International Journal of Mathematics And Its Applications
Vol.2 No.4 (2014), pp.25-31.
ISSN: 2347-1557(online)

Minimum Broadcast time for Sierpiński Gasket Rhombus graphs

D. Antony Xavier[†], M. Rosary[†,1] and Andrew Arokiaraj[†]

[†] *Department of Mathematics, Loyola College, Chennai, India.*

Abstract : Broadcasting is a fundamental operation extensively used in various linear algebra algorithms, transitive closure algorithms, database queries and linear programming algorithms. *Sierpiński Gasket Rhombus graph* is formed by identifying two copies of *Sierpiński Gasket graphs* along their side edges. In this paper, we compute the broadcast time in *Sierpiński Gasket Rhombus graph* when either $SR_{n,L,L}$ or $SR_{n,R,R}$ is the source node.

Keywords : Sierpiński Gasket Rhombus graph, broadcasting, broadcast time.

AMS Subject Classification: 05C85, 05C90.

1 Introduction

Broadcasting is the message dissemination process in a communication network where all vertices become informed of a message by calls over lines in the network. Given an undirected graph $G = (V, E)$ consists of a vertex set V and an edge set E. The process of disseminating the information is discussed by the following limitations:

(i). Each call involves two neighbouring vertices.

(ii). Each call requires one time unit.

(iii). Each vertex can take part in atmost one call per time unit.

Broadcast scheme refers to the set of calls used to disseminate the information. Dalal and Metcalfe [6] proposed the multi-destination broadcasting problem. The minimum number of time units required to complete broadcasting

[1]Corresponding author E-Mail:roserosary19@gmail.com (M. Rosary)

from vertex w is called the broadcast time of the vertex w and is denoted by $b(w)$. The broadcast time of a graph G, $b(G)$ is the maximum broadcast time of any vertex w in G. Broadcasting in graphs of bounded degree has been discussed in many papers. A minimum broadcast tree to be a rooted tree with n vertices and root x such that $b(x)=\lceil log_2\ n\rceil$ is given in [3]. In any connected graph G, the dissemination of information from a vertex x determines a rooted spanning tree of G. Hence, in a minimal broadcast graph G, every vertex is the root of a minimum broadcast tree which includes all the vertices of G. Broadcasting also finds an application in Internet messaging, rumours and virus spreading and supercomputing [4].

For surveys of results on broadcasting and other related problems, refer [11, 8]. The problem of finding the minimum broadcast time problem in general graphs is NP-Complete [10] and thus it is not likely to be solved exactly. Broadcasting in trees and grid graphs is studied in [13, 7]. E.A. *Stöhr* has given an upper bound on the broadcast time of the butterfly graph [12]. Hedetniemi et al. has given a detailed discussion of broadcasting and related topics [11]. An exact algorithm and many heuristic algorithms for the Minimum Broadcast Time problem was developed by Scheuermann and Wu [14]. Some of the properties of *Sierpiński Gasket graphs* is widely studied in [15]. Broadcasting in *Sierpiński Gasket graph* is studied in [16]. *Sierpiński Gasket Rhombus graph* is obtained by identifying two copies of *Sierpiński Gasket graphs* along their side edges leading to Rhombus like structure. Topological properties of *Sierpiński Gasket Rhombus graph* is studied in [1]. In the following sections, we study the broadcasting for *Sierpiński Gasket Rhombus graph*.

2 Sierpiński Gasket graph S_n

Definition 2.1 ([15]). *Sierpiński Gasket graphs S_n, $n \geq 1$, are defined geometrically as the graphs whose vertices are the intersection points of the line segments of the finite sierpiński gasket σ_n and line segments of the gasket as edges.*

S_{n+1} consists of three attached copies of S_n which is referred as the *top*, $bottomleft$ and $bottomright$ components of S_{n+1} and denote by $S_{n+1,T}$, $S_{n+1,L}$ and $S_{n+1,R}$ respectively.

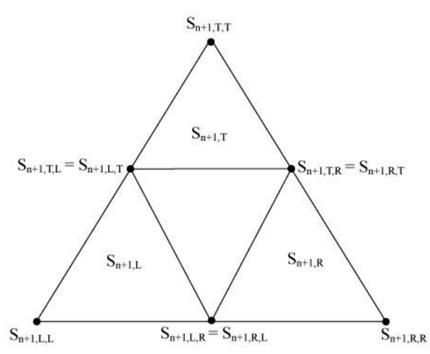

Figure 1. S_{n+1}.

Sierpiński Gasket graphs S_n has $\frac{3}{2}(3^{n-1}+1)$ vertices and 3^n edges. S_n is properly three colorable for each n. The diameter of S_n is 2^{n-1}. The hamiltonicity, pancyclicity, domination numbers and pebbling numbers of S_n are discussed in [15]. The broadcast time for n-dimensional *Sierpiński Gasket graphs* is $2^{n-1} + (n-2)$, $n \geq 4$ [16].

3 Sierpiński Gasket Rhombus SR_n

Definition 3.1 ([1]). *A Path − Sum is a way of combining two graphs by gluing them together along a path. If two graphs G and H, each contains a path of equal length, the Path − Sum of G and H is formed from their disjoint union by identifying pair of vertices in these two paths to form a single shared path.*

Definition 3.2 ([1]). *A Sierpiński Gasket Rhombus of level n [denoted by SR_n], is obtained by identifying the edges in two Sierpiński Gasket graphs S_n along one of their side.*

Sierpiński Gasket Rhombus SR_n is composed of two copies of *Sierpiński Gasket* S_n. SR_n has four extreme vertices namely $SR_{n,T,T}$, $SR_{n,R,R}$, $SR_{n,B,B}$ and $SR_{n,L,L}$. SR_n can be divided into four parts: The top *Sierpiński Gasket graph* of level $n-1$ [denoted by ($S_{n,T}$)], the left *Sierpiński Gasket Rhombus* of level $n-1$ [denoted by ($SR_{n,L}$)], the right *Sierpiński Gasket Rhombus* of level $n-1$ [denoted by ($SR_{n,R}$)] and the bottom *Sierpiński Gasket graph* of level $n-1$ [denoted by ($S_{n,B}$)]. *Sierpiński Gasket Rhombus* SR_n has $3^n-2^{n-1}+2$ vertices and $2{\cdot}3^n-2^{n-1}$ edges. Quotient labeling of SR_n is discussed in [1]. The diameter of SR_n is 2^n, for $n \geq 1$ and it is not Eulerian since it has odd degree vertices. SR_n is hamiltonian and pancyclic for each n [1]. When the center vertex is a source node, an upper bound for broadcasting in *Sierpiński Gasket Rhombus graph* SR_n, $n > 2$ is $b(SR_n) \leq 2^{n-1} + 1$ [2].

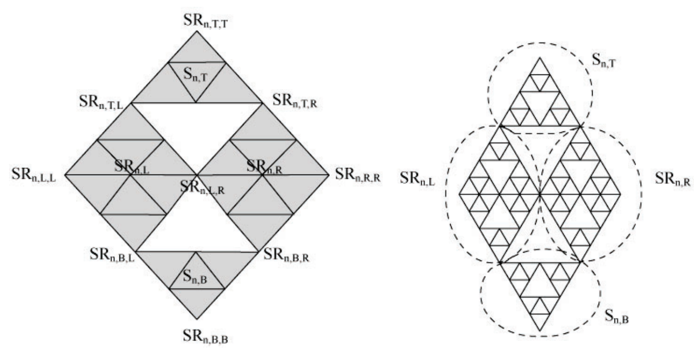

Figure 2. *Sierpiński Gasket Rhombus SR_n.*

We give an algorithm to prove an upper bound for the broadcast time of SR_n.

Procedure BROADCASTING IN SR_n

Input: An n-dimensional *Sierpiński Gasket Rhombus graph* SR_n, $n \geq 4$ when either $SR_{n,L,L}$ or $SR_{n,R,R}$ is the source node.

Algorithm:

Step 1: SR_n consists of two *Sierpiński Gasket graphs* of dimension n say $S'_{n,T}$ and $S'_{n,B}$. We construct a BFS (Breadth-First Search) tree for the top *Sierpiński Gasket graph* of dimension n, $S'_{n,T}$ of SR_n with $SR_{n,L,L}$ as the root. Since bottom *Sierpiński Gasket graph* of dimension n, $S'_{n,B}$ is the mirror image of $S'_{n,T}$, the same BFS tree can be constructed for $S'_{n,B}$. We construct a spanning tree for SR_n by identifying these two BFS trees (See Figure 3).

Step 2: If a vertex in level L_i is labeled as x and has three childrens then label its children in level L_{i+1} from top to bottom as x + 2, x + 1 and x + 3.

Step 3: In $S'_{n,T}$, if a vertex in level L_i is labeled as x and has two childrens then label its children in level L_{i+1} from top to bottom as x + 1 and x + 2. Similarly, in $S'_{n,B}$ if a vertex in L_i is labeled as x and has two childrens then label its children in level L_{i+1} from top to bottom as x + 2 and x + 1. In both $S'_{n,T}$ and $S'_{n,B}$, if a vertex in level L_i is labeled as x and has one children then label its children in level L_{i+1} as x + 1.

Output: The broadcast time of SR_n, $n \geq 4$ is $b(SR_n) \leq 2^{n-1} + (n-1)$.

Proof of Correctness: The extreme left vertex labeled as x at level i, (i = 1,3,...,2n − 1) has three childrens to be labeled as x + 2, x + 1 and x + 3 from top to bottom. In top and bottom *Sierpiński Gasket graph* of dimension n, at level i, i = 3,5,...,2n + 1 has two childrens to be labeled as x + 1 and x + 2 from top to bottom and bottom to top respectively, where as those at level i, i = 2,4,...,2n has one child to be labeled as x + 1. Then all the vertices in SR_n, $n \geq 4$ will be informed in $2^{n-1} + (n-1)$ unit time. Thus, $b(SR_n) \leq 2^{n-1} + (n-1)$.

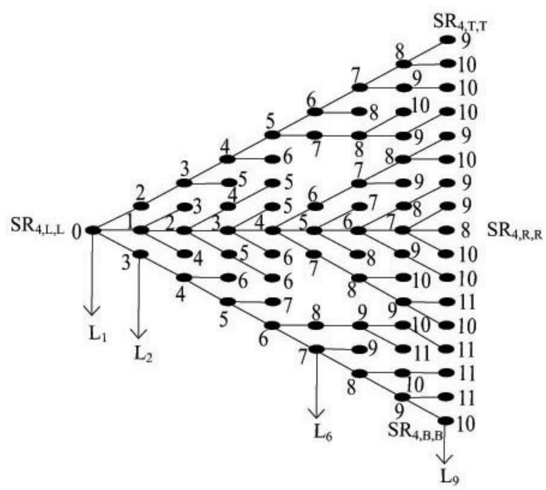

Figure 3. Broadcasting in SR_4.

Lemma 3.1 ([16])**.** *The broadcast time of an n-dimensional Sierpiński Gasket graph* S_n, $b(S_n) \geq 2^{n-1} + 1$, $n \geq 1$

Theorem 3.2. *The broadcast time of an* $n-$*dimensional Sierpiński Gasket Rhombus graph* SR_n, $n \geq 4$ *is* $b(SR_n) \geq 2^{n-1} + (n-1)$ *when either* $SR_{n,L,L}$ *or* $SR_{n,R,R}$ *is a source node.*

Proof. Consider $SR_{n,L,L}$ as the source node with three adjacent vertices namely left, middle and right. At first unit time, the right vertex is informed. In second unit time, the middle and left vertex is informed. SR_n is composed of top and bottom *Sierpiński Gasket graph* of dimension n say, $S'_{n,T}$ and $S'_{n,B}$. By the structure of SR_n, the broadcasting in SR_n from the left extreme vertex $SR_{n,L,L}$ to all vertices of $S'_{n,T}$ and $S'_{n,B}$ must pass through successive lower dimensional *Sierpiński Gasket graphs*, S_i, $1 \leq i \leq n-1$.

Thus, $b(SR_n) \geq 1 + b(S_{n-1}) + b(S_{n-2}) + \ldots + b(S_1)$

$\geq 1 + (2^{n-2} + 1) + (2^{n-3} + 1) + \ldots + (2^1+1) + (2^0+1)$ [Using Lemma 3.1]

$= 2^{n-1} + (n-1)$. □

From the above algorithm and theorem 3.2, we give the following result.

Theorem 3.3. *The broadcast time of an $n-$dimensional Sierpiński Gasket Rhombus graph SR_n, $n \geq 4$ is $b(SR_n)$ $= 2^{n-1} + (n-1)$ when either $SR_{n,L,L}$ or $SR_{n,R,R}$ is a source node.*

Remark 3.4. *When $1 \leq n \leq 3$, the broadcast time for Sierpiński Gasket Rhombus graph SR_n is 2, 4 and 6*

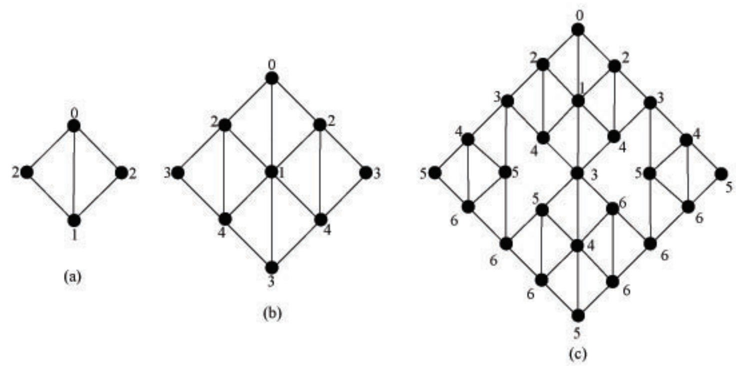

Figure 4. (a)$b(SR_1)$ (b)$b(SR_2)$ (c)$b(SR_3)$

4 Comparison of broadcast time of SR_n and S_n

In this section, we compare the broadcast time of *Sierpiński Gasket Rhombus graph SR_n* with the other well-known graph named as *Sierpiński Gasket* (when the source node for SR_n is either $SR_{n,L,L}$ or $SR_{n,R,R}$ and for S_n is either $S_{n,T,T}$ or $S_{n,L,L}$ or $S_{n,R,R}$). Even though the number of vertices in *Sierpiński Gasket Rhombus graph SR_n* is more than the number of vertices in *Sierpiński Gasket graph S_n*, the broadcast time of *Sierpiński Gasket Rhombus graph SR_n*, $n \geq 4$ is slightly greater than one, when compared to the broadcast time of *Sierpiński Gasket graph S_n*, $n \geq 4$.

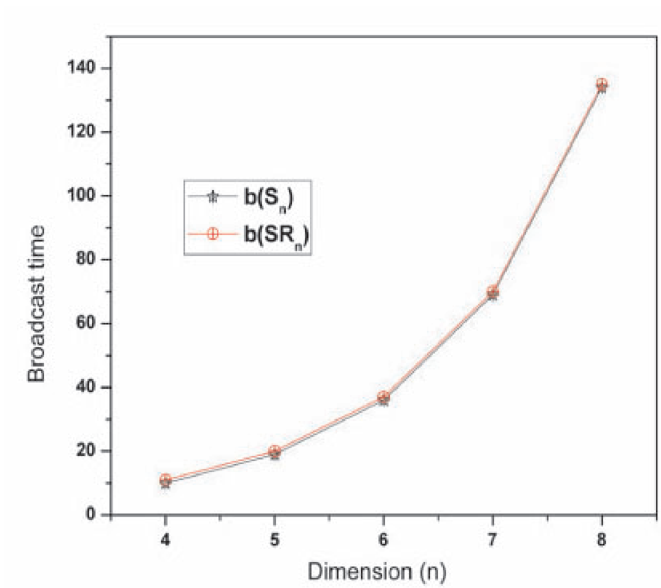

Figure 5. Comparison of broadcast time.

5 Conclusion

In this paper, we find the broadcast time for *Sierpiński Gasket Rhombus graph* SR_n, $n \geq 4$. Finding broadcasting in other interconnection networks is quite challenging. $k-$ broadcasting and other graph theory problems in SR_n is under study.

Acknowledgement

This work is supported by Maulana Azad National Fellowship F1-17.1/2013-14/MANF-2013-14-CHR-TAM-29053 of the University Grants Commission, New delhi, India.

References

[1] D. Antony Xavier, M. Rosary and Andrew Arokiaraj, *Topological properties of Sierpiński Gasket Rhombus Graphs*, International Journal of Mathematics and Soft Computing, 4(2) (2014) 95-104.

[2] D. Antony Xavier, M. Rosary, Christina and Elizabeth Thomas, *Broadcasting in Sierpiński Gasket Rhombus Graphs*, Proceedings of International Conference on Mathematical Sciences, (2014) 521-524.

[3] Arthur Farley, Stephen Hedetniemi, Sandra Mitchell and Andrzej Proskurowski, *NOTE Minimum Broadcast graphs*, Discrete Mathematics, 25 (1979) 189-193.

[4] Canlin Dan Morosan, *Studies of Interconnection networks with application in broadcasting*, Ph.D thesis, (2007).

[5] E. Cockayne and S. Hedetniemi, *A Conjucture concerning broadcasting in m-dimensional grid graphs*, CS-TR-78-14, Computer Science Department, University of Oregon.

[6] Y.K. Dalal and R.M. Metcalfe, *Reverse Path Forwarding of Broadcast Packets*, Communications ofthe ACM, 21(12), 1040-1048.

[7] A.M. Farley and S. T. Hedetniemi, *Broadcasting in grid graphs*, Proceedings 9th S-E Conference on Combinatorics, Graph theory and Computing, FL Congressus Numerantium, (1978) XXI:275-288.

[8] P. Fraigniaud and E. Lazard, *Methods and problems of communication in usual networks*, Discrete Applied Mathematics, 53 (1994) 79-133.

[9] Guy Kortsaz and David Peleg, *Approximation Algorithms for Minimum Time Broadcast*, SIAM Journal on Discrete Methods, 8 (1995) 401427.

[10] M.R. Garery and D.S. Johnson, *Computers and Intractability: A Guide to the theory of NP-Completeness*, W.H. Freeman and Company, (1979).

[11] S.T. Hedetniemi, S.M. Hedetniemi and A.L. Liestman, *A Survey of broadcasting and gossiping in Communication networks*, Networks, 18 (1988) 319-349.

[12] E.A. Stöhr, *Broadcasting in the Butterfly network*, Information Processing Letters, 39 (1991) 41-43.

[13] P.J. Slater, E.J. Cockayne and T. Hedetniemi, *Information dissemination in trees*, SIAM J. on Comput., 10 (1981).

[14] P. Scheuermann and G. Wu, *Heuristic Algorithms for Broadcasting in Point-to-Point Computer Networks*, IEEE Trans. on Computers, Vol. C-33, No. 9, 804-811.

[15] A.M. Teguia and A.P. Godbole, *Sierpiński gasket graphs and some of their properties*, Australas. J. Combin, 35 (2006) 181-192.

[16] A. Shanthakumari, *Cycle cover and broadcasting in certain interconnection networks*, Ph.D thesis, (2012).