International Journal of Mathematics And Its Applications

Vol.2 No.3(2014), pp.53-62.

ISSN: 2347-1557(online)

Coupled Coincidence Fixed Point Theorem for Two Pairs in G-Metric Space

Manish Kumar[1]

Department of Mathematics, Government College Nanauta,

Saharanpur (U.P.), India.

manishrajput04@yahoo.co.uk

Abstract : In the present paper, we prove a coupled coincidence fixed point theorem in the setting of two pairs of mappings in G-metric space. The main result is illustrated by an example.

Keywords : coupled coincidence point, common fixed point, G metric space.

1 Introduction

The study of common fixed points of mappings satisfying certain contractive condition has been carried out by many mathematician because of its wide application in mathematics and applied sciences. In this series, coincidence point theory also plays a major role see [1, 2, 3, 4, 6, 7, 8, 10, 15, 16, 21]. In 2003, Mustafa and Sims [12] introduced a new notion of generalized metric space called G metric space. A number of fixed point theorems have been studied on G metric spaces [11, 13, 14, 18, 19, 20]. V. Laxmikantham et al.[5, 9] introduced the concept of coupled coincidence point of mapping F from $X \times X$ into X and g from X into X, and developed fixed point results in partial metric spaces. In [22], W.

[1]Corresponding author E-Mail: manishrajput04@yahoo.co.uk (Manish Kumar)

AMS Subject Classification: 54H25, 47H10, 54E50.

Shantanawi proved a coupled coincidence theorem in G metric space. All coupled coincidence theorems have been established in the setting of pair of maps F, g. The aim of the present paper is to prove a coupled coincidence theorem for two pairs of such mappings $\{F, h\}$ and $\{S, g\}$ in G metric space.

2 Basic Concept

Definition 2.1 ([12]). Let X be a nonempty set and $G : X \times X \times X \to R^+$ a function satisfying the following properties:

(G1) $G(x, y, z) = 0$ if $x = y = z$,

(G2) $0 < G(x, x, y)$, for all $x, y \in X$ with $x \neq y$,

(G3) $G(x, x, y) \leq G(x, y, z)$ for all $x, y, z \in X$ with $z \neq y$,

(G4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = ...$, (symmetry)

(G5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$ for all $x, y, z, a \in X$.

Then the function G is called a generalized metric or more specifically, a G-metric on X , and the pair (X, G) is called a G-metric space.

Definition 2.2 ([12]). Let (X, G) be a G-metric space and (x_n) a sequence of points of X. A point $x \in X$ is said to be the limit of the sequence (x_n), if $\lim_{n,m \to +\infty} G(x, x_n, x_m) = 0$, and we say that the sequence (x_n) is G-convergent to x or that (x_n) G-converges to x.

Thus, $x_n \to x$ in a G-metric space (X, G) if for any $\varepsilon > 0$, there exists $k \in N$ such that $G(x, x_n, x_m) < \varepsilon$ for all $m, n \geq k$.

Proposition 2.3 ([12]). Let (X, G) be a G-metric space. Then the following are equivalent:

(1) (x_n) is G-convergent to x.

(2) $G(x_n, x_n, x) \to 0$ as $n \to +\infty$.

(3) $G(x_n, x, x) \to 0$ as $n \to +\infty$.

(4) $G(x_n, x_m, x) \to 0$ as $n, m \to +\infty$.

Definition 2.4 ([10]). Let (X, G) be a G-metric space. A sequence (x_n) is called G-Cauchy if for every $\varepsilon > 0$, there is $k \in N$ such that $G(x_n, x_m, x_l) < \varepsilon$, for all $n, m, l \geq k$; that is $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to +\infty$.

Proposition 2.5 ([12]). Let (X, G) be a G-metric space. Then the following are equivalent:

(1) The sequence (x_n) is G-Cauchy.

(2) For every $\varepsilon > 0$, there is $k \in N$ such that $G(x_n, x_m, x_m) < \varepsilon$, for all $n, m \geq k$.

Definition 2.6 ([12]). Let (X, G) and (X', G') be G-metric spaces and $f : (X, G) \to (X', G')$ a function. Then f is said to be G-continuous at a point $a \in X$ if and only if for every $\varepsilon > 0$, there is $\delta > 0$ such that $x, y \in X$ and $G(a, x, y) < \delta$ implies $G'(f(a), f(x), f(y)) < \varepsilon$. A function f is G-continuous at X if and only if it is G- continuous at all $a \in X$.

Proposition 2.7 ([12]). Let (X, G) be a G-metric space. Then the function $G(x, y, z)$ is jointly continuous in all three of its variables.

The followings are examples of G-metric spaces.

Example 2.8 ([12]). Let (R, d) be the usual metric space. Define G_s by $G_s(x, y, z) = d(x, y) + d(y, z) + d(x, z)$ for all $x, y, z \in R$. Then it is clear that (R, G_s) is a G-metric space.

Example 2.9 ([12]). Let $X = \{a, b\}$. Define G on $X \times X \times X$ by $G(a, a, a) = G(b, b, b) = 0, G(a, a, b) = 1, G(a, b, b) = 2$ and extend G to $X \times X \times X$ by using the symmetry in the variables. Then it is clear that (X, G) is a G-metric space.

Definition 2.10 ([12]). A G-metric space (X, G) is called G-complete if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).

Definition 2.11 ([5]). An element $(x, y) \in X \times X$ is called a coupled fixed point of a mapping $F : X \times X \to X$ if $F(x, y) = x$ and $F(y, x) = y$.

Definition 2.12 ([9]). An element $(x, y) \in X \times X$ is called a coupled coincidence point of the mappings $F : X \times X \to X$ and $g : X \to X$ if $F(x, y) = gx$ and $F(y, x) = gy$.

Definition 2.13 ([9]). Let X be a nonempty set. Then we say that the mappings $F : X \times X \to X$ and $g : X \to X$ are commutative if $gF(x, y) = F(gx, gy)$.

In [22], W. Shantanawi proved the following theorem
"Let (X, G) be a G- metric space. Let $F : X \times X \to X$ and $g : X \to X$ be mappings such that $G(F(x, y), F(u, v), F(z, w)) \leq k(G(gx, gu, gz) + G(gy, gv, gw))$, for all $x, y, z, u, v, w \in X$. Assume that F and g satisfy the following conditions

(1) $F(X \times X) \subseteq g(X)$,

(2) $g(X)$ is complete,

(3) g is G- continuous and commutes with F.

If $k \in (0, \frac{1}{2})$, then there is a unique x in X such that $F(x, x) = g(x) = x$".

3 Main Result

Lemma 3.1. Let (X, G) be a G-metric space. Let $F, S : X \times X \to X$ and $g, h : X \to X$ be mappings such that

$$G(F(x,y), S(u,v), S(z,w)) \leq k(G(hx, gu, gz) + G(hy, gv, gw)) \tag{3.1}$$

for all $x, y, z, w, u, v \in X$. Assume that (x, y) is a coupled coincidence point of the pairs of mappings $\{F, h\}$ and $\{S, g\}$ and $gx = hx$ and $gy = hy$. If $k \in [0, \frac{1}{8})$, then $S(x,y) = gx = gy = S(y,x)$ and $F(x,y) = hx = hy = F(y,x)$.

Proof. Since (x, y) is a coupled coincidence point of pairs of mappings $\{F, h\}$ and $\{S, g\}$, we have $hx = F(x,y)$, $hy = F(y,x)$ and $gx = S(x,y), gy = S(y,x)$.
Assume $gx \neq gy$. Then by (3.1), we get

$$\begin{aligned} G(gx, gy, gy) &= G(F(x,y), S(y,x), S(y,x)) \\ &\leq k(G(hx, gy, gy) + G(hy, gx, gx)) \\ &= k(G(gx, gy, gy) + G(gy, gx, gx)). \end{aligned}$$

Also by (3.1), we have

$$\begin{aligned} G(gy, gx, gx) &= G(F(y,x), S(x,y), S(x,y)) \\ &\leq k(G(hy, gx, gx) + G(hx, gy, gy)) \\ &= k(G(gy, gx, gx) + G(gx, gy, gy)). \end{aligned}$$

Therefore $G(gx, gy, gy) + G(gy, gx, gx) \leq 2k(G(gx, gy, gy) + G(gy, gx, gx))$. Since $2k < 1$, we get $G(gx, gy, gy) + G(gy, gx, gx) < G(gx, gy, gy) + G(gy, gx, gx)$, which is a contradiction. So $gx = gy$, and hence $S(x,y) = gx = gy = S(y,x)$ and $F(x,y) = hx = hy = F(y,x)$. Thus the lemma is proved □

Theorem 3.2. Let (X, G) be a G- metric space. Let $F, S : X \times X \to X$ and $g, h : X \to X$ be mappings such that Let (X, G) be a G- metric space. Let $F, S : X \times X \to X$ and $g, h : X \to X$ be mappings such that

$$G(F(x,y), S(u,v), S(z,w)) \leq k(G(hx, gu, gz) + G(hy, gv, gw)) \tag{3.2}$$

for all $x, y, z, w, u, v \in X$. Assume that F, S and g, h satisfy the following conditions:

(1) $F(X \times X) \subseteq g(X)$ and $S(X \times X) \subseteq h(X)$

(2) g(X) or h(X) is complete

(3) g and h are G- continuous and pairs $\{F, h\}$ and $\{S, g\}$ are of commuting mappings.

If $k \in (0, \frac{1}{8})$, then there is a unique x in X such that $F(x,x) = S(x,x) = g(x) = h(x) = x$.

Proof. Let $x_0, y_0 \in X$. Since $F(X\times X) \subseteq g(X)$, we can choose $x_1, y_1 \in X$ such that $u_1 = gx_1 = F(x_0, y_0)$ and $v_1 = gy_1 = F(y_0, x_0)$. Again since $S(X \times X) \subseteq h(X)$, we can choose x_2, y_2 in X such that $u_2 = hx_2 = S(x_1, y_1)$ and $v_2 = hy_2 = S(y_1, x_1)$. Continuing this process, we can construct two sequences $\{u_n\}$ and $\{v_n\}$ in X such that $u_{2n+1} = gx_{2n+1} = F(x_{2n}, y_{2n}), v_{2n} = gy_{2n+1} = F(y_{2n+1}, x_{2n+1})$ and $u_{2n+2} = hx_{2n+2} = S(x_{2n+1}, y_{2n+1}), v_{2n+2} = hy_{2n+2} = S(y_{2n+1}, x_{2n+1})$ for all $n \in N$. From

$$\begin{aligned} G(u_{2n+1}, u_{2n+2}, u_{2n+2}) &= G(F(x_{2n}, y_{2n}), S(x_{2n+1}, y_{2n+1}), S(x_{2n+1}, y_{2n+1})) \\ &\le k\{G(hx_{2n}, gx_{2n+1}, gx_{2n+1}) + G(hy_{2n}, gy_{2n+1}, gy_{2n+1)}\} \\ &= k\{G(u_{2n}, u_{2n+1}, u_{2n+1}) + G(v_{2n}, v_{2n+1}, v_{2n+1)}\} \end{aligned} \tag{3.3}$$

and similarly

$$G(v_{2n+1}, v_{2n+2}, v_{2n+2}) \le k\{G(v_{2n}, v_{2n+1}, v_{2n+1}) + G(u_{2n}, u_{2n+1}, u_{2n+1})\} \tag{3.4}$$

we have

$$\begin{aligned} &G(u_{2n+1}, u_{2n+2}, u_{2n+2}) + G(v_{2n+1}, v_{2n+2}, v_{2n+2}) \\ &\qquad\le 2k\{G(u_{2n}, u_{2n+1}, u_{2n+1}) + G(v_{2n}, v_{2n+1}, v_{2n+1})\} \\ &\qquad\le 8k\{G(u_{2n}, u_{2n+1}, u_{2n+1}) + G(v_{2n}, v_{2n+1}, v_{2n+1})\} \end{aligned} \tag{3.5}$$

holds for all $n \in N$. Again from

$$\begin{aligned} G(u_{2n}, u_{2n+1}, u_{2n+1}) &\le 2G(u_{2n+1}, u_{2n}, u_{2n}) \\ &= 2G(F(X_{2n}, y_{2n}), S(x_{2n-1}, y_{2n-1}), S(x_{2n-1}, y_{2n-1})) \\ &\le 2k\{G(hx_{2n}, gx_{2n-1}, gx_{2n-1}) + G(hy_{2n}, gy_{2n-1}, gy_{2n-1})\} \\ &= 2k\{G(u_{2n}, u_{2n-1}, u_{2n-1}) + G(v_{2n}, v_{2n-1}, v_{2n-1})\} \\ &\le 4k\{G(u_{2n-1}, u_{2n}, u_{2n}) + G(v_{2n-1}, v_{2n}, v_{2n})\} \end{aligned} \tag{3.6}$$

and

$$\begin{aligned} G(v_{2n}, v_{2n+1}, v_{2n+1}) &\le 2G(v_{2n+1}, v_{2n}, v_{2n}) \\ &= 2G(F(y_{2n}, x_{2n}), S(y_{2n-1}, x_{2n-1}), S(y_{2n-1}, x_{2n-1})) \\ &\le 2k\{G(hy_{2n}, gy_{2n-1}, gy_{2n-1}) + G(hx_{2n}, gx_{2n-1}, gx_{2n-1})\} \\ &= 2k\{G(v_{2n}, v_{2n-1}, v_{2n-1}) + G(u_{2n}, u_{2n-1}, u_{2n-1})\} \\ &\le 4k\{G(u_{2n-1}, u_{2n}, u_{2n}) + G(v_{2n-1}, v_{2n}, v_{2n})\}, \end{aligned} \tag{3.7}$$

we have

$$G(u_{2n}, u_{2n+1}, u_{2n+1}) + G(v_{2n}, v_{2n+1}, v_{2n+1}) \le 8k\{G(u_{2n-1}, u_{2n}, u_{2n}) + G(v_{2n-1}, v_{2n}, v_{2n})\} \tag{3.8}$$

holds for all $n \in N$. Thus, using (3.5) and (3.8) in (3.3), we get

$$\begin{aligned} G(u_{2n+1}, u_{2n+2}, u_{2n+2}) &\leq k8k\{G(u_{2n-1}, u_{2n}, u_{2n}) + G(v_{2n-1}, v_{2n}, v_{2n})\} \\ &\leq k(8k)^2\{G(u_{2n-2}, u_{2n-1}, u_{2n-1}) + G(v_{2n-2}, v_{2n-1}, v_{2n-1})\} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\leq k(8k)^{2n}\{G(u_0, u_1, u_1) + G(v_0, v_1, v_1)\} \\ &\leq (8k)^{2n+1}\{G(u_0, u_1, u_1) + G(v_0, v_1, v_1)\} \end{aligned}$$

and also, using (3.5) and (3.8) in (3.6),

$$\begin{aligned} G(u_{2n}, u_{2n+1}, u_{2n+1}) &\leq 4k(8k)\{G(u_{2n-2}, u_{2n-1}, u_{2n-1}) + G(v_{2n-2}, v_{2n-1}, v_{2n-1})\} \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\leq (8k)^{2n}\{G(u_0, u_1, u_1) + G(v_0, v_1, v_1)\}. \end{aligned}$$

Thus for each $n \in N$ we have

$$G(u_n, u_{n+1}, u_{n+1}) \leq (8k)^n\{G(u_0, u_1, u_1) + G(v_0, v_1, v_1)\}. \tag{3.9}$$

Let $m, n \in N$ with $m > n$. By Axiom G5 of the definition of G–metric space, we have

$$G(u_n, u_m, u_m) \leq G(u_n, u_{n+1}, u_{n+1}) + G(u_{n+1}, u_{n+2}, u_{n+2}) + ... + G(u_{m-1}, u_m, u_m).$$

Since $8k < 1$, by (3.9) we get that

$$\begin{aligned} G(u_n, u_m, u_m) &\leq \sum_{i=n}^{m-1} (8\mathrm{k})^{\mathrm{i}} \{G(u_0, u_1, u_1) + G(v_0, v_1, v_1)\} \\ &\leq \frac{(8k)^n}{(1-8k)}\{G(u_0, u_1, u_1) + G(v_0, v_1, v_1)\}. \end{aligned}$$

Letting $m, n \to +\infty$, we have $\lim_{m,n \to +\infty} G(u_n, u_m, u_m) = 0$.
Thus $\{u_n\}$,and any subsequence thereof, is a G- Cauchy sequence in X. Similarly we may show that $\{v_n\}$,and any subsequence thereof, is G-Cauchy in X. suppose g(X) is complete then subsequence $\{u_{2n+1}\}$ = $\{gx_{2n+1}\}$ and $\{v_{2n+1}\}$ =$\{gy_{2n+1}\}$ are G- convergent to some $x \in X$ and $y \in X$ respectively. We know that every subsequence and the sequence itself of a G-Cauchy sequence are convergent to the same point. Consequently the sub sequences $\{u_{2n}\}$ = $\{hx_{2n}\}$ and $\{v_{2n}\}$ = $\{hy_{2n}\}$ are also convergent to x and y respectively. Since g and h are G- continuous, we have

$$\{ggx_{2n+1}\} \to gx, \{hgx_{2n+1}\} \to hx, \{ghx_{2n}\} \to gx, \{hhx_{2n}\} \to hx$$

and

$$\{ggy_{2n+1}\} \to gy, \{hgy_{2n+1}\} \to hy, \{ghy_{2n}\} \to gy, \{hhy_{2n}\} \to hy.$$

Since pairs $\{F, h\}$ and $\{S, g\}$ are of commutative mappings, we have

$$hgx_{2n+1} = hf(x_{2n}, y_{2n}) = F(hx_{2n}, hy_{2n}) \text{ and } ghx_{2n} = gS(x_{2n-1}, y_{2n-1}) = S(gx_{2n-1}, gx_{2n-1}).$$

Thus

$$\begin{aligned} G(hgx_{2n+1}, ghx_{2n}, ghx_{2n}) &= G(F(hx_{2n}, hy_{2n}), S(gx_{2n-1}, gy_{2n-1}), S(gx_{2n-1}, gy_{2n-1})) \\ &\leq k\{G(hhx_{2n}, ggx_{2n-1}, ggx_{2n-1}) + G(hhy_{2n}, ggy_{2n-1}, ggy_{2n-1})\}. \end{aligned}$$

Letting $n \to +\infty$, we have $G(hx, gx, gx) = k\{G(hx, gx, gx) + G(hy, gy, gy)\}$. In the same way, we may show that $G(hy, gy, gy) = k\{G(hy, gy, gy) + G(hx, gx, gx)\}$. Thus

$$G(hx, gx, gx) + G(hy, gy, gy) = 2k\{G(hx, gx, gx) + G(hy, gy, gy)\}.$$

Since $2k < 8k < 1$, the last inequality happens only if $G(hx, gx, gx) = G(hy, gy, gy) = 0$. Hence $hx = gx$ and $hy = gy$. Again

$$\begin{aligned} G(hgx_{2n+1}, S(x, y), S(x, y)) &= G(F(hx_{2n}, hy_{2n}), S(x, y), S(x, y)) \\ &\leq k\{G(hhx_{2n}, gx, gx) + G(hhy_{2n}, gy, gy)\}. \end{aligned}$$

Letting $n \to +\infty$, we have

$$G(hx, S(x, y), S(x, y)) \leq k\{G(hx, gx, gx) + G(hy, gy, gy)\} = 0.$$

Thus we get $G(hx, S(x, y), S(x, y)) = 0$ which immediately yields $S(x, y) = hx$. Similarly we may show that $S(y, x) = hy$. In the same manner

$$\begin{aligned} G(F(x, y), ghx_{2n}, ghx_{2n}) &= G(F(x, y), S(gx_{2n-1}, gy_{2n-1}), S(gx_{2n-1}, gy_{2n-1})) \\ &\leq k\{G(hx, ggx_{2n-1}, ggx_{2n-1}) + G(hy, ggy_{2n-1}, ggy_{2n-1})\} \end{aligned}$$

Letting $n \to +\infty$, we have

$$G(F(x, y), gx, gx) = k\{G(hx, gx, gx) + G(hy, gy, gy)\} = 0.$$

Thus we get $G(F(x, y), gx, gx) = 0$ which implies that $F(x, y) = gx$. Similarly we may show that $F(y, x) = gy$. Therefore we obtain $gx = hx, gy = hy$ and $F(x, y) = gx$, $F(y, x) = gy$, $S(x, y) = hx$, $S(y, x) = hy$ which, by an application of Lemma 3.1, yields

$$F(x, y) = gx = gy = F(y, x) = S(x, y) = hx = hy = S(y, x).$$

Now

$$\begin{aligned} G(gx_{2n+1}, gx, gx) &= G(F(x_{2n}, y_{2n}), S(x, y), S(x, y)) \\ &\leq k\{G(hx_{2n}, gx, gx) + G(hy_{2n}, gy, gy)\}. \end{aligned}$$

Letting $n \to +\infty$, we have $G(x, gx, gx) = k\{G(x, gx, gx) + G(y, gy, gy)\}$. Similarly, we may show that $G(y, gy, gy) = k\{G(y, gy, gy) + G(x, gx, gx)\}$. Thus $G(x, gx, gx) + G(y, gy, gy) = 2k\{G(x, gx, gx) + G(y, gy, gy)\}$. Since $2k < 8k < 1$, the last inequality happens only if $G(x, gx, gx) = 0$ and $G(y, gy, gy) = 0$. Hence $gx = x$ and $gy = y$. Thus we get $F(x, x) = S(x, x) = gx = hx = x$. To prove the uniqueness , let $z \in X$ with $z \neq x$ such that $F(z, z) = S(z, z) = gz = hz = z$. Then

$$\begin{aligned} G(x, z, z) &= G(F(x, x), S(z, z), S(z, z)) \\ &\leq k\{G(hx, gz, gz) + G(hx, gz, gz)\} \\ &= k\{G(x, z, z) + G(x, z, z)\} \\ &= 2kG(x, z, z). \end{aligned}$$

Since $2k < 8k < 1$, we get $G(x, z, z) < G(x, z, z)$, which is a contradiction. Thus F, S, g, h have a unique common fixed point □

Corollary 3.3. Let (X, G) be a G- metric space. Let $F, S : X \times X \to X$ and $g, h : X \to X$ be mappings such that

$$G(F(x, y), S(u, v), S(u, v)) = k(G(hx, gu, gu) + G(hy, gv, gv))$$

for all $x, y, u, v \in X$. Assume that F, S and g, h satisfy the following conditions:

(1) $F(X \times X) \subseteq g(X)$ and $S(X \times X) \subseteq h(X)$

(2) $g(X)$ or $h(X)$ is complete

(3) g and h are G- continuous and pairs $\{F, h\}$ and $\{S, g\}$ are of commuting mappings.

If k $\in (0, \frac{1}{8})$, then there is a unique $xinX$ such that $F(x, x) = S(x, x) = g(x) = h(x) = x$.

Example 3.4. Let $x = [0, 1]$. Define $G : X \times X \times X \to R^+$ by $G(x, y, z) = |x - y| + |y - z| + |z - x|$ for all $x, y, z \in X$. Define mappings $F, S : X \times X \to X$ and $g, h : X \to X$ by

$$F(x, y) = \frac{1}{36}xy, S(x, y) = \frac{1}{144}xy \text{ and } gx = \frac{1}{4}x, hx = \frac{1}{2}x.$$

Since $|xy - uv| = |x - u| + |y - v|$ holds for all $x, y, u, v \in X$, we have

$$\begin{aligned} G(F(x, y), S(u, v), S(z, w)) &= |\frac{1}{36}xy - \frac{1}{144}uv| + |\frac{1}{144}uv - \frac{1}{144}zw| + |\frac{1}{144}zw - \frac{1}{36}xy| \\ &\leq \frac{1}{9}\{|\frac{1}{2}x - \frac{1}{4}u| + |\frac{1}{4}u - \frac{1}{4}z| + |\frac{1}{4}z - \frac{1}{2}x| \\ &+ |\frac{1}{2}y - \frac{1}{4}v| + |\frac{1}{4}v - \frac{1}{4}w| + |\frac{1}{4}w - \frac{1}{2}y|\} \\ &= \frac{1}{9}\{G(hx, gu, gz) + G(hy, gv, gw)\} \end{aligned}$$

holds for all $x, y, z, u, v, w \in X$. it is easy to see that F, S, g, h satisfiy all hypothesis of Theorem 3.2. Thus F, S, g, h have a unique common fixed point. Here $F(0, 0) = S(0, 0) = g0 = h0 = 0$.

References

[1] M. Abbas and G. Jungck, *Common fixed point results for noncommuting mappings without continuity in cone metric spaces* J. Math. Anal. Appl., 341(2008), 416–420.

[2] M. Abbas and G.E. Rhoades, *Common fixed point results for noncommuting mapping without continuity in generalized metric spaces*, Applied Mathematics and Computation, 215(2009), 262–269.

[3] Th.Abdeljawad, *Completion of cone metric spaces*, Hacet. J. Math. Stat., 39(2010), 67–74.

[4] I. Beg and M. Abbas, *Coincidence point and invariant approximation for mappings satisfying generalized weak contractive condition*, Fixed Point Theor. Appl.,(2006), 1–7.

[5] T.G. Bhaskar and G.V. Lakshmikantham, *Fixed point theorems in partially ordered metric spaces and applications*, Nonlinear Analysis, 65(2006), 1379–1393.

[6] L.G. Haung and X. Zhang, *Cone metric spaces and fixed point theorems of contractive mappings*, J. Math. Anal. Appl., 332(2007), 1468–1476.

[7] G. Jungck, *Commuting maps and fixed points*, Am. Math. Monthly, 83(1976), 1976.

[8] G. Jungck and N. Hussain, *Compatible maps and invariant approximations*, J. Math. Anal. Appl., 325(2007), 1003–1012.

[9] V. Lakshmikantham, V. Ciric and B. Lj, *Coupled fixed point theorems for nonlinear contrac- tions in partially ordered metric spaces*, Nonlinear Analysis, 70(2009), 4341–4349.

[10] P.P. Murthy and K. Tas, *New common fixed point theorems of Gregus type for R- weakly commuting mappings in 2-metric spaces*, Hacet. J. Math. Stat., 38(2009), 285–291.

[11] Z. Mustafa and B. Sims, *Some Remarks concerning D-metric spaces*, Proc. Int. Conf. on Fixed Point Theor. Appl.,(2003), 189–198.

[12] Z. Mustafa and B. Sims, *A new approach to generalized metric spaces*, J. Nonlinear Convex Anal., 7 (2006), 289–297.

[13] Z. Mustafa, W. Shatanawi and M. Bataineh, *Existence of fixed point results in G-metric spaces*, International Journal of Mathematics and Mathematical Sciences , Article ID 283028(2009).

[14] Z. Mustafa, H. Obiedat and F. Awawdeh, *Some common fixed point theorem for mapping on complete G-metric spaces*, Fixed Point Theor. Appl., Article ID 189870(2008).

[15] V. Popa and M. Mocanu, *Altering distance and common fixed points under implicit rela- tions*, Hacet. J. Math. Stat., 38(2009), 329–337.

[16] I. Sahin and M. Telci, *Fixed points of contractive mappings on complete cone metric spaces*, Hacet. J. Math. Stat., 38(2009), 59–67.

[17] S. Sedghi, I. Altun and N. Shobe, *Coupled fixed point theorems for contractions in fuzzy metric spaces*, Nonlinear Analysis, 72(2010) 1298–1304.

[18] W. Shatanawi, *Common fixed point result for two self-maps in G-metric space*, submitted.

[19] W. Shatanawi, *Some fixed point theorems in ordered G-metric spaces and applications*, Abstr. Appl. Anal., Article ID 126205(2011).

[20] W.Shatanawi, *Fixed point theory for contractive mappings satisfying* Φ*-maps in G- metric spaces*, Fixed Point Theory Appl., Article ID 181650(2010).

[21] W. Shatanawi, *Some common coupled fixed point results in cone metric spaces*, Int. J. Math. Analysis, 4(48)(2010), 2381–2388.

[22] W. Shantanawi, *Coupled fixed point theorems in generalized metric spaces*, Hacet. J. Math. Stat. 40(3)(2011), 441 – 447.