of 200 hz (our brain has the spectrum of perception of 100 hz to 200 hz) or Tara Shadja again. Ratio again 2/1

(5). "5th Harmonic" or "Gandhar" : Immediately thereafter, as the energy put in the string for plucking reduces further, the string starts vibrating in 5 parts. This produces a sound of 500 hz called as 5th Harmonic. This is perceived by our brain as equivalent of 250 hz or 125 hz or Gandhar. Ratio will be 5/4



After this stage, the energy put in the string for plucking reduces so much that further harmonics (6th, 7th, 8th and so on) are barely heard.

***Harmonics :** The Sounds created which are in multiple proportions to the frequency of the Fundamental or Shadja (100,200,300,400,500 as above), are called as "Harmonics", because they are in "harmony" with the Shadja. They sound very pleasant to the human ear. We all have experienced that Pancham (3rd harmonic) and Gandhar (5th harmonic) are most pleasant. Tanpura creates an atmosphere of dominant Shadja-Gandha-Pancham.

5. Ratio Calculations from S:G:P

(1). Frequencies S:G:P natural shuddha swara is 100:125:150 or 4:5:6 and ratio will be 1/1, 5/4 and 3/2.

(2). Treating upper Sa (S') as pancham, shadja will be at M (133*33) and gandhar will be at Dha and therefore M:D:S' will be 133*33 : 166:66 : 200 or 4:5:6.

(3). Take P(150) as shadja Gandhar will be at (187.5) and pancham will be R' (225) or 4:5:6 when R' 225 R will be 112.50

(4). From above the Natural shuddha saptak will be

	S	R	G	M	P	D	N	S'
	100	112.50	125	133.33	150	166.66	187.50	200
Ratio	1/1	9/8	5/4	4/3	3/2	5/3	15/8	2/1

In terms of Shrutis

	S	R_1	G_1	M_1	P	D_1	N_1	S'
Ratio	1/1	9/8	5/4	4/3	3/2	5/3	15/8	2/1

Calculations for remaining shrutis:

- m_1 (Ma Teevra): ma teevara will be natural Gandhar of R_1, therefore

$$m_1 = R_1 \times \frac{5}{4} = \frac{9}{8} \times \frac{5}{4} = \frac{45}{32}$$

- n_1 (Ati Komal Nishad): n_1 will be madham of madhyam, therefore

$$n_1 = M_1 \times \frac{4}{3} = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$$

- r_2 (Komal Re): Natural Gandhar of Komal Re will be madhyam (M_1), therefore

$$r_2 \times \frac{5}{4} = \frac{4}{3} \quad or \quad r_2 = \frac{16}{15}$$

- g_2 (Komal Gandhar): natural gandhar of a komal gandhar will be pacham (P) therefore

$$g_2 \times \frac{5}{4} = \frac{3}{2} \quad or \quad g_2 = \frac{12}{10} = \frac{6}{5}$$

- Dhaiwat (d_2): komal dhaiwat has its natural gandhar at S' (upper sa), therefore

$$d_2 \times \frac{5}{4} = \frac{2}{1} \quad Or \quad d_2 = \frac{8}{5}$$

- Teevra Dhaiwat (D_2): Acording to shruti chakra of pancham, D_2 will be pancham of R_2, therefore

$$D_2 = R_2 \times \frac{3}{2} = \frac{9}{8} \times \frac{3}{2} = \frac{27}{16}$$

- Komal Dhaivat (d_2): According to shruti chakra or pancham d_2 will be pancham of r_2, therefore

$$\begin{aligned} d_2 &= r_2 \times \frac{3}{2} \\ &= \frac{16}{15} \times \frac{3}{2} = \frac{8}{5} \end{aligned}$$

- Komal Nishad (n_2): According to shruti chakra of pancham n_2 will be pancham of g_2, therefore

$$n_2 = g_2 \times \frac{3}{2} = \frac{6}{5} \times \frac{3}{2} = \frac{9}{5}$$

- Shuddha Rishabh (R_1): Acording to shruti Chakra of pancham D_1 will be pancham of R_1, therefore

$$R_1 \times \frac{3}{2} = D_1 = \frac{5}{3} \Rightarrow R_1 = \frac{5}{3} \times \frac{2}{3} = \frac{10}{9}$$

- Ati Komal Gandhar (g_1): Ati Komal Gandhar has its pancham at n_1 (Ati Komal Nishad), therefore

$$g_1 \times \frac{3}{2} = n_1 = \frac{16}{9} \Rightarrow g_1 = \frac{16}{9} \times \frac{2}{3} = \frac{32}{27}$$

- Ati Komal dhaiwat (d_1) : Ati Komal dhaiwat will become madhyam of Ati Komal Gandhar, therefore

$$g_1 \times \frac{4}{3} = d_1 \Rightarrow \frac{32}{27} \times \frac{4}{3} = d_1 = \frac{128}{81}$$

- Ati Komal Rishabh (r_1): Pancham of Ati Komal Rishabh will be at d_1 (Ati Komal Dhaiwat), therefore

$$r_1 \times \frac{3}{2} = d_1 = \frac{128}{81} \quad Or \quad r_1 = \frac{128}{81} \times \frac{2}{3} = \frac{256}{243}$$

- G_2 (Teevra Gandhar): G_2 has its madhyam at D_2, therefore

$$G_2 \times \frac{4}{3} = \frac{27}{16} \Rightarrow G_2 = \frac{27}{4} \times \frac{3}{16} = \frac{81}{64}$$

- N_2 (Teevra Nishad): N_2 (Tivra Nishad) will be the puncham of G_2, therefore

$$G_2 \times \frac{3}{2} = n_2 \quad Or \quad \frac{81}{64} \times \frac{3}{2} = N_2 = \frac{243}{128}$$

- m_2 (Tivratam Madhyam) has its madhyam at N_2, therefore

$$m_2 \times \frac{4}{3} = \frac{243}{128} \quad Or \quad m_2 = \frac{729}{512}$$

Thus we have calculated all ratios of 22 shruties tabulating them we get as following assuring S=100

S	r_1	r_2	R_1	R_2	g_1	g_2	G_1	G_2
1/1	256/243	16/15	10/9	9/8	32/27	6/5	5/4	81/64
100	105.35	106.66	111.11	112.50	118.51	120	125	126.56

M_1	M_2	m_1	m_2	P	d_1	d_2	D_1	D_2
4/3	27/20	45/32	729/512	3/2	128/81	8/5	5/3	27/16
133.33	135	140.68	142.32	150	158.02	160	166.66	168.75

n_1	n_2	N_1	N_2	S'
16/9	9/5	15/8	243/128	2/1
177.77	180	187.50	189.84	200

6. Calculations of Prarnan Shruti, Neun Shruti and Pooma Shruti Differences Between the Shrutis

From above calculations we get,

(1). $r_{1/S} = g_{1/K2} = M_{1/G_1} = P_{/m_2} = d_{1/P} = M_{1/M_2} = n_{1/P2} = S'_{/N_2} = 256/243 = 1.0534.$

This ratio is called praman shruti difference

(2). $R_{1/r_2} = G_{1/g_2} = M_{1/M_2} = D_{1/d_2} = N_{1/n_2} = 25/24 = 1.041666.$

This ratio is neun shruti difference

(3). $r_{2/r_1} = R_{2/R_1} = g_{2/g_1} = G_{2/G_1} = M_{1/M_2} = m_{1/m_2} = d_{2/d_1} = D_{2/D_1} = 91/80 = 1.0125.$

This ratio is praman shruti difference. In an octave from above shruti differences. The 22 shruti differences can be give as 1 3 2 3 1 3 2 3 1 3 2 3 1 1 3 2 3 1 3 2 3 1.

No.	Shruti	Description	Frequency	=	Ratio
1	S	Shadja	100	=	1/1
2	r1	Atikomal Rishabh (Lower)	105.3497942	=	256/243
3	r2	Komal Rishabh(Higher)	106.666666	=	16/15
4	R1	Shuddha Rishabh (Lower)	111.111111	=	10/9
5	R2	Teevra Rishabh (Higher)	112.5	=	9/8
6	g1	Atikoml Gandhar (Lower)	118.51851	=	32/27
7	g2	Komal Gandhar (Higher)	120	=	6/5
8	G1	Shuddha Gandhar (Lower)	125	=	5/4
9	G2	Teevra Gandhar (Higher)	126.5625	=	81/64
10	M1	Shuddha Madhyam (Lower)	133.33333	=	4/3
11	M2	Ekashruti Madhyam (Higher)	135	=	27/20
12	m1	Teevra Madhyam (Lower)	140.625	=	45/32
13	m2	Teevratama Madhyam (Higher)	142.3828125	=	729/512
14	P	Pancham	150	=	3/2
15	d1	Atikomal Dhaivat (Lower)	158.0246913	=	128/81
16	d2	Komal Dhaivat (Higher)	160	=	8/5
17	D1	Shuddha Dhaivat (Lower)	166.666666	=	5/3
18	D2	Teevra Dhaivat (Higher)	168.75	=	27/16
19	n1	Atikomal Niashad (Lower)	177.777777	=	16/9
20	n2	Komal Nishad (Higher)	180	=	9/5
21	N1	Shuddha Nishad (Lower)	187.5	=	15/8
22	N2	Teevra Nishad (Higher)	189.84375	=	243/128

7. Implementation of 22 Shruties in 22 Shruti Harmonium

Dr. Vidhyadhar Oke, Researeher and manu facturer of 22 shruti harmoniums has established the ration's for shrutis in 22 shruti harmonium and patented it (India patent No 250197). Hurdle was how to tune 22 reeds within 12 keys in a Saptak. This was made possible by providing 4 reedboards so that 4 reeds came under 1 key. Further, a mechanical modification in the structure of Harmonium, so that a pair of tuned reeds could be played by selection with the help of a knob provided under every key. Figures showing the photograph of Melodium (under Patent) with knobs under every key.

The mechanical modification of knobs under every key allows any one pair of reeds (with a higher of lower pitch) as required in raga to be selectively played. The Melodium player can even do these adjustments while continuing to play the Melodium, although this may not be practically necessary. 2 reedboards are tuned to higher Shrutis and the other 2 reedboards are tuned to lower Shrutis. This new version of Harmonium is now under patent with my name as inventor. Figures showing Melodium (under Patent) while Playing (First, without opening the keys; Second, after keys are opened).

8. Conclusion

Research is a orgoing process this work clarifies many issues and precise numbers regarding 22 shrutis. This research work provides the logic and a firm mathematical principal with a combination of 3 rations 256/243, 81/80,25/24 which sequentially and accurately provides the precise positions of all the Indian 22 shruties. This shrutis come sequnentially giving accurate positions on the way of the principal notes such as gandhar (5/4) madhyam (4/3), pancham (3/2). Latter part of work is application, (ie) establishing these shruties (22) in Harmonium. These is solved by providing additional Readboards and new harmonium modification can play 22 shruties, completely transforming the European tunning and nature of the instrument to the Indian Instrument.

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