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# Topological Energies of Some Standard Graphs

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**Abstract:** Let $G$ be simple, finite, undirected graph (molecular graph) with vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\}$ and edge set $E = \{e_1, e_2, e_3, \ldots, e_n\}$. The energy concept originated in chemistry and was introduced by I Gutman in 1978. Similar to the energies like dominating energy, minimum dominating energy, minimum total dominating energy, Zagreb energy is having its own importance in the study of physic-chemical properties of chemical compound that are often modeled by means of topological indices. In this paper, we find, first Zagreb energy [1] (introduced by Gutman), second Zagreb energy and hyper Zagreb energy of some standard graphs like Complete graph, complete bipartite graph, star graph and crown graph.

**MSC:** 05C69.

**Keywords:** Graph, Topological Indices, Topological Energy, Zagreb matrix, Zagreb energy.

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## 1. Introduction

Mathematical Chemistry is a part of theoretical chemistry [2-4] where we analyze and predict the chemical structure using mathematical concept. Again chemical graph theory is a branch of mathematical chemistry. By applying the graph theoretical concept chemical phenomenon can be modeled. Throughout this paper, graph (molecular graph) referred is simple, finite, undirected and connected one. Let $G$ be graph with vertex set $V(G) = \{v_1, v_2, v_3, \ldots, v_n\}$ and edge set $E(G) = \{e_1, e_2, e_3, \ldots, e_n\}$. If $v_i$ and $v_j$ are the adjacent vertices in $G$ then $v_iv_j$ denotes an edge connecting vertices $v_i$ and $v_j$. The properties of chemical compounds are often studied and modeled by molecular graph based structure descriptors called as topological indices. [5-11] Particularly the degree based topological indices are widely studied and are called as Zagreb indices. Corresponding to Zagreb indices [12, 13] the Zagreb matrices [15] are defined. And they are

$$\text{First Zagreb matrix} = Z^{(1)} = z_{ij} = \begin{cases} d_{ii}^2, & \text{for } v_i \in V(G) \\ 0, & \text{otherwise} \end{cases} \tag{1}$$

$$\text{Second Zagreb matrix} = Z^{(2)} = z_{ij} = \begin{cases} d_id_j, & \text{for } v_iv_j \in E(G) \\ 0, & \text{for } i = j \\ 0, & \text{otherwise} \end{cases} \tag{2}$$

$$\text{Hyper Zagreb matrix} = Z^{(H)} = z_{ij} = \begin{cases} (d_i + d_j)^2, & \text{for } {}_iv_j \in E(G) \\ 0, & \text{for } i = j \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

Using these Zagreb matrices, Zagreb energies [14] of standard graphs are obtained.

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## 2. Zagreb Energy of a Graph

For a simple connected graph $G$, using the definition (1), (2) and (3) of Zagreb matrices are constructed. Let $Z^{(n)} = (z_{ij})$ for $n = 1, 2$ and $H$, represents first Zagreb matrix, second Zagreb matrix and hyper Zagreb matrix of a graph $G$ respectively. Let $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ be the eigen values of particular Zagreb matrix of a graph $G$. The values are assumed to be in the increasing order, that is $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_n$. Since $Z(G)$ is real and symmetric. Its eigen values are real number. The Zagreb energy $ZE(G)$ of graph is defined to the sum of absolute values of its eigen values of graph $G$ That is $ZE(G) = \sum\limits_{1}^{n} |\lambda_i|$.

**Example 2.1.** *Let $G$ be the graph with 4-vertices say $v_1, v_2, v_3, v_4$ as shown in the Figure 1*

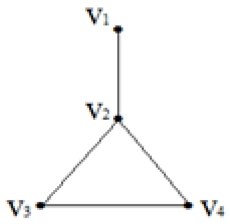


**Figure 1.** **Simple Graph with 4 vertices**

*The first Zagreb Matrix of a graph $G$ is* $Z^{(1)}M(G) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 9 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ *since $Z^1M(G)$ is a diagonal matrix, diagonal elements represent eigen values of the matrix. Therefore first Zagreb energy,* $Z^{(1)}E(G) = |1|(1) + |9|(1) + |4|(2) = 18$.

**Example 2.2.** *For the graph $G$ shown in Figure 1. Second Zagreb matrix of $G$ is* $Z^{(2)}M(G) = \begin{bmatrix} 0 & 3 & 0 & 0 \\ 3 & 0 & 6 & 6 \\ 0 & 6 & 0 & 4 \\ 0 & 6 & 4 & 0 \end{bmatrix}$ *reducing to a diagonal matrix* $Z^{(2)}M(G) = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix}$ *the eigen values are $3, 3, 4$ and $4$. Therefore second Zagreb energy,* $Z^{(2)}E(G) = |3|(2) + |4|(2) = 14$.

**Example 2.3.** *For the graph $G$ shown in Figure 1. Hyper Zagreb Matrix of $G$ is* $Z^{(H)}M(G) = \begin{bmatrix} 0 & 16 & 0 & 0 \\ 16 & 0 & 25 & 25 \\ 0 & 25 & 0 & 16 \\ 0 & 25 & 16 & 0 \end{bmatrix}$ *reducing to a diagonal matrix* $Z^{(H)}M(G) = \begin{bmatrix} 16 & 0 & 0 & 0 \\ 0 & 16 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 16 \end{bmatrix}$. *Therefore hyper Zagreb energy,* $Z^{(H)}E(G) = |16|(4) = 64$.

## 3. First Zagreb Energy of Some Standard Graphs

Based on the first Zagreb matrix, First Zagreb energy for complete graphs, complete bipartite graph, star graph and Crown graph are obtained in the following theorems.

**Theorem 3.1.** *If $K_p$ is a complete graph with $p \geq 3$ vertices then $Z^{(1)}E(K_p) = p(p-1)^2$.*

*Proof.* Let $K_p$ be a complete graph with vertex set $V = \{v_1, v_2, \ldots, v_p\}$. The first Zagreb Matrix corresponding to $K_p$ is

$$Z^{(1)}M(K_p) = \begin{bmatrix} (p-1)^2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & (p-1)^2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & (p-1)^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & (p-1)^2 & 0 \\ 0 & 0 & 0 & \cdots & 0 & (p-1)^2 \end{bmatrix}_{p\times p}$$

Since matrix is a diagonal matrix the elements along the diagonal represents eigen values. The characteristic polynomial is given by $[\lambda - (p-1)^2]^p = 0$. Therefore eigen values of first Zagreb matrix are $\lambda = (p-1)^2 \ \ \{p - times\}$. Then, first Zagreb energy of $K_p$ is $Z^{(1)}E(K_p) = \left|(p-1)^2\right|(p) = p(p-1)^2$. □

**Theorem 3.2.** *If $K_{p,p}$ is a complete bipartite graph with $p \geq 2$ vertices then $Z^{(1)}E(K_{p,p}) = 2p^3$.*

*Proof.* Let $V = \{v_1, v_2, \ldots, v_p, u_1, u_2, \ldots, u_p\}$. be the vertex set of complete bipartite graph $K_{p,p}$, The first Zagreb Matrix corresponding to $K_{p,p}$ is

$$Z^{(1)}M(K_{p,p}) = \begin{bmatrix} p^2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & p^2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & p^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & p^2 & 0 \\ 0 & 0 & 0 & \cdots & 0 & p^2 \end{bmatrix}_{2p\times 2p}$$

The characteristic polynomial of the matrix is given by $[\lambda - p^2]^{2p} = 0$. Eigen values of first Zagreb matrix $Z^{(1)}M(K_{p,p})$ are $\lambda = p^2\{2p - times\}$. Then first Zagreb energy of $K_{p,p}$ is $Z^{(1)}E(K_{p,p}) = |p^2|2p = 2p^3$. □

**Theorem 3.3.** *If $K_{1,p-1}$ is a star graph with $p \geq 3$ vertices then $Z^{(1)}E(K_{1,p-1}) = p(p-1)$.*

*Proof.* Let $K_{1,p-1}$ be a star graph with vertex set $V = \{v_1, v_2, \ldots, v_p\}$. First Zagreb Matrix corresponding to $K_{1,p-1}$ is

$$Z^{(1)}M(K_{1,p-1}) = \begin{bmatrix} (p-1)^2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}_{2p\times 2p}$$

The characteristic polynomial of the matrix is given by $\left[\lambda - (p-1)^2\right](\lambda - 1)^{p-1} = 0$. Eigen values of first Zagreb matrix $Z^{(1)}M(K_{1,p-1})$ are $\lambda = (p-1)^2 \ \ \{once\}$ and $\lambda = 1 \ \ \{(p-1) \ \ times\}$. Then first Zagreb energy of $K_{1,p-1}$ is $Z^{(1)}E(K_{1,p-1}) = \left|(p-1)^2\right|(1) + |1|(p-1) = p(p-1)$. □

**Theorem 3.4.** *If $S_p$ is a crown graph with $p \geq 4$ vertices then* $Z^{(1)}E\left(S_p\right) = p\left(\frac{p}{2}-1\right)^2$.

*Proof.* Let $S_p$ be a crown graph with vertex set $V = \{v_1, v_2, \ldots, v_p\}$. First Zagreb Matrix corresponding to $S_p$ is

$$Z^{(1)}M\left(S_p\right) = \begin{bmatrix} \left(\frac{p}{2}-1\right)^2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \left(\frac{p}{2}-1\right)^2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \left(\frac{p}{2}-1\right)^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & \left(\frac{p}{2}-1\right)^2 & 0 \\ 0 & 0 & 0 & \cdots & 0 & \left(\frac{p}{2}-1\right)^2 \end{bmatrix}_{p\times p}$$

The characteristic polynomial of the matrix is given by $\left[\lambda - \left(\frac{p}{2}-1\right)^2\right]^p = 0$. Eigen values of first Zagreb matrix of $Z^{(1)}M\left(S_p\right)$ are $\lambda = \left(\frac{p}{2}-1\right)^2\{ptimes\}$. Then first Zagreb energy of $S_p$ is $Z^{(1)}E\left(S_p\right) = \left|\left(\frac{p}{2}-1\right)^2\right|(p - times) = p\left(\frac{p}{2}-1\right)^2$.
**Graphically:** The first Zagreb energy of $K_p$, $K_{p,p}$, $K_{1,p-1}$ and $S_p$ are as follows

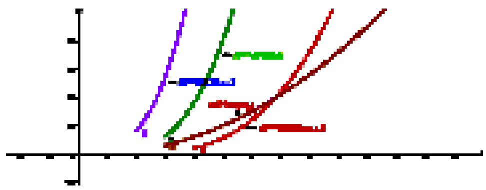

**Figure 2.** **Comparison of first Zagreb energy of standard graphs**

Graph shows that First Zagreb energy corresponding to complete bipartite graph increases rapidly with the increase in the number of vertices as compared to first Zagreb energy of other graphs. ☐

## 4. Second Zagreb Energy of Some Standard Graphs

By the definition of second Zagreb matrix, second Zagreb energy for complete graph, complete bipartite graph, star graph and Crown graph are obtained in the following theorems.

**Theorem 4.1.** *If $K_p$ is a complete graph with $p \geq 3$ vertices then* $Z^{(2)}E\left(K_p\right) = 2\left(p-1\right)^3$.

*Proof.* For $K_p$ being a complete graph with vertex set $V = \{v_1, v_2, \ldots, v_p\}$. Second Zagreb Matrix corresponding to $K_p$ is

$$Z^{(2)}M\left(K_p\right) = \begin{bmatrix} 0 & A & A & \cdots & A & A \\ A & 0 & A & \cdots & A & A \\ A & A & 0 & \cdots & A & A \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ A & A & A & \cdots & 0 & A \\ A & A & A & \cdots & A & 0 \end{bmatrix}_{p\times p}$$

where $A = (p-1)^2$. The characteristic polynomial is given by $(-1)^p[\lambda + (p-1)^2]^{p-1}[\lambda - (p-1)^3] = 0$. Eigen values of Second Zagreb matrix are $\lambda = -(p-1)^2\{(p-1)\ times\}$ And $\lambda = (p-1)^3\ \{once\}$. Therefore second Zagreb energy of $K_p$ is $Z^{(2)}E\left(K_p\right) = \left|(p-1)^2\right|(p-1) + \left|(p-1)^3\right|(1) = 2(p-1)^3$. ☐

**Theorem 4.2.** *If $K_{p,p}$ is a complete bipartite graph with $p \geq 2$ vertices then $Z^{(2)}E\left(K_{p,p}\right) = 2p^3$.*

*Proof.* For $K_{p,p}$ being a complete bipartite graph with vertex set $V = \{v_1, v_2, \ldots, v_p, u_1, u_2, \ldots, u_p\}$. Second Zagreb Matrix corresponding to $K_{p,p}$ is

$$Z^{(2)}M\left(K_{p,p}\right) = \begin{bmatrix} p^2 & 0 & 0 & \cdots & 0 & 0 \\ 0 & p^2 & 0 & \cdots & 0 & 0 \\ 0 & 0 & p^2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & p^2 & 0 \\ 0 & 0 & 0 & \cdots & 0 & p^2 \end{bmatrix}_{2p \times 2p}$$

The characteristic polynomial of the matrix is given by $\lambda^{2(p-1)}[\lambda^2 - (\lambda^3)^2] = 0$. Eigen values of second Zagreb matrix $Z^{(2)}M\left(K_{p,p}\right)$ are $\lambda = 0 \quad \{2\left(p-1\right)\ times\}$, $\lambda = (p^3)^2 \quad \{once\}$ and $\lambda = -(p^3)^2 \quad \{once\}$. Then second Zagreb energy of $K_{p,p}$ is $Z^{(2)}E\left(K_{p,p}\right) = |0|\left(2\left(p-1\right)\right) + \left|p^3\right|(1) + \left|-p^3\right|(1) = 2p^3$. $\square$

**Theorem 4.3.** *If $K_{1,p-1}$ is a star graph with $p \geq 3$ vertices then $Z^{(2)}E\left(K_{1,p-1}\right) = 2(p-1)\sqrt{p-1}$.*

*Proof.* Let $V = \{v_1, v_2, \ldots, v_p\}$. be the vertex set of star graph $K_{1,p-1}$. The second Zagreb Matrix corresponding to $K_{1,p-1}$ is

$$Z^{(2)}M\left(K_{1,p-1}\right) = \begin{bmatrix} (p-1)^2 & (p-1)^2 & (p-1)^2 & \cdots & (p-1)^2 & (p-1)^2 \\ (p-1)^2 & 0 & 0 & \cdots & 0 & 0 \\ (p-1)^2 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ (p-1)^2 & 0 & 0 & \cdots & 0 & 0 \\ (p-1)^2 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{p \times p}$$

The characteristic polynomial of the matrix is given by $\lambda^{p-2}\left[\lambda - (p-1)\sqrt{p-1}\right]\left[\lambda + (p-1)\sqrt{p-1}\right] = 0$. Eigen values of second Zagreb matrix $Z^{(2)}M\left(K_{1,p-1}\right)$ are $\lambda = 0 \quad \{(p-2)\ \ times\}$, $\lambda = (p-1)\sqrt{p-1} \quad \{once\}$ and $\lambda = -\left(p-1\right)\sqrt{p-1} \quad \{once\}$. Then second Zagreb energy of $K_{1,p-1}$ is

$$\begin{aligned} Z^{(2)}E\left(K_{1,p-1}\right) &= |0|\left((p-2)\right) + \left|(p-1)\sqrt{p-1}\right|(1) + \left|-\left(p-1\right)\sqrt{p-1}\right|(1) \\ &= 2\left(p-1\right)\sqrt{p-1} \end{aligned}$$

$\square$

**Theorem 4.4.** *If $S_p$ is a crown graph with $p \geq 4$ vertices then $Z^{(2)}E\left(S_p\right) = 4\left(\frac{p}{2}-1\right)^3$.*

*Proof.* Let $S_p$ be a crown graph with vertex set $V = \{v_1, v_2, \ldots, v_p\}$. Second Zagreb Matrix corresponding to $S_p$ is

$$Z^{(2)}M\left(S_p\right) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & \left(\frac{p}{2}-1\right)^2 & \left(\frac{p}{2}-1\right)^2 \\ 0 & 0 & 0 & \cdots & \left(\frac{p}{2}-1\right)^2 & 0 & \left(\frac{p}{2}-1\right)^2 \\ 0 & 0 & 0 & \cdots & \left(\frac{p}{2}-1\right)^2 & \left(\frac{p}{2}-1\right)^2 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \left(\frac{p}{2}-1\right)^2 & \left(\frac{p}{2}-1\right)^2 & \cdots & 0 & 0 & 0 \\ \left(\frac{p}{2}-1\right)^2 & 0 & \left(\frac{p}{2}-1\right)^2 & \cdots & 0 & 0 & 0 \\ \left(\frac{p}{2}-1\right)^2 & \left(\frac{p}{2}-1\right)^2 & 0 & \cdots & 0 & 0 & 0 \end{bmatrix}_{p \times p}$$

The characteristic polynomial of the matrix is $\left[\lambda^2-\left(\frac{p}{2}-1\right)^2\right]^{\left(\frac{p}{2}-1\right)}\left[\lambda^2-\left(\frac{p}{2}-1\right)^3\right]^2=0$. Eigen values of second Zagreb matrix of $Z^{(2)}M\left(S_p\right)$ are

$$\lambda=\left(\frac{p}{2}-1\right)^2 \quad \left\{\left(\frac{p}{2}-1\right) \ times\right\}$$
$$\lambda=-\left(\frac{p}{2}-1\right)^2 \quad \left\{\left(\frac{p}{2}-1\right) \ times\right\}$$
$$\lambda=\left(\frac{p}{2}-1\right)^3 \quad \{once\}$$
$$\lambda=-\left(\frac{p}{2}-1\right)^3 \quad \{once\}$$

Then second Zagreb energy of $S_p$ is

$$\begin{aligned}Z^{(2)}E\left(S_p\right)&=\left|\left(\frac{p}{2}-1\right)^2\right|\left(\frac{p}{2}-1\right) \ times+\left|-\left(\frac{p}{2}-1\right)^2\right|\left(\frac{p}{2}-1\right) \ times+\left|\left(\frac{p}{2}-1\right)^3\right|(1)+\left|\left(\frac{p}{2}-1\right)^3\right|(1)\\&=4\left(\frac{p}{2}-1\right)^3\end{aligned}$$

**Graphically:** The second Zagreb energy of $K_p$, $K_{p,p}$, $K_{1,p-1}$ and $S_p$ are as follows

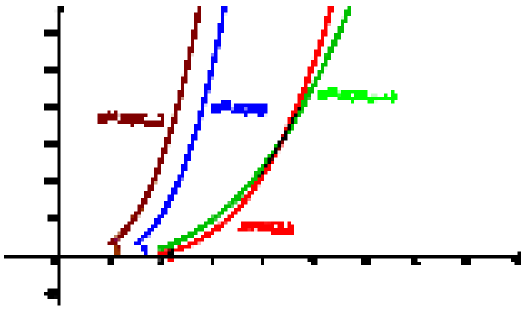

**Figure 3.** **Comparison of second Zagreb energy of standard graphs**

Graph shows that second Zagreb energy corresponding to complete bipartite graph increases rapidly with the increase in the number of vertices as compared to second Zagreb energy of other graphs. □

## 5. Hyper Zagreb energy of some standard graphs

Using the definition of Hyper Zagreb matrix, Hyper Zagreb energry for complete graph, complete bipartite graph, star graph and Crown graph are obtained in the following theorems.

**Theorem 5.1.** *If $K_p$ is a complete graph with $p \geq 3$ vertices then $Z^{(H)}E\left(K_p\right)=8(p-1)^3$.*

*Proof.* For $K_p$ being a complete graph with vertex set $V=\{v_1,v_2,\ldots,v_p\}$. Hyper Zagreb Matrix corresponding to $K_p$ is

$$Z^{(H)}M\left(K_p\right)=\begin{bmatrix}0&B&B&\cdots&B&B\\B&0&B&\cdots&B&B\\B&B&0&\cdots&B&B\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots\\B&B&B&\cdots&0&B\\B&B&B&\cdots&B&0\end{bmatrix}_{p\times p}$$

where $B=[2\,(p-1)]^2$. The characteristic polynomial is given by $(-1)^p[\lambda+\{2(p-1)\}^2]^{p-1}[\lambda-(p-1)\{2(p-1)\}^3]=0$. Eigen values of Hyper Zagreb matrix are $\lambda=-\{2\,(p-1)\}^2\;\;\{(p-1)\;\;times\}$ and $\lambda=(p-1)\,\{2(p-1)\}^3\;\;\{once\}$. Therefore Hyper Zagreb energy of $K_p$ is $Z^{(H)}E\,(K_p)=\left|-2(p-1)^2\right|(p-1)+\left|(p-1)\,\{2(p-1)\}^3\right|(1)=8(p-1)^3$. □

**Theorem 5.2.** *If $K_{p,p}$ is a complete bipartite graph with $p\geq 2$ vertices then $Z^{(H)}E\,(K_{p,p})=8p^3$.*

*Proof.* For $K_{p,p}$ being a complete bipartite graph with vertex set $V=\{v_1,v_2,\ldots,v_p,u_1,u_2,\ldots,u_p\}$. Hyper Zagreb Matrix corresponding to $K_{p,p}$ is

$$Z^{(H)}M\,(K_{p,p})=\begin{bmatrix}0&0&0&\cdots&C&C&C\\0&0&0&\cdots&C&C&C\\0&0&0&\cdots&C&C&C\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots\\C&C&C&\cdots&0&0&0\\C&C&C&\cdots&0&0&0\\C&C&C&\cdots&0&0&0\end{bmatrix}_{2p\times 2p}$$

where $C=p[2p]^2$. The characteristic polynomial is given by $\lambda^{2p-2}[\lambda-(p)\{2p\}^2][\lambda+(p)\{2p\}^2]=0$. Eigen values of Hyper Zagreb matrix are $\lambda=0\;\;\{(2p-2)\;\;times\}$, $\lambda=p\{2p\}^2\;\;\{once\}$ and $\lambda=-p\{2p\}^2\;\;\{once\}$ Therefore Hyper Zagreb energy of $K_{p,p}$ is $Z^{(H)}E\,(K_p)=|0|\,(2p-2)+\left|(p)\,\{2p\}^2\right|(1)+\left|-(p)\,\{2p\}^2\right|(1)=8p^3$. □

**Theorem 5.3.** *If $K_{1,p-1}$ is a star graph with $p\geq 3$ vertices then $Z^{(H)}E\,(K_{1,p-1})=2p^2\sqrt{p-1}$.*

*Proof.* For $K_{1,p-1}$ being a star graph with vertex set $V=\{v_1,v_2,\ldots,v_p\}$. Hyper Zagreb Matrix corresponding to $K_{1,p-1}$ is

$$Z^{(H)}M\,(K_{1,p-1})=\begin{bmatrix}0&p^2&p^2&\cdots&p^2&p^2&p^2\\p^2&0&0&\cdots&0&0&0\\p^2&0&0&\cdots&0&0&0\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots\\p^2&0&0&\cdots&0&0&0\\p^2&0&0&\cdots&0&0&0\\p^2&0&0&\cdots&0&0&0\end{bmatrix}_{p\times p}$$

The characteristic polynomial is $(-1)^p\lambda^{p-2}[\lambda-p^2\sqrt{p-1}][\lambda+p^2\sqrt{p-1}]=0$. Eigen values of Hyper Zagreb matrix are $\lambda=0\;\;\{(p-2)\;\;times\}$, $\lambda=p^2\sqrt{p-1}\;\;\{once\}$ and $\lambda=-p^2\sqrt{p-1}\;\;\{once\}$. Therefore Hyper Zagreb energy of $K_{1,p-1}$ is $Z^{(H)}E\,(K_p)=|0|\,(p-2)+\left|p^2\sqrt{p-1}\right|(1)+\left|-p^2\sqrt{p-1}\right|(1)=2p^2\sqrt{p-1}$. □

**Theorem 5.4.** *If $S_p$ is a crown graph with $p\geq 4$ vertices then, $Z^{(H)}E\,(S_p)=2p^3-12p^2+24p-16$.*

*Proof.* Let $S_p$ is a crown graph with vertex set $V=\{v_1,v_2,\ldots,v_p\}$. Hyper Zagreb Matrix corresponding to $S_p$ is

$$Z^{(H)}M\,(S_p)=\begin{bmatrix}0&0&0&\cdots&0&(p-2)^2&(p-2)^2\\0&0&0&\cdots&(p-2)^2&0&(p-2)^2\\0&0&0&\cdots&(p-2)^2&(p-2)^2&0\\\vdots&\vdots&\vdots&\ddots&\vdots&\vdots&\vdots\\0&(p-2)^2&(p-2)^2&\cdots&0&0&0\\(p-2)^2&0&(p-2)^2&\cdots&0&0&0\\(p-2)^2&(p-2)^2&0&\cdots&0&0&0\end{bmatrix}_{p\times p}$$

The characteristic polynomial of the matrix is

$$
\begin{aligned}
\left[\lambda-\left\{2\left(\frac{p}{2}-1\right)\right\}^2\right]^{\left(\frac{p}{2}-1\right)}\left[\lambda+\left\{2\left(\frac{p}{2}-1\right)\right\}^2\right]^{\left(\frac{p}{2}-1\right)} &= 0 \\
&= \left[\lambda-(p^2-4p+4)\right]^{\left(\frac{p}{2}-1\right)}\left[\lambda+(p^2-4p+4)\right]^{\left(\frac{p}{2}-1\right)} \\
&\quad \left[\lambda+\left(\frac{p^3}{2}-3p^2+6p-4\right)\right]\left[\lambda-\left(\frac{p^3}{2}-3p^2+6p-4\right)\right] \\
&= 0
\end{aligned}
$$

Eigen values of Hyper Zagreb matrix of $Z^{(H)}M\left(S_p\right)$ are

$$
\begin{aligned}
\lambda &= \left(p^2-4p+4\right) \quad \left\{\left(\frac{p}{2}-1\right)\ times\right\}, \\
\lambda &= -\left(p^2-4p+4\right) \quad \left\{\left(\frac{p}{2}-1\right)\ times\right\}, \\
\lambda &= \left(\frac{p^3}{2}-3p^2+6p-4\right) \quad \{once\}, \\
\lambda &= -\left(\frac{p^3}{2}-3p^2+6p-4\right) \quad \{once\ \}
\end{aligned}
$$

Then Hyper Zagreb energy of $S_p$ is

$$
\begin{aligned}
Z^{(H)}E\left(S_p\right) &= \left|\left(p^2-4p+4\right)\right|\left(\frac{p}{2}-1\right)\ times+\left|-\left(p^2-4p+4\right)\right|\left(\frac{p}{2}-1\right)\ times \\
&\quad +\left|\left(\frac{p^3}{2}-3p^2+6p-4\right)\right|(1)+\left|-\left(\frac{p^3}{2}-3p^2+6p-4\right)\right|(1) \\
&= 2p^3-12p^2+24p-16
\end{aligned}
$$

**Graphically:** The Hyper Zagreb energy of $K_p$, $K_{p,p}$, $K_{1,p-1}$ and $S_p$ are as follows

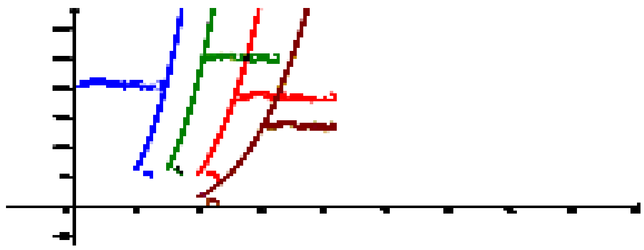

**Figure 4.** **Comparison of Hyper Zagreb energy of standard graphs**

Graph shows that Hyper Zagreb energy corresponding to complete bipartite graph increases rapidly with the increase in the number of vertices as compared to Hyper Zagreb energy of other graphs. First Zagreb energy, Second Zagreb energy and Hyper Zagreb energy of $K_p$, $K_{p,p}$, $K_{1,p-1}$ and $S_p$ are together shown in the following table

| | First Zagreb Energy | Second Zagreb Energy | Hyper Zagreb Energy |
|---|---|---|---|
| Complete Graph | $p(p-1)^2$ | $2(p-1)^3$ | $8(p-1)^3$ |
| Complete Bipartite graph | $2p^3$ | $2p^3$ | $8p^3$ |
| Star Graph | $p(p-1)$ | $2(p-1)\sqrt{p-1}$ | $2p^2\sqrt{p-1}$ |
| Crown Graph | $p\left(\frac{p}{2}-1\right)^2$ | $4\left(\frac{p}{2}-1\right)^3$ | $2p^3-12p^2+24p-16$ |

**Table 1.** **First, Second and Hyper Zagreb energies of standard graphs**

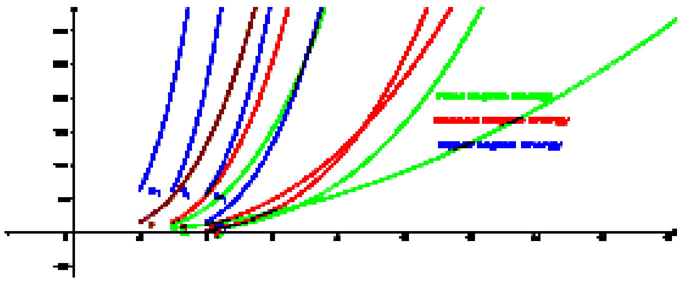

**Figure 5.** **Graphically Comparison of Zagreb energies of standard graphs**

□

## 6. Conclusion

Here we have computed First Zagreb energy, Second Zagreb energy and Hyper Zagreb energy of some standard graphs and they have been compared graphically.

### References


[1] Nadar Jafari, Akbar Jahanbani and Ivan Gutman, *Zagreb energy and Zagreb Estrada index of graph*, MATCH Communication in Mathematical and In Computer Chemistry, 79(2018), 371-386.

[2] Gutman and Trinajstic, *Graphs and molecular orbital's*, Chem. Phy. Lett., 17, 535-538.

[3] Mehar Ali Malik and Muhammad Imran, *On Multiple Zagreb Indices of* $TiO_2$ *Nanotubes*, Acta Chim., 62(2015), 973-976.

[4] M. Eliasi, A. Iranmasnesh and I. Gutman, *Multiplicative Versions of First Zagreb Index*, MATCH Communication in Mathematical and In Computer Chemistry, 68(2012), 217-230.

[5] A. R. Asharafi, A. Hamzeh and S. Hossein-zadeh, *Calculation of some topological indices of splices and links of graphs*, Jour. of Appl. Math. and Informatics, 9(1)(2011), 327-335

[6] G. H. Shirdel, H. Rezapour and A. M. Sayadi, *The Hyper Zagreb index of Graph Operation*, Iranian Journal of Mathematical Chemistry, 4(2)(2013), 213-220.

[7] Ivan Gutman and Ch. Kinkar, *The First Zagreb Index after 30 years*, MATCH Commun. Math. Comut. Chem., 50(2003), 103-112.

[8] M. R. Farahani, *On Multiple Zagreb indices of Circumcoronene Homologous series of Benzenoid*, Chemical Physics Research Journal, 7(2)(2014), 277-28.

[9] Mohamed Essalih, Mohamed El Marraki and Gabr El hargi, *Calculation of Some Topological Indices of Graphs* , Journal of Theoretical and Applied Technology, 30(2)(2011), 122-127.

[10] Ramin Kazani, *Note on Multiplicative Zagreb indices*, Discrete Applied Mathematics, 198(2016), 147-154.

[11] U. Mary and Anju Antony *The First and Second Zagreb Indices for Some Special Graphs*, S K International Jour. of Multidisciplinary Research Hub, 2(10)(2015), 1-7.

[12] B.Basavanagoud and S. Patil, *A note on Hyper Zagreb index of Graph operations*, Iranian Journal of mathematical Chemistry, 7(1)(2016), 89-92.

[13] B. Zhou and I. Gutman, *Further Properties of Zagreb Indices*, MATCH Commun. Math. Comut. Chem., 54(2005), 233-239.

[14] M. El. Marraki and A. Modabish, *Wiener index of planar maps*, Journal of Theoretical and Applied Information Technology, 18(2010), 7-10.

[15] Dusanka, Ante Milicevic, Sonja Nikolic and Trinajstic, *Graph theoretical matrices in chemistry*, University of Kragujevac, Serbia, (2006).