International Journal of Mathematics And Its Applications
Vol.1 No.1 (2013), pp.31-37
ISSN: 2347-1557(online)

Stochastic Analysis of Manpower and Business

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Abstract : In this paper we consider a business organization under fluctuating conditions of availability of man power and business with a special emphasis given to a new and prevailing idea of business to go off with the manpower leading to crisis state. The different states have been discussed under the assumption that changes from availability to shortage and shortage to availability occur in random times with general distributions. Time dependent and time independent probabilities of the states of the system are presented. Two models are treated. In model1, manpower and business fluctuate independently and in model 2, present business goes off with manpower. Numerical results are presented.

Keywords : Manpower planning, State probabilities and integral equations.

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AMS Subject Classification: 90B05.

1 Introduction

Nowadays we find that labor has become a buyers market as well as sellers market. Any company which normally runs on commercial basis wishes to keep only the optimum level of any resources needed to meet companys requirement at any time during the course of the business and manpower is not an exception. This is spelt in the sense that a company does not want to keep manpower more than what is required. Hence, retrenchment and recruitment are common and frequent in most of the companies now. Recruitment is done when the business is busy and shed manpower when the business is lean. Equally true with the labor, has the option to switch over to other jobs because of better working condition, better emolument, proximity to their living place or other reasons. Under such situations the company may face crisis because business may be there but manpower may not be available. If skilled laborers and technically qualified persons leave the business the seriousness is worst felt and the company has to hire paying heavy price or pay overtime to employees.

Approach to manpower problems have been dealt in very many different ways as early as 1947 by Vajda [10] and others. Models in manpower planning has been dealt in depth in Barthlomew [1], Grinold & Marshal [3] and Vajda [10] The methods to compute wastages (Resignation, dismissal and death) and promotion intensities which produce the proportions corresponding to some desired planning proposals has been dealt by Lesson [4]. Markov models are designed for wastages and promotion in manpower system by Vassilou [11]. V. Subramaniam [9] in his thesis has made an attempt to provide optimal policy for recruitment training, promotion, and wastages in man power planning models with special provisions such as time bound promotions, cost of training and voluntary retirement scheme. For application of Markov chains in a manpower system with efficiency and seniority and Stochastic structures of graded size in manpower planning systems one may refer to Setlhare [8]. A two unit stand by system has been investigated by Chandrasekar and Natrajan [2] with confidence limits under steady state. For n unit standby system one may refer to Ramanarayanan and Usha [7]. Yadhavalli and Botha [12] have examined the same for two unit system with introduction of preparation time for the service facility and the confidence limits for stationary rate of disappointment of an intermittently used system. For three characteristics system involving manpower, money and machine one may refer to C. Mohan and R. Ramanarayanan [6] For the study of Semi Markov Models for Manpower planning one may refer to the paper by Sally Meclean [5].

In this paper we consider two characteristics namely manpower and business. We derive formulas for the time dependent and time independent state probabilities. The situations may be that the manpower be fully available or hardly available and business may fluctuate between full availability to nil availability. In model 1, business and manpower fluctuate independently. In model 2, it goes off when the manpower becomes nil. The employees may take the business along with them or those who have brought good will to the concern may carry the clients off the concern. The business depends fully on the availability of manpower. The probabilities describing the transitions in various states are derived for general distributions using integral equations applying renewal theoretic arguments. Numerical illustrations are provided.

2 Model 1: Business and Manpower with Independent Fluctuation

Assumptions

(1). Business [Manpower] fluctuates between two states 0 and 1 where 0 indicates the business [manpower] is not available and 1 indicates it is fully available.

(2). The transition time of business, [manpower]) from entry into state 0 to entry into state 1has Cdf. $F_0(x)$, $[G_0(x)]$ with pdf $f_0(x)$, $[g_0(x)]$ and mean b_0, $[m_0]$.

(3). The transition time of business, [manpower]) from entry into state 1 to entry into state 0 has Cdf. $F_1(x)$, $[G_1(x)]$ with pdf $f_1(x)$, $[g_1(x)]$ and mean b_1, $[m_1]$.

(4). Manpower and business fluctuations are independent of each other. At time 0 both manpower and business enter state 0 and the system starts.

Analysis

We define

$$\begin{aligned}
B_0(x) &= P(\text{Business is at state 0 at time } x| \text{ It enters state 0 at time 0})\\
B_1(x) &= P(\text{Business is at state 1 at time } x| \text{ It enters state 0 at time 0})\\
M_0(x) &= P(\text{Manpower is at state 0 at time } x| \text{ It enters state 0 at time 0})\\
M_1(x) &= P(\text{Manpower is at state 1 at time } x| \text{ It enters state 0 at time 0})
\end{aligned} \tag{2.1}$$

We note here

$$B_0(x) = \bar{F}_0(x) + \int_0^x [\int_0^v f_0(u) f_1(v-u)du] B_0(x-v)dv \tag{2.2}$$

The first term of the right side is the probability that the business remains at state 0 during (0, x) and the second term is the probability that the business from state 0 at time 0 moves to state 1, enters state 0 at time v and is at state o at time x. Taking Laplace transform of (2), we get

$$B_0^*(s) = \bar{F}_0^*(s) + f_0^*(s) f_1^*(s) B_0^*(s) \tag{2.3}$$

Simplifying (2.3), we get

$$B_0^*(s) = \frac{\bar{F}_0^*(s)}{1 - f_0^*(s) f_1^*(s)} \tag{2.4}$$

Taking inversion of (2.4), we get

$$B_0(t) = \bar{F}_0(t) \copyright \sum_0^{\infty} [f_0(t) \copyright f_1(t)]n \tag{2.5}$$

where © refers to convolution of functions. Similarly we can get,

$$M_0^*(s) = \frac{\bar{G}_0^*(s)}{1 - g_0^*(s) g_1^*(s)}$$

and

$$M_0(t) = \bar{G}_0(t)© \sum_0^\infty [g_0(t)©g_1(t)]_n \tag{2.6}$$

By independence we get using equations (2.5) and (2.6) P (Business and Manpower at state (0 0) at time $x|$at time 0 they enter (0 0)) $= B_0(t)$

$$M_0(t) = \{\bar{F}_0(t)© \sum_0^\infty [f_0(t)©f_1(t)]_n\}\{\bar{G}_0(t)© \sum_0^\infty [g_0(t)©g_1(t)]_n\} \tag{2.7}$$

Now

$$B_1(x) = \int_0^x f_0\bar{F}_1(x-u)du + \int_0^x [\int_o^v f_0(u)f_1(v-u)du]B_1(x-u)dv \tag{2.8}$$

Taking Laplace transform of (2.8), we get

$$B_1^*(s) = \frac{f_0^*(s)\bar{F}_1^*(s)}{1-f_0^*(s)f_1^*(s)} \tag{2.9}$$

Taking inversion of (2.9) we get,

$$B_1(t) = f_0(t)© \sum_0^\infty [f_0(t)©f_1(t)]_n©\bar{F}_1(t) \tag{2.10}$$

By similar method we can get $M_1^*(s) = \frac{g_0^*(s)\bar{G}_1^*(s)}{1-g_0^*(s)g_1^*(s)}$

$$M_1(t) = g_0(t)© \sum_0^\infty [g_0(t)©g_1(t)]_n©\bar{G}_1(t) \tag{2.11}$$

Again by independence P (Business and Manpower at state $(1,1)$ at time $x|$ at time 0 they enter (0 0)) = $B_1(t)$

$$M_1(t) = \{\bar{f}_0(t)© \sum_0^\infty [f_0(t)©f_1(t)]_n\}\{\bar{g}_0(t)© \sum_0^\infty [g_0(t)©g_1(t)]_n©\bar{G}_1(t)\} \tag{2.12}$$

Using initial value theorem of Laplace transform and LHospital rule, we get

$$\begin{aligned}
\lim_{t\to\infty} B_0(t) = B_0 = \lim_{s\to 0} sB_0(s) = \frac{b_0}{b_0+b_1}\\
\lim_{t\to\infty} B_1(t) = B_1 = \lim_{s\to 0} sB_1(s) = \frac{b_1}{b_0+b_1}\\
\lim_{t\to\infty} M_0(t) = M_0 = \lim_{s\to 0} sM_0(s) = \frac{m_0}{m_0+m_1}\\
\lim_{t\to\infty} M_1(t) = M_1 = \lim_{s\to 0} sM_1(s) = \frac{m_1}{m_0+m_1}
\end{aligned} \tag{2.13}$$

3 Model II: Business to go off with Manpower

Assumptions

(1). Business [Manpower] fluctuates between two states 0 and 1 where 0 indicates the business [manpower] is not available and 1 indicates it is fully available. Business fluctuations occur from level 0 to level full and from level full to level 0 only when manpower is available. When manpower becomes 0 the business operation goes off. When manpower becomes available (enters state 1) the business starts afresh from level 0 irrespective of whether the business was in level 0 or level full when it went off.

(2). The transition time of manpower, [business during the continuous availability of manpower] from entry into state 0 to entry into state 1 has Cdf.$G_0(x)$, $[F_0(x)]$ with pdf $g_0(x)$, $[f_0(x)]$ and mean m_0, $[b_0]$.

(3). The transition time of manpower, [business during the continuous availability of manpower] from entry into state 1 to entry into state 0 has Cdf.$G_1(x)$, $[F_1(x)]$ with pdf $g_1(x)$, $[f_1(x)]$ and mean m_1, $[b_1]$.

Analysis

We define

$M_0(x) =$ P(Business is at state 0 at time $x|$ At time 0 manpower enters state 0)

$M_1(x) =$ P(Business is at state 1 at time $x|$ At time 0 manpower enters state 0)

Let (i, j) indicate that manpower is at state i and business is at state j for $i, j = 0, 1$.

Now $M_{i,j}(x) =$ P[Manpower and business at state (i, j) at time x) | Manpower and business enter state (0, 0) at time 0], for $i, j = 0, 1$. We get

$$M_{1,1}(x) = \int_0^x \{g_0(u) © \sum_0^\infty [g_1(u) © g_0(u)]_n\}\{\bar{G}_1(x-u)B_1(x-u)\}du. \tag{3.1}$$

To write the above probability we consider the case that the manpower moves to state 1, it moves from state 1 to state 0 and from state 0 to state 1, n times after which it stays there, the business enters state 0 at time u when manpower enters state 1last to stay put there and the business is in state 1 at time x. Since when manpower is continuously available during $(0, x-u)$ using the arguments given for model 1, $B_1(t)$ here also may be obtained as given by equation (2.10).

Now taking Laplace transform of equation (3.1)

$$M_{1,1}(s) = \frac{g_0^*(s)}{1 - g_0^*(s)g_1^*(s)} \int_0^\infty e^{-st}\bar{G}_1(t)B_1(t)dt \tag{3.2}$$

Using initial value theorem of Laplace transform, we get

$$\lim_{t\to\infty} M_{1,1}(t) = \lim_{s\to 0} sM_{1,1}(s) = M_{1,1} = \frac{\int_0^\infty \bar{G}_1(t)B_1(t)dt}{m_0 + m_1} \tag{3.3}$$

We note that $M_{1,0}(t) = M_1(t) - M1,1(t)$.

Further we may find that $M_1(t)$ and $M_0(t)$ may be obtained using model 1 arguments as given by equations (2.6) and (2.11). We note that since when manpower is nil, business goes off and $M_{0,1}(t) = 0$. In addition we may note that $M_{0,0}(t) = M_0(t)$. Now

$$M_{1,0} = M_1 - M_{1,1} = \frac{m_1}{m_0 + m_1} - \frac{\int_0^\infty \bar{G}_1(t)B_1(t)dt}{m_0 + m_1} \tag{3.4}$$

Special Case-All Exponential

We now present a model in which all the distributions are exponential as follows. Let

$$G_0(x) = 1 - e^{-\mu x},\ G_1(x) = 1 - e^{-\lambda x},\ F_0(x) = 1 - e^{-\beta x},\ \text{and } F_1(x) = 1 - e^{-\lambda x}. \tag{3.5}$$

For model 1 we get the following results using Laplace inversion.

$$\begin{aligned} B_1 &= \frac{\beta}{\alpha+\beta} - \frac{\beta}{\alpha+\beta}e^{-(\alpha+\beta)t}, \\ B_0 &= \frac{\alpha}{\alpha+\beta} + \frac{\beta}{\alpha+\beta}e^{-(\alpha+\beta)t}, \\ M_1 &= \frac{\mu}{\lambda+\mu} - \frac{\mu}{\lambda+\mu}e^{-(\lambda+\mu)t} \text{ and} \\ M_0 &= \frac{\lambda}{\lambda+\mu} + \frac{\mu}{\lambda+\mu}e^{-(\lambda+\mu)t} \end{aligned} \tag{3.6}$$

The limits c-an be obtained by putting $t = \infty$. For model 2 we get the following results using Laplace inversion.

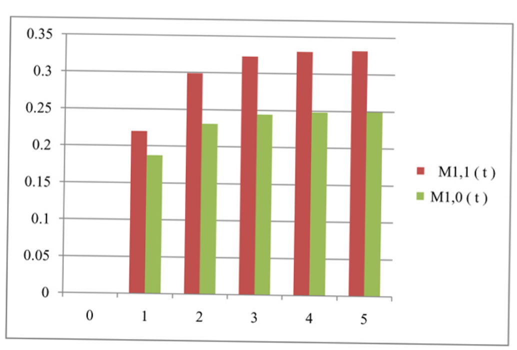
$$\begin{aligned} M_{1,1} &= \frac{\beta\mu}{(\lambda+\mu)(\lambda+\beta+\alpha)} - \frac{\beta\mu}{(\lambda+\mu)(\alpha+\beta-\mu)}e^{-(\lambda+\mu)t} + \frac{\beta\mu}{(\lambda+\beta+\alpha)(\alpha+\beta-\mu)}e^{-(\lambda+\beta+\alpha)t} \\ M_{1,0} &= \frac{\mu(\lambda+\alpha)}{(\lambda+\mu)(\lambda+\beta+\alpha)} + \frac{\beta\mu}{(\lambda+\mu)(\alpha+\beta-\mu)}e^{-(\lambda+\mu)t} - \frac{\beta\mu}{(\lambda+\beta+\alpha)(\alpha+\beta-\mu)}e^{-(\lambda+\beta+\alpha)t} \\ &\quad - \frac{\mu}{\lambda+\mu}e^{-(\lambda+\mu)t}, \end{aligned} \tag{3.7}$$

$M_{0,1}(t) = 0$ for every t and $M_{0,0}(t) = M_0(t)$ which is as given by equation (3.6). The probability values are calculated by assigning values to the parameters and at different timings

(i). $\lambda = 0.50,\ \mu = 0.70,\ \alpha = 4,\ \beta = 6$:

t	(i) $M_{1,1}(t)$	(ii) $M_{1,0}(t)$
0	0	0
1	0.2200	0.1876
2	0.2992	0.2312
3	0.3230	0.2444
4	0.3302	0.2483
5	0.3323	0.2495

The graph of the above is given below:



[t - $M_{1,1}$ (t), $M_{1,0}$ (t)] graph

Observation

It is observed from the above that the probabilities of business and manpower to be full at different timings is more than their corresponding probabilities when manpower is full but no business under certain values of the parameters. The graph makes this fact clear.

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