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Reflection and transmission phenomenon of elastic waves at viscous liquid and fluid saturated porous solid interface

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Abstract : A study of the reflection and transmission of elastic waves from a plane surface separating a viscous liquid half space and a fluid saturated porous half space is presented when a longitudinal or a transverse wave impinge obliquely at the interface. Amplitude ratios of various reflected and transmitted waves are obtained using suitable boundary conditions at the interface. The amplitude ratios have been computed numerically for a particular model and the results obtained are presented graphically. It is observed that the amplitude ratios depend not only on the angle of incidence of the incident wave, but also on material properties of the medium through which the waves traversed. Special cases with empty porous half space medium and inviscid liquid half space has been obtained from the present investigation.

Keywords : Reflection and transmission of elastic waves, viscous liquid.

1 Introduction

Due to the importance of porous media in different branches of study like material science, petroleum industry, soil mechanics, geophysics, seismology, earth sciences, earthquake engineering and civil engineering etc., many researchers made a considerable work on porous medium by taking different theories for the porous medium. Based on the work of Von Terzaghi ([20, 21]), Biot [2] proposed a general theory of three dimensional deformations of fluid saturated porous solids. Biot ([3, 4, 5]) showed the propagation of two dilatational waves and one rotational elastic wave in fluid saturated porous solids. Many researchers worked by taking the Biot theory of Poroelasticity, e.g. Berryman [1], Vashishth, Sharma, and Gogna [22], Kumar, Miglani and Garg [15] and Sharma [18] etc.

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Biot's theory was based on the assumption of compressible constituents. But there are sufficient reasons for considering the fluid saturated porous constituents as incompressible, for example, soil in which both the solid as well as liquid constituents are incompressible. Bowen [6] and de Boer & Ehlers ([8, 9]) developed an interesting theory for porous medium in which all the constituents are incompressible. The assumption of incompressible constituents resembles the properties appearing in many porous media materials, which are of use in many branches of study stated above. It also avoids the introduction of many complicated material parameters as considered in the Biot theory. Based on this theory, many researchers like de Boer & Didwania [7], de Boer & Liu([11, 12]), Kumar & Hundal [13], Tajuddin & Hussaini [19] and Kumar et.al.[16] etc. studied some problems of wave propagation in fluid saturated porous media.

In the present study, i.e., the reflection and transmission of longitudinal and transverse waves at an interface between a viscous liquid half space and a fluid saturated porous half space,we have used the porous media theory given by de Boer and Ehlers [9]. The model considered is assumed to exist in the oceanic crust part of the earth and the propagation of wave through such a model will be of great use in the fields related to earth sciences. Special cases of the problem are also solved by taking the fluid saturated porous half space to be empty porous solid and viscous liquid half space to be inviscid liquid half space. Amplitude ratios of various reflected and transmitted waves are computed for a particular model and the results are shown graphically to discuss them.

2 Basic equations

The equations governing the deformation of an incompressible porous medium saturated with non-viscous fluid in the absence of body forces are given by de Boer and Ehlers [9] as

$$\nabla .\left(\eta^S \dot{\mathbf{u}}^S + \eta^F \dot{\mathbf{u}}^F\right) = 0, \tag{2.1}$$

$$\left(\lambda^S + \mu^S\right) \nabla \left(\nabla .\mathbf{u^S}\right) + \mu^S \nabla^2 \mathbf{u^S} - \eta^S \nabla p - \rho^S \ddot{\mathbf{u}}^S + S_v \left(\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S\right) = 0 \tag{2.2}$$

$$\eta^F \nabla p + \rho^F \ddot{\mathbf{u}}^F + S_v \left(\dot{\mathbf{u}}^F - \dot{\mathbf{u}}^S\right) = 0, \tag{2.3}$$

$$\mathbf{T^S_E} = 2\mu^S \mathbf{E^S} + \lambda^S \left(E^S.\mathbf{I}\right) \mathbf{I} \tag{2.4}$$

$$\mathbf{E^S} = \frac{1}{2}\left(grad\ \mathbf{u^S} + grad^T \mathbf{u^S}\right) \tag{2.5}$$

where $\mathbf{u}^{\mathrm{i}},\ \dot{\mathbf{u}}^{\mathrm{i}},\ \ddot{\mathbf{u}}^{\mathrm{i}},\quad \mathrm{i = F, S}$ denote the displacement, velocity and acceleration of fluid and solid phases, respectively and p is the effective pore pressure of the incompressible pore fluid. ρ^{S} and $\rho^{\mathrm{F}}$are the densities of the solid and fluid constituents, respectively. $\mathbf{T^S_E}$ is the effective stress in the solid phase and $\mathbf{E^S}$ is the linearized langrangian strain tensor. λ^{S} and μ^{S} are the macroscopic Lame's parameters of the porous solid and η^{S} and η^{F} are the volume fractions satisfying

$$\eta^S + \eta^F = 1. \tag{2.6}$$

In the case of isotropic permeability, the tensor $\mathbf{S}_{\mathrm{v}}$ describing the coupled interaction between the solid and fluid is given by de Boer and Ehlers [9] as

$$\mathbf{S}_v = \frac{\left(\eta^F\right)^2 \gamma^{FR}}{K} \mathbf{I} \tag{2.7}$$

where γ^{FR} is the specific weight of the fluid and K is the Darcy's permeability coefficient of the porous medium and $\mathbf{I}$ stands for unit vector. Assuming the displacement vector $\mathbf{u}^{\mathrm{i}}$ $(\mathrm{i} = \mathrm{F}, \mathrm{S})$ as

$$\mathbf{u}^i = \left(u^i, 0, w^i\right), \text{where } \; i = F, S, \tag{2.8}$$

and hence Equation (2.1)- (2.3) give the equations of motion for fluid saturated incompressible porous medium in the component form as

$$\left(\lambda^S + \mu^S\right) \frac{\partial \theta^S}{\partial x} + \mu^S \nabla^2 u^{mS} - \eta^S \frac{\partial p}{\partial x} - \rho^S \frac{\partial^2 u^S}{\partial t^2} + S_v \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{2.9}$$

$$\left(\lambda^S + \mu^S\right) \frac{\partial \theta^S}{\partial z} + \mu^S \nabla^2 w^S - \eta^S \frac{\partial p}{\partial z} - \rho^S \frac{\partial^2 w^S}{\partial t^2} + S_v \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{2.10}$$

$$\eta^F \frac{\partial p}{\partial x} + \rho^F \frac{\partial^2 u^F}{\partial t^2} + S_v \left[\frac{\partial u^F}{\partial t} - \frac{\partial u^S}{\partial t}\right] = 0, \tag{2.11}$$

$$\eta^F \frac{\partial p}{\partial z} + \rho^F \frac{\partial^2 w^F}{\partial t^2} + S_v \left[\frac{\partial w^F}{\partial t} - \frac{\partial w^S}{\partial t}\right] = 0, \tag{2.12}$$

$$\eta^S \left[\frac{\partial^2 u^S}{\partial x \partial t} + \frac{\partial^2 w^S}{\partial z \partial t}\right] + \eta^F \left[\frac{\partial^2 u^F}{\partial x \partial t} + \frac{\partial^2 w^F}{\partial z \partial t}\right] = 0, \tag{2.13}$$

where

$$\theta^S = \frac{\partial \left(u^S\right)}{\partial x} + \frac{\partial \left(w^S\right)}{\partial z}, \tag{2.14}$$

and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \tag{2.15}$$

Using the Helmholtz decomposition of displacement vector, the displacement components u^{i} and w^{i} are related to the potential functions ϕ^i and ψ^i as

$$u^i = \frac{\partial \phi^i}{\partial x} + \frac{\partial \psi^i}{\partial z}, \quad w^i = \frac{\partial \phi^i}{\partial z} - \frac{\partial \psi^i}{\partial x}, \qquad i = F, S. \tag{2.16}$$

Using, (2.16) in Equations (2.9)–(2.13), we obtain the following equations:

$$\nabla^2 \phi^S - \frac{1}{C^2} \frac{\partial^2 \phi^S}{\partial t^2} - \frac{S_v}{\left(\lambda^S + 2\mu^S\right) \left(\eta^F\right)^2} \frac{\partial \phi^S}{\partial t} = 0 \tag{2.17}$$

$$\phi^F = -\frac{\eta^S}{\eta^F} \phi^S, \tag{2.18}$$

$$\mu^S \nabla^2 \psi^S - \rho^S \frac{\partial^2 \psi^S}{\partial t^2} + S_v \left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}\right] = 0, \tag{2.19}$$

$$\rho^F \frac{\partial^2 \psi^F}{\partial t^2} + S_v \left[\frac{\partial \psi^F}{\partial t} - \frac{\partial \psi^S}{\partial t}\right] = 0, \tag{2.20}$$

$$\left(\eta^F\right)^2 p - \eta^S \rho^F \frac{\partial^2 \phi^S}{\partial t^2} - S_v \frac{\partial \phi^S}{\partial t} = 0, \tag{2.21}$$

where

$$C = \sqrt{\frac{\left(\eta^F\right)^2 \left(\lambda^S + 2\mu^S\right)}{\left(\eta^F\right)^2 \rho^S + \left(\eta^S\right)^2 \rho^F}} \tag{2.22}$$

Also, the normal and tangential stresses in the solid phase takes the form,

$$t_{zz}^S=\lambda^S\left(\frac{\partial^2\phi^S}{\partial x^2}+\frac{\partial^2\phi^S}{\partial z^2}\right)+2\mu^S\left(\frac{\partial^2 ?^S}{\partial z^2}-\frac{\partial^2\psi^S}{\partial x\partial z}\right), \tag{2.23}$$

$$t_{zx}^S=\mu^S\left(2\frac{\partial^2\phi^S}{\partial x\partial z}+\frac{\partial^2\psi^S}{\partial z^2}-\frac{\partial^2\psi^S}{\partial x^2}\right). \tag{2.24}$$

The time harmonic solution of the system of Equations (2.17)–(2.21) can be considered as

$$\left(\phi^S,\phi^F,\psi^S,\psi^F,p\right)=\left(\phi_1^S,\phi_1^F,\psi_1^S,\psi_1^F,p_1\right)\exp\left(i\omega t\right), \tag{2.25}$$

where ω is the complex circular frequency. Making use of (2.25) in Equations (2.17)–(2.21), we obtain

$$\left[\nabla^2+\frac{\omega^2}{{C_1}^2}-\frac{i\omega S_v}{\left(\lambda^S+2\mu^S\right)\left(\eta^F\right)^2}\right]{\phi_1}^S=0 \tag{2.26}$$

$$\left[\mu^S\nabla^2+\rho^S\omega^2-i\omega S_v\right]{\psi_1}^S=-i\omega S_v{\psi_1}^F \tag{2.27}$$

$$\left[-\omega^2\rho^F+i\omega S_v\right]{\psi_1}^F-i\omega S_v{\psi_1}^S=0 \tag{2.28}$$

$$\left(\eta^F\right)^2 p_1+\eta^S\rho^F\omega^2\phi_1^S-i\omega S_v\phi_1^S=0 \tag{2.29}$$

$$\phi_1^F=-\frac{\eta^S}{\eta^F}{\phi_1}^S. \tag{2.30}$$

Equation (2.26) represents the propagation of a longitudinal wave with velocity V_1, given by

$$V_1^2=\frac{1}{G_1}, \tag{2.31}$$

where

$$G_1=\left[\frac{1}{{C_1}^2}-\frac{iS_v}{\omega\left(\lambda^S+2\mu^S\right)\left(\eta^F\right)^2}\right]. \tag{2.32}$$

From equations (2.27) and (2.28), we obtain

$$\left[\nabla^2+\frac{\omega^2}{{V_2}^2}\right]{\psi_1}^S=0 \tag{2.33}$$

Equation (2.33) corresponds to the propagation of a transverse wave with velocity V_2, given by $V_2^2=\frac{1}{G_2}$, where

$$G_2=\left\{\frac{\rho^S}{\mu^S}-\frac{iS_v}{\mu^S\omega}-\frac{{S_v}^2}{\mu^S\left(-\rho^S\omega^2+i\omega S_v\right)}\right\}, \tag{2.34}$$

Further, following Fehler (1982) the equations of motion for a viscous liquid medium in terms of the potential functions ϕ' and ψ' corresponding to longitudinal and transverse waves are as

$$k'\nabla^2\phi'+\frac{4}{3}\eta\frac{\partial}{\partial t}\nabla^2\phi'=\rho'\frac{\partial^2\phi'}{\partial t^2},\qquad \eta\frac{\partial}{\partial t}\nabla^2\psi'=\rho'\frac{\partial^2\psi'}{\partial t^2}, \tag{2.35}$$

where k' is the bulk modulus, ρ' is the fluid density, η is the fluid viscosity. The components of displacements and stresses are given by

$$u'=\frac{\partial\phi'}{\partial x}-\frac{\partial\psi'}{\partial z},\qquad w'=\frac{\partial\phi'}{\partial z}+\frac{\partial\psi'}{\partial x},\qquad {t_{zx}}'=\eta\frac{\partial}{\partial t}\left[2\frac{\partial^2\phi'}{\partial x\partial z}+\frac{\partial^2\psi'}{\partial x^2}-\frac{\partial^2\psi'}{\partial z^2}\right], \tag{2.36}$$

$${t_{zz}}'=\left[k'-\frac{2}{3}\eta\frac{\partial}{\partial t}\right]\left[\frac{\partial^2\phi'}{\partial x^2}+\frac{\partial^2\phi'}{\partial z^2}\right]+2\eta\frac{\partial}{\partial t}\left[\frac{\partial^2\psi'}{\partial x\partial z}-\frac{\partial^2\phi'}{\partial z^2}\right]. \tag{2.37}$$

3 Formulation of the problem

Consider a viscous liquid half space medium M_2 lying over an incompressible fluid saturated porous half space medium M_1. Taking the Cartesian coordinate system in such a way that the interface between the two half-spaces represents the xy-plane and z-axis is pointed vertically downward in the lower half space medium M_1, so the medium M_1 is represented as $z \geq 0$ and medium M_2 as $z \leq 0$. A plane wave (P or SV-wave) propagates through the half space medium M_1 and incident at the plane interface z=0, making an angle θ_0 with normal to the surface. Corresponding to each incident wave (P or SV-wave), we get reflected P and SV- waves in the medium M_1, and transmitted P and SV-waves in medium M_2. The problem considered is a two dimensional problem with $\frac{\partial}{\partial y} \equiv 0$.

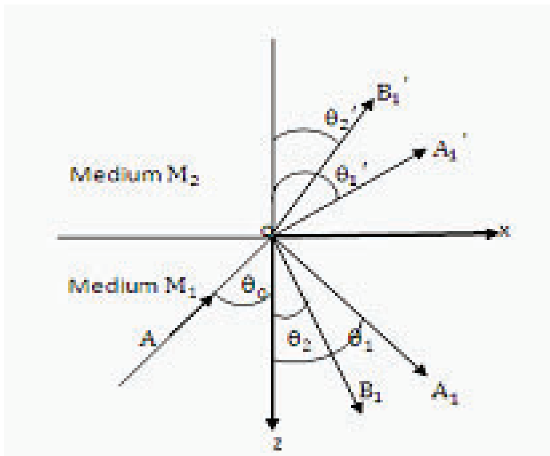

Fig.1– Geometry of the problem

4 Solution of the problem

The potential function solution of the equations (2.17)–(2.21) can be taken as

$$\begin{aligned}\{\phi^S, \phi^F, p\} = \{1, m_1, m_2\}[&A_{01} \exp\{ik_1(x \sin\theta_0 - z\cos\theta_0) + i\omega_1 t\} \\ &+ A_1 \exp\{ik_1(x\sin\theta_1 + z\cos\theta_1) + i\omega_1 t\}],\end{aligned} \tag{4.1}$$

$$\begin{aligned}\{\psi^S, \psi^F\} = \{1, m_3\}[&B_{01} \exp\{ik_2(x \sin\theta_0 - z\cos\theta_0) + i\omega_2 t\} \\ &+ B_1 \exp\{ik_2(x\sin\theta_2 + z\cos\theta_2) + i\omega_2 t\}],\end{aligned} \tag{4.2}$$

where

$$m_1 = -\frac{\eta^S}{\eta^F}, \quad m_2 = -\left[\frac{\eta^S\omega_1^2\rho^F - i\omega_1 S_v}{(\eta^F)^2}\right], \quad m_3 = \frac{i\omega_2 S_v}{i\omega_2 S_v - \omega_2^2\rho^F}, \tag{4.3}$$

A_{01} and B_{01} are the amplitudes of the incident P and SV-waves, respectively, and A_1, B_1 are the amplitudes of the reflected P and SV-waves, respectively. Following Kumar & Tomar [17], the solution of the system of Equation (2.35) is taken in the form

$$\phi^{'}=\mathrm{A_1}^{'}\ \exp\left\{-\mathrm{i}\left(\mathrm{K}^{'2}-\mathrm{k_1}^{'2}\sin^2\theta_1^{'}\right)^{\frac{1}{2}}\mathrm{z}\right\}\exp\left\{\mathrm{i}\left(\omega_1^{'}\ \mathrm{t}+\mathrm{k_1}^{'}\mathrm{x}\ \sin\theta_1^{'}\right)\right\} \tag{4.4}$$

$$\psi^{'}=\mathrm{B_1}^{'}\ \ \exp\left\{-\mathrm{i}\left(\frac{\mathrm{i}\omega}{\nu}+\mathrm{k_2}^{'2}\sin^2\theta_2^{'}\right)^{\frac{1}{2}}\mathrm{z}\right\}\exp\left\{\mathrm{i}\left(\omega_2^{'}\ \mathrm{t}+\mathrm{k_2}^{'}\mathrm{x}\ \sin\theta_2^{'}\right)\right\}, \tag{4.5}$$

where A_1' and B_1' are the amplitudes of transmitted P- and SV-waves. Also

$$\mathrm{K}^{'}=\frac{\omega}{\mathrm{c}^{'}}\left(1+\frac{4}{3}\frac{\mathrm{i}\omega\nu}{\mathrm{c}^{'2}}\right)^{-1/2}, \qquad \mathrm{c}^{'2}=\frac{\mathrm{k}^{'}}{\rho^{'}}, \qquad \nu=\frac{\eta}{\rho^{'}}, \tag{4.6}$$

k_1' and k_2' are the wave numbers of transmitted P- and SV-waves, respectively.

5 Boundary conditions

The appropriate boundary conditions at the interface $z = 0$ are the continuity of displacements and stresses. Mathematically, these boundary conditions can be expressed as:

$$t_{zz}^{S} - p = t_{zz}', \ \ t_{zx}^{S} = t_{zx}', \ \ w^{s} = w', \ \ u^{s} = u'. \tag{5.1}$$

Snell's law:

$$\frac{\sin\theta_0}{V_0} = \frac{\sin\theta_1}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\theta_1'}{V_1'} = \frac{\sin\theta_2'}{V_2'}, \tag{5.2}$$

where

$$V_1' = \left[\frac{k'}{\rho'}\left(1+\frac{4}{3}\frac{i\omega\eta}{k'}\right)\right]^{\frac{1}{2}}, \ \ V_2' = \left[\frac{i\omega\eta}{\rho'}\right]^{\frac{1}{2}}. \tag{5.3}$$

Also

$$k_1V_1 = k_2V_2 = k_1'V_1' = k_2'V_2' = \omega, \ \ \text{at} \ \ z = 0, \tag{5.4}$$

For incident longitudinal P-wave, we take

$$V_0 = V_1, \ \ \theta_0 = \theta_1, \tag{5.5}$$

For incident transverse SV-wave, we have

$$V_0 = V_2, \ \ \theta_0 = \theta_2, \tag{5.6}$$

For incident longitudinal wave at the interface $z = 0$, we put $B_{01} = 0$ in equation (4.1) and for incident transverse wave, we put $A_{01} = 0$ in equation (4.1). Now substitute the expressions of potentials given by (4.1)–(4.1) in equations (2.16) and then in (2.23)–(2.24), and also substitute (4.4)–(4.5) in (2.36)–(2.37). Then using all these in boundary conditions (5.1), we get a system of four non homogeneous equations which can be written as

$$\sum_{j=1}^{4} a_{ij}Z_j = Y_i, \ \ (i = 1, 2, 3, 4), \tag{5.7}$$

where

$$Z_1 = \frac{A_1}{A}, \ \ Z_2 = \frac{A_2}{A}, \ \ Z_3 = \frac{A_1'}{A}, \ \ Z_4 = \frac{B_1'}{A}, \tag{5.8}$$

where A is the amplitude of incident wave. Coefficients a_{ij} in non dimensional form are obtained as

$$
\begin{aligned}
a_{11} &= \frac{\lambda^s}{\mu^s} + 2\cos^2\theta_1 + \frac{m_2}{\mu^S k_1^2},\\
a_{12} &= -2\sin\theta_2\cos\theta_2\frac{k_2^2}{k_1^2},\\
a_{13} &= \frac{1}{\mu^S k_1^2}\left[K'^2\left(\frac{8}{3}i\eta\omega_1' - k'\right) - 2i\eta\omega_1' {k_1'}^2 \sin^2{\theta_1}'\right],\\
a_{14} &= 2i\eta{\omega_2}'{k_2}'\sin{\theta_2}'\left(\frac{i\omega}{\nu} + {k_2'}^2\sin^2{\theta_2}'\right)^{\frac{1}{2}},\\
a_{21} &= 2\sin\theta_1\cos\theta_1,\\
a_{22} &= \frac{k_2^2}{k_1^2}\left(\cos^2\theta_1 - \sin^2\theta_2\right),\\
a_{23} &= \frac{1}{\mu^S k_1^2}\left[2i\eta\omega_1' k_1' \sin{\theta_1}'\left(K'^2 - {k_1'}^2\sin^2{\theta_1}'\right)^{\frac{1}{2}}\right],\\
a_{24} &= \frac{1}{\mu^S k_1^2}\left[i\eta\omega_2'\left(\frac{i\omega}{\nu}\right)\right],\\
a_{31} &= \cos\theta_1,\\
a_{32} &= -\frac{k_2}{k_1}\sin\theta_2,\\
a_{33} &= \frac{1}{k_1}\left(K'^2 - k_1'^2\sin^2{\theta_1}'\right)^{\frac{1}{2}},\\
a_{34} &= -\frac{k_2'}{k_1}\sin{\theta_2}',\\
a_{41} &= \sin\theta_1,\\
a_{42} &= -\frac{k_2}{k_1}\cos\theta_2,\\
a_{43} &= -\frac{k_1'}{k_1}\sin{\theta_1}',\\
a_{44} &= -\frac{1}{k_1}\left(\frac{i\omega}{\nu} + k_2'^2\sin^2{\theta_2}'\right)^{\frac{1}{3}}.
\end{aligned}
\tag{5.9}
$$

For incident longitudinal wave, we have

$$A = A_{01},\ Y_1 = -a_{11},\ Y_2 = a_{21},\ Y_3 = a_{31},\ Y_4 = -a_{41}, \tag{5.10}$$

For incident transverse wave, we have

$$A = B_{01},\ Y_1 = a_{12},\ Y_2 = -a_{22},\ Y_3 = -a_{32},\ Y_4 = a_{42}, \tag{5.11}$$

6 Particular cases

Case-1

If pores are absent or gas is filled in the pores then ^{F} is very small as compared to ^{S} and can be neglected, so the relation (2.22) gives us

$$C = \sqrt{\frac{\lambda^S + 2\mu^S}{\rho^S}}, \tag{6.1}$$

and the coefficients a_{11} in (5.9) changes to

$$a_{11} = \frac{\lambda^s}{\mu^s} + 2\cos^2\theta_1 \tag{6.2}$$

Case-2

Neglecting the viscous effect of the liquid, that is, we take $\eta = 0$, then medium M_2 becomes inviscid liquid half space and the problem reduces to the problem of inviscid liquid half space over incompressible porous solid half space. Boundary conditions for this case reduces to

$$t^S_{zz} - p = t'_{zz},\ t^S_{zx} = t'_{zx},\ w^s = w', \tag{6.3}$$

and hence we obtain a system of three non-homogeneous equations which can be written as

$$\sum_{j=1}^{3} a_{ij} Z_j = Y_i, \quad (i = 1, 2, 3), \tag{6.4}$$

where a_{ij} are obtained as under

$$\begin{aligned} a_{11} &= \frac{\lambda^s}{\mu^s} + 2\cos^2\theta_1 + \frac{m_2}{\mu^S k_1^2}, \\ a_{12} &= -2\sin\theta_2\cos\theta_2\frac{k_2^2}{k_1^2}, \quad a_{13} = -\rho'\omega_1'^2, \\ a_{21} &= 2\sin\theta_1\cos\theta_1, \quad a_{22} = \frac{k_2^2}{k_1^2}\left(\cos^2\theta_1 - \sin^2\theta_2\right), \\ a_{23} &= 0, \quad a_{31} = \cos\theta_1, \\ a_{32} &= -\frac{k_2}{k_1}\sin\theta_2, \quad a_{33} = \frac{k_1'}{k_1}\cos\theta_1' \end{aligned} \tag{6.5}$$

which can also be obtained directly from (5.9) by taking $\eta = 0$.

7 Numerical results and discussion

The theoretical results obtained above indicate that the amplitude ratios Z_i $(i = 1, 2, 3)$ depend on the angle of incidence of incident wave and material properties. The behaviour of various amplitude ratios are observed through numerical computations by considering a particular model, for which the numerical values of the parameters are taken as under:-

In medium M_1, the physical constants for fluid saturated incompressible porous medium are taken from de Boer, Ehlers and Liu [10] as $\eta^s = 0.67$, $\eta^F = 0.33$, $\rho^s = 1.34$ Mg/m^3, $\rho^F = 0.33$ Mg/m^3, $\lambda^s = 5.5833$ MN/m^2, K^F=0.01m/s, $\gamma^{FR} = 10.00$KN/m^3, $\mu^s = 8.3750$ N/m^2, $\omega^* = 10/s$. In medium M_2, $k' = 0.119 \times 10^{11}$ dyne/cm^2, $\rho' = 1.01$g/cm^3, $\eta = 0.0014$ poise

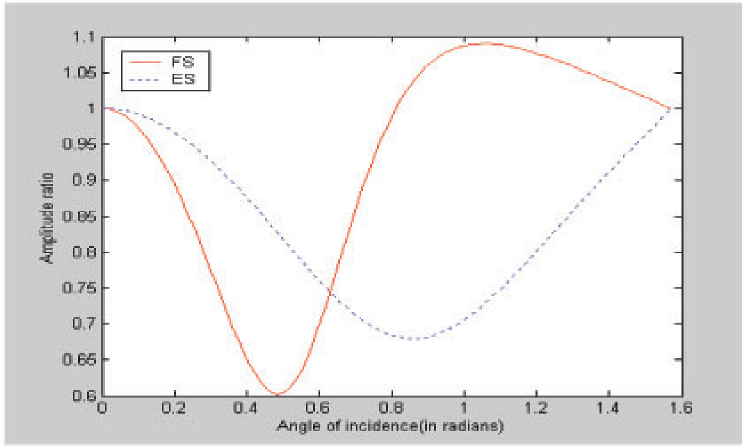

Fig.2– Variation of the amplitude ratio $|Z_1|$ of the reflected P-wave for an incident P-wave

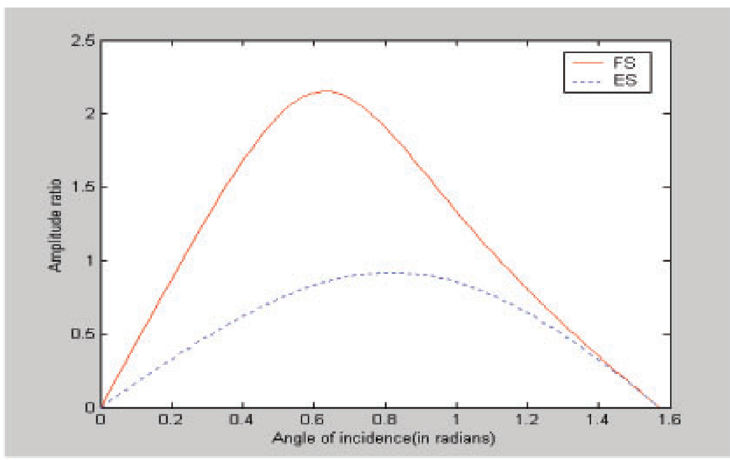

Fig.3– Variation of the amplitude ratio $|Z_2|$ of the reflected SV-wave for an incident P-wave

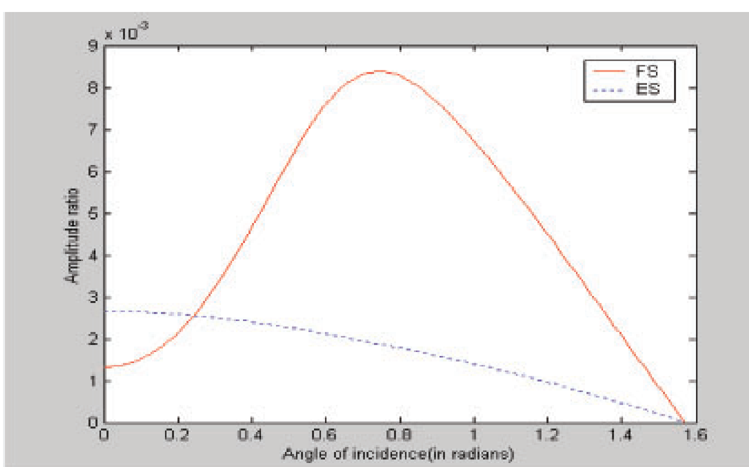

Fig.4– Variation of the amplitude ratio $|Z_4|$ of the transmitted P-wave for an incident P-wave

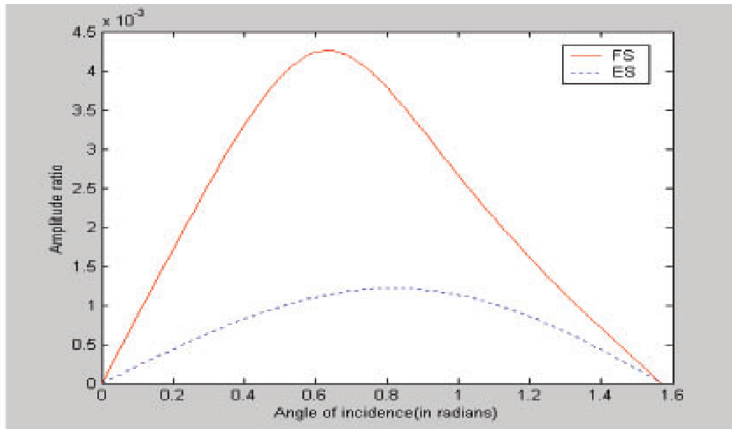

Fig.5– Variation of the amplitude ratio $|Z_4|$ of the transmitted SV-wave for an incident P-wave

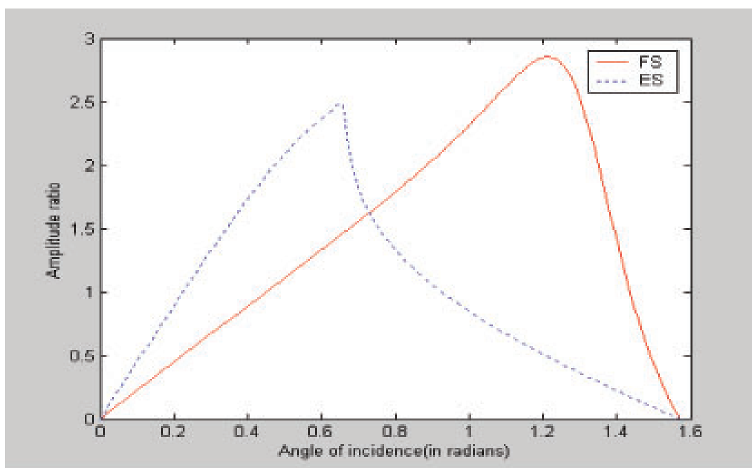

Fig.6– Variation of the amplitude ratio $|Z_1|$ of the reflected P-wave for an incident SV-wave

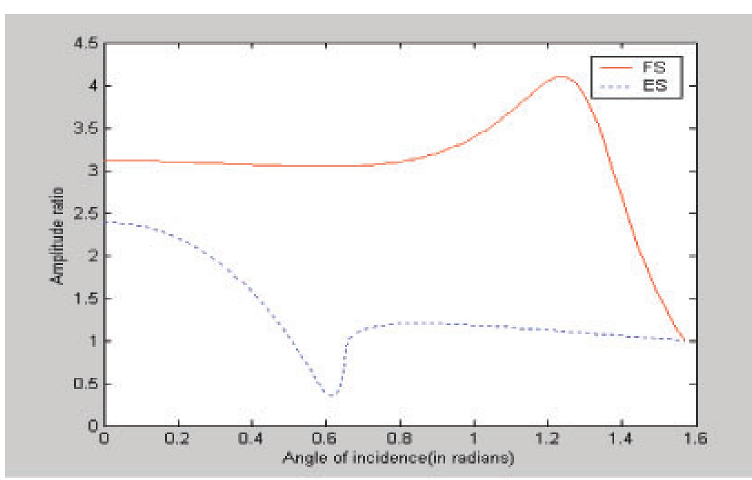

Fig.7– Variation of the amplitude ratio $|Z_2|$ of the reflected SV-wave for an incident SV-wave

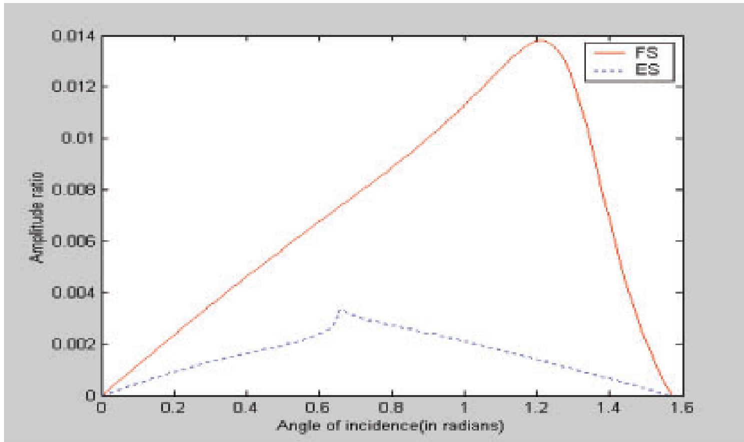

Fig.8– Variation of the amplitude ratio $|Z_3|$ of the transmitted P-wave for an incident SV-wave

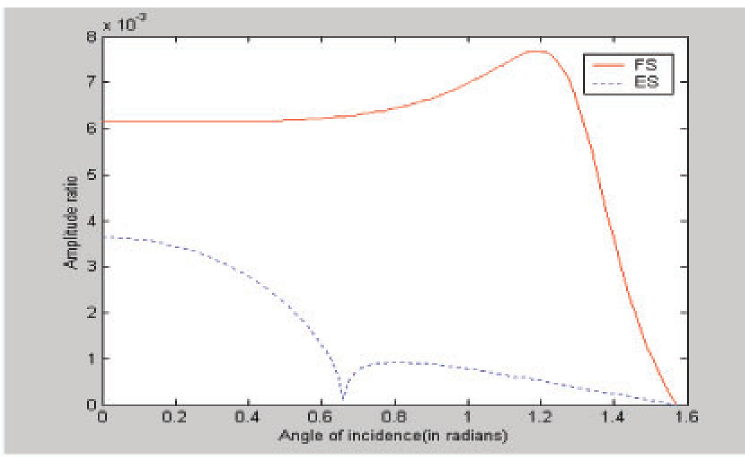

Fig.9– Variation of the amplitude ratio $|Z_4|$ of the transmitted SV-wave for an incident SV-wave

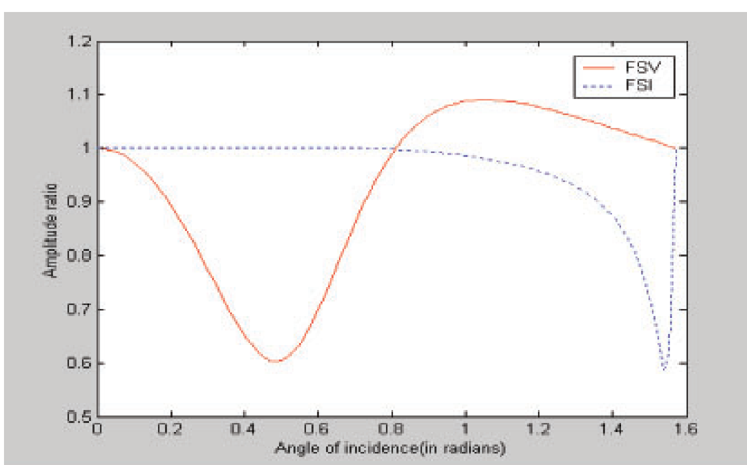

Fig.10– Variation of the amplitude ratio $|Z_1|$ of the reflected P-wave for an incident P-wave

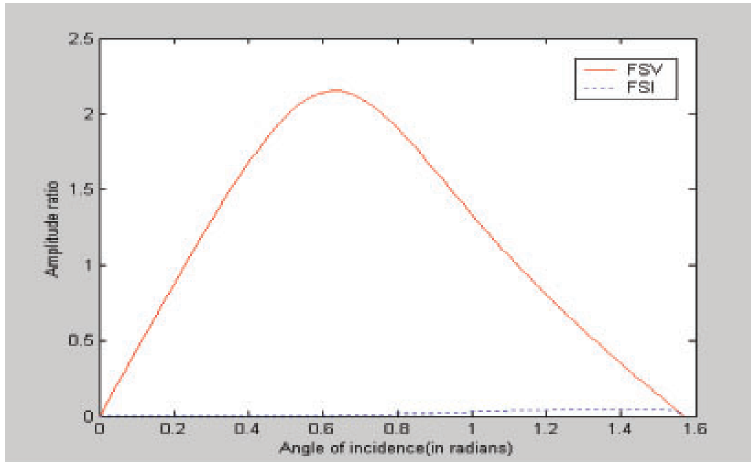

Fig.11– Variation of the amplitude ratio $|Z_2|$ of the reflected SV-wave for an incident P-wave

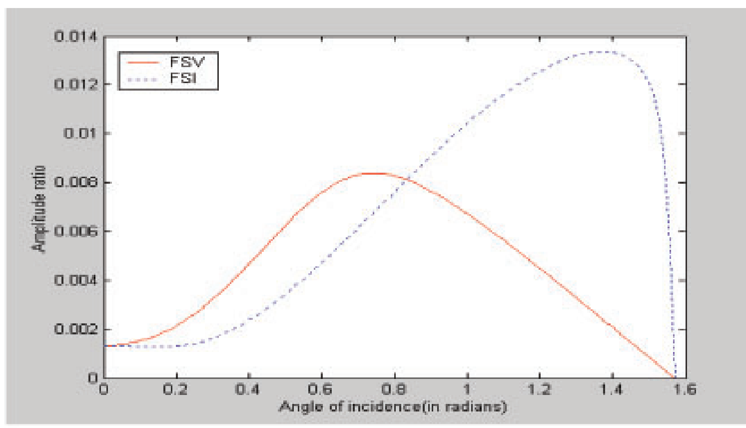

Fig.12– Variation of the amplitude ratio $|Z_3|$ of the transmitted P-wave for an incident P-wave

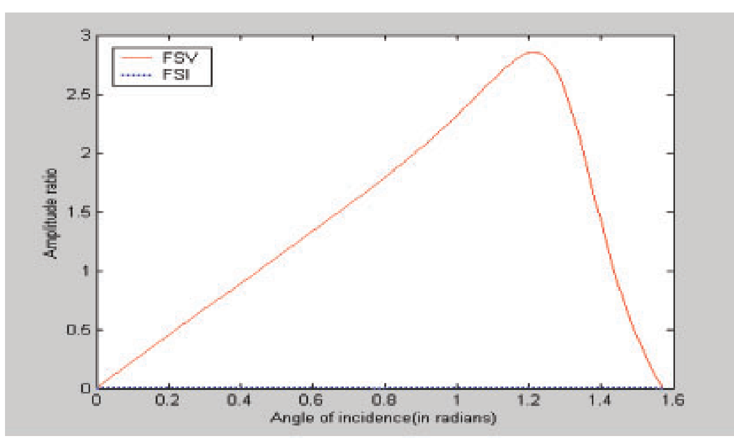

Fig.13– Variation of the amplitude ratio $|Z_1|$ of the reflected P-wave for an incident SV-wave

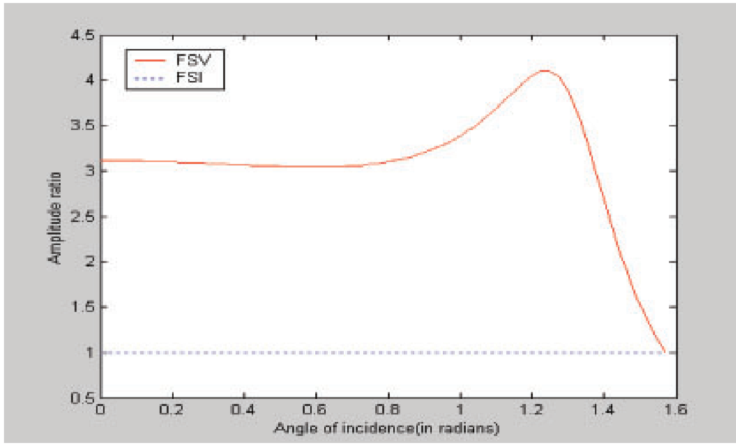

Fig.14– Variation of the amplitude ratio $|Z_2|$ of the reflected SV-wave for an incident SV-wave

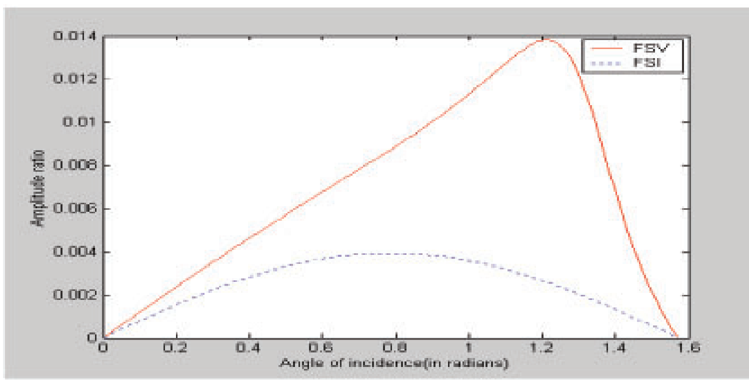

Fig.15– Variation of the amplitude ratio $|Z_3|$ of the transmitted P-wave for an incident SV-wave

Figures (2)-(9) represent the case of either fluid saturated porous half space or empty porous solid half space lying under viscous liquid half space. The case of incident P-wave is shown in figures (2)-(5) and that of incident SV-wave in figures (6)-(9).The solid curves correspond to the case of fluid saturated porous medium whereas the dotted lines correspond to the case of empty porous medium. From figures (2)-(9), the effect of fluid present in the pores of fluid saturated porous half space can be observed. It is clear from these figures that it plays an important role in both the situations whether P-wave is incident or SV-wave is incident. For the case of incident P-wave in figures (2)-(5), it is observed that the value of amplitude ratio in case of reflected P-wave is one for extreme value of incident angle i.e. $\theta = 0^0$ (0 radians), and $\theta = 90^0$ (1.5750 radians),whereas it is zero for the case of reflected SV-wave. In between, the value changes continuously to give absolute maximum with the change in angle of incidence before reaching the ultimate value. Also, it is seen that the magnitude of amplitude ratio in case of reflected SV-wave is large as compared to reflected P-wave. Further, absolute maximum for fluid saturated case is larger than empty porous solid for both the reflected waves. Similar trends are observed from figures (4) and (5) for transmitted P and SV waves with change in curves and values are very very small in comparison to its reflected counterparts. Also at $\theta = 0^0$ (0 radians), the value of amplitude ratio is finite for the case of transmitted P-wave. Similar distribution is noticed from figures (6)-(9) in case of incident SV-wave where the change in magnitude values in different situations is observed with the exception that there is a specific value of at which the curve change its trend suddenly for empty porous solid case in all the four cases of reflected and transmitted P and SV waves.

Figures (10)-(15) show the variation of amplitude ratios when fluid saturated porous half space lies under either the viscous liquid half space or inviscid liquid half space. In these figures solid lines represent the case of viscous liquid and dotted lines for inviscid liquid. From figures (10)-(15), the effect of viscosity of viscous liquid is observed for both cases of incident P-wave and SV-wave. These figures depict the effect of viscosity on amplitude ratios. Further from figures (10)-(15), which depicts the effect of viscosity, it is observed that the curves for reflected P and SV waves remain confined to value 1 or 0 for almost all the values of except for large values of in case of incident P-wave. From the above observations, it is concluded that the amplitudes ratios of various reflected and transmitted waves depend on the angle of incidence of the incident wave, the kind of incident wave and the material properties of the medium through which they travel. The fluid filled in the pores of incompressible fluid saturated porous medium and viscosity of liquid have significant effect on different amplitude ratios. The model presented in this paper is an earth model which is more realistic. Such a situation can be thought of in the earths crust.

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