Int. J. Math. And Appl., 6(4)(2018), 221–230
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of *Mathematics And its Applications*

Algebraic Expression: Identification, Sequencing & Application of its Elementary Concepts

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Abstract: In this paper, total 312 elementary concepts of 'algebraic expression' have been identified and sequenced. Out of these only 181 is found to be considered by National Council of Educational Research and Training (N.C.E.R.T.) and 110 by West Bengal Board of Secondary Education (W.B.B.S.E.) in their prescribed text books. The number of use of these elementary concepts together with their frequency of use in the West Bengal, The Council for the Indian School Certificate Examinations (C.I.S.C.E.) and Visva-Bharati final examinations have been identified. Two lists of elementary concepts of algebraic expression, one of the three Boards combined and the other only WB.B.S.E. respectively have been done. Afterwards, a test was developed on the identified top 12 elementary concepts of algebraic expression according to the frequency of their use in Madhyamik Pariksha (M.P.) under W.B.B.S.E. The test was then applied on X^{th}-Standard 194 students (boys-107, girls-87). The sample was selected randomly from Bengali medium schools of Burdwan district and affiliated to W.B.B.S.E. Several materials and methods have been adopted for the study which are mentioned above.

MSC: 97A99, 97C99, 97D70, 97H20.

Keywords: Algebraic expression, major concept, elementary concept, identification, sequencing, frequently used elementary concepts, text book scanning process, task analysis technique, achievement test.

 Accepted on: 03.12.2018

1. Introduction

Considering the importance of the branch of mathematics named algebra, generally it has been introduced at the beginning of upper primary level i.e. at class-VI at the age level of 11-12 years in our country. But the scenario of the subject, mathematics as well as algebra was not satisfactory to the students, teachers, parents and other stakeholders. Since, the researcher is a secondary level teacher of mathematics under W.B.B.S.E. He tried to find out the causes to avoid this unwanted situation. There are several causes are responsible behind this situation, Lacuna in content organization is one of them. In this regard, several mathematicians and research demand the logical sequencing of mathematical concept [1-4, 28-30, 32]. So, he stressed on the identification and sequencing the major concept and its elementary concepts to frame the flowless syllabus. The author has done several works entitled by 'A Study of Major Concepts in Mathematics at Secondary Level' on this area [33-36]. In this context, the elementary concepts of algebraic expression have not been published. So, the author has emphasized to present these concepts sequentially in this paper. At the same time, it is also important that which concepts of algebraic expression are most used in different Board examinations, specially Madhyamik Pariksha on account of the special care needed to the learners. This area has been considered in this study and after that he also stressed on to find out the learning competencies of the X^{th}-Standard learners on most frequently usable elementary concepts of algebraic

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expression used in Madhyamik Pariksha. Now, it is clear that every student should acquire the learning competencies on these most usable concepts of algebraic expression to solve some of the algebraic problems smoothly. But it is mentioned before, there is no good teaching-learning ambience, specially, in rural area in respect of conceptual gaping of this. So, it is important to know the knowledge of difficulty levels of most frequently used these top basic mathematical concepts that will be helpful to formulate strategies for mastery learning of these concepts. Thus, an investigation has been undertaken to determine the difficulty level of most frequently used these elementary concepts of algebraic expression to solve the algebra group problem of mathematics of M.P. (1999 & 2000).

2. Objective of the Study

(i). Identification of different sub-concepts and their sequential order of the major concept: algebraic expression.

(ii). Identification of the elementary concepts of algebraic expression among thirty most usable elementary concepts of algebra in respect of their use in public examinations of three Boards viz. W.B.B.S.E., C.I.S.C.E. and Visva-Bharati combined.

(iii). Identification of the elementary concepts of algebraic expression in mathematics of M.P. under W.B.B.S.E.

(iv). To identify the order of difficulty of the elementary concepts of algebraic expression in mathematics of M.P. under W.B.B.S.E.

3. Definitions

Mathematical concept: Schiralli and Sinclair (2003) have defined mathematical concept as "A mathematical concept is a publicly accessible tool-with a history and a future-involved in pursuing, representing, exploring and manipulating patterns and pattern possibilities. This tool may continue to have utility in its present form, may be improved in future, or even supplanted by newer tools as yet unrepresented. The significance of a mathematical concept lies in the way it connects with related concepts-with the logical patterns of the connective rules functioning as the medium within which mathematical enquiry publicly proceeds [31]".

Major concept and Sub-concepts: A Major concept is an idea which is complete in itself and is comprehended through a sequential process of step-by-step partial comprehension of its related concepts. These related concepts are called sub-concepts which are not complete in itself but are parts of the major concept. Examples of major concepts are 'addition of whole numbers including the familiarity of the numbers', 'subtraction of whole numbers', 'measurement of length', 'fraction', 'decimal' etc.

For the major concept such as 'Addition of whole numbers including the familiarity of the numbers', its sub-concepts are 'addition of one-digit numbers including zero where the sum is one-digit number', 'addition of one-digit numbers where the sum is two-digit number' etc.,

First level sub-concepts: The sub-concepts which are obtained after immediate derivation of a major concept are called first level sub-concepts.

Second level sub-concepts and others: When the derivation of first level sub-concepts are continued, the other sub-concepts so obtained stage by stage are called second level, third level etc.

Elementary concept: A sub-concept which cannot be split further will be called an 'elementary concept'.

4. Materials and Methods

The following procedure as given below has been followed for fulfill the objectives of the study.

(i). **Collection of text books:** The prescribed text books of mathematics from class-I to X of W.B.B.P.E., W.B.B.S.E., N.C.E.R.T. and other available books from the market were collected in the first stage [5-27].

(ii). **Analysis of text books:** These books were analyzed to identify the major concept: algebraic expression and their elementary concepts of algebra mainly.

(iii). **Task analysis of question papers:** For identification of elementary concepts of algebraic expression, task analysis technique has also been applied on arithmetical and algebraic problems of mathematics question papers of Madhyamik Pariksha (M.P.) (1999, 2000) of W.B.B.S.E., Indian Certificate of Secondary Education (I.C.S.E.) Examination of (2005, 2006) of C.I.S.C.E. & School Certificate Examination (S.C.E.) (1999, 2000) of Visva-Bharati (V.B.).

(iv). **Addition of concepts by the researcher:** After collection of elementary concepts of algebraic expression from the text books as well as task analysis of question papers, the researcher had identified some concept gaps. So, some new elementary concepts were added by the researcher to fill in the gaps.

(v). **Sequencing of concepts:** All the identified elementary concepts of the major concept: algebraic expression were sequenced keeping in view the logical order of mathematics and the psychological order of learners by the researcher himself at the initial stage.

(vi). **Experts' opinion:** All elementary concepts of this major concept with examples were given to experts for their comments. The experts were requested to add or omit or alter the sequence of concepts as they felt necessary. Finally, the sequential form of the major concept: algebraic expression and their elementary concepts was developed incorporating the experts' opinion.

(vii). **Identification of top most frequently used elementary concepts according to three Boards combined:** After completion of task analysis which is mentioned above in iii), all elementary concepts of algebra which are used in M.P. (1999 & 2000), I.C.S.E. Examination (2005, 2006) & S.C.E. (1999, 2000) were counted. Then, top thirty most frequently used elementary concepts of algebra were determined according to three Boards combined. After all, top most frequently used elementary concepts of algebraic expression among the top thirty most frequently used elementary concepts of algebra were determined.

(viii). **Identification of top most frequently used elementary concepts according to W.B.B.S.E.:** Top thirty most frequently used elementary concepts of algebra which are used in M.P. (1999 & 2000) were determined. Then, top most frequently used elementary concepts of algebraic expression among the top thirty most frequently used elementary concepts of algebra which are used in M.P. (1999 & 2000) were determined.

(ix). **Test Construction:** Top most frequently used elementary concepts of algebraic expression, their frequency of use and rank (R_1) in respect of frequency of use are shown in column-2, 3 and 4 of Table-5. Only one item was included in the question paper for testing one elementary concept. All items were prepared for these most frequently used elementary concepts for the study. Prepared items were given to experts for validity of the test. Experts were requested to add or omit or alter the items in light of appropriateness testing of elementary concepts whenever was necessary. The answer scripts were assessed according to scoring key (1mark for 1 correct response item and mark 0 (zero) for wrong response)

after completion the test. Then all the correct responses of each item were computed and sums were converted into percentage.

5. Application

The developed test was applied on X-standard 194 students (boys-107, girls-87) in the session 2010-11. The sample was selected randomly from rural area of Bengali-medium school of Burdwan district under WBBSE.

6. Data

(i). The frequency of most usable elementary concepts of algebraic expression among the top thirty most frequently used elementary concepts of algebra which were used in M.P.

(ii). The correct responses of each item of 194 students.

7. Salient Points of the Study

I. Identification of different sub-concepts and their sequential order of the major concept: algebraic expression: The major concept 'algebraic expression' has been divided under 4 first level sub-concepts which are given below in Table-1. Each first level sub-concept has been divided into different levels of sub-concepts. Total 312 elementary concepts under each of the 4 first level sub-concepts are shown in Table-2.

1	2
S. Nos.	Name of the first level sub-concept
1	Positive integer and zero (whole number) and its properties
2	Integer and its properties
3	Formation of algebraic expression through translation of problems
4	Algebraic expression and its value using whole number

Table 1: List of first level sub-concepts of the major concept of algebra: Algebraic expression [33].

1	2	3	4	5	6	7	8
First level sub-concept S. Nos	Number of second level sub-concept	Number of third level sub-concept	Number of fourth level sub-concept	Number of fifth level sub-concept	Number of sixth level sub-concept	Number of seventh level sub-concept	Total number of elementary concepts
1	5	5+10+8+13+9	7+10+8+13+9	13+10+8+13+9	–	–	53
2	12	2+1+1+1+4 +4+5+4+ 4 +5+7+4	3+1+1+1+13 +11+8 +4+ 32+27 +7+29	3+1+1+1+17 +11+8 +4+64 +27 +7+29	3+1+1+1+17 +11+8 +4+64 +27 +7+29	3+1+1+1 +17 +11+8 +4+81 +27 +7+29	190
3	2	6+6	16+16	22+22	–	–	44
4	5	7+2+5 +1+2	9+2+5+1+2	11+2+ 5+1+2	15+2 +5+1+2	–	25
Grand Total							312

Table 2: Detailed list of different levels of sub-concepts of the major concept: Algebraic expression [33].

The process of detailing out of the first level sub-concept-4: Algebraic expression and its value using whole number only has been done due to economy of space. Here, the elementary concepts are elaborated providing the question (Q).
In the following sections:

$*$: represents the concepts which have been introduced in the prescribed text books of N.C.E.R.T. and W.B.B.S.E.

** : represents the concepts which have been introduced in the prescribed text books of N.C.E.R.T. only but not in W.B.B.S.E.

* * * : represents the concepts which have been introduced in the prescribed text books of W.B.B.S.E. only.

* * ** : represents the concepts which have not been introduced in the prescribed text books of W.B.B.S.E. and N.C.E.R.T. both.

The second level sub-concepts of sub-concept no. 4: Algebraic Expression and its value using whole number:

4.1 Different types of expressions.

4.2 Concept of co-efficient.

4.3 Determination of value of an expression using whole number.

4.4 Determination of value of algebraic expression when the unknown is given in terms of an algebraic equation (****).

Q. Find the value of $\frac{x+y}{z}$ when $x = a + b$, $y = a - b$ and $z = 2a$.

4.5 Expression of binomial, trinomial etc.

The third level sub-concepts of sub-concept no. 4.1: Different types of expressions:

4.1.1 Terms of an expression (*).

The portions of expression which are connected by '+', '-' signs are called terms.

In the expression $2a + 3b - 4c$ each of $2a$, $3b$, $4c$ is a term.

4.1.2 Monomial (*).

Q. Give an example of monomial.

4.1.3 Binomial (*).

Q. Give an example of binomial.

4.1.4 Polynomial.

4.1.5 Rational expression (****).

Q. Give an example of rational expression.

4.1.6 Expression using radical sign (****).

Q. Give an example of expression using radical sign.

4.1.7 Identification of equal expressions (**).

Q. Identify equal expressions.

$2x^3 - y^3$, $x^3 - 2y^3$, $-y^3 + 2x^3$.

The fourth level sub-concepts of sub-concept no. 4.1.4: Polynomial:

4.1.4.1 Definition (*).

Expression having three or more terms is called polynomial.

Q. Give an example of polynomial.

4.1.4.2 Types of polynomials according to number of variables.

The fifth level sub-concepts of sub-concept no.4.1.4.2: Types of polynomials according to number of variables:

4.1.4.2.1 Single variable (****).

Q. Give an example of a polynomial of single variable.

4.1.4.2.2 More than one variable (****).

Q. Give an example of a polynomial of more than one variable.

The fifth level sub-concepts of sub-concept no. 4.1.4.3: Degree of polynomials:

4.1.4.3.1 Concept of degree of polynomial (****).

Q. What is the degree of a polynomial?

4.1.4.3.2 Different types of polynomial according to its degree.

The sixth level sub-concepts of sub-concept no.4.1.4.3.2: Different types of polynomial according to its degree:

4.1.4.3.2.1 First degree or linear (****).

Q. Give an example of first-degree polynomial.

4.1.4.3.2.2 Quadratic (****).

Q. Give an example of quadratic polynomial.

4.1.4.3.2.3 Cubic (****).

Give an example of cubic polynomial.

4.1.4.3.2.4 Bi-Quadratic (****).

Q. Give an example of bi-quadratic polynomial.

4.1.4.3.2.5 Polynomial of degree higher than four (****).

Q. Give an example of a polynomial of degree higher than four.

The third level sub-concepts of sub-concept no. 4.2: Concept of co-efficient:

4.2.1 Numeral co-efficient (***).

Q. What is the numeral co-efficient of 2xy?

4.2.2 Symbolic co-efficient (*).

Q. What is the co-efficient of x of 2xy?

The third level sub-concepts of sub-concept no. 4.3: Determination of value of an algebraic expression when the unknown is positive whole number:

4.3.1 Finding the value of monomial (power of each unknown is one) (**).

Q. If $a = 3$, find the value of $4a$.

4.3.2 Finding the value of binomial (power of each unknown is one) (*).

Q. If $a = 2$, $b = 3$ and $c = 4$, find the value of $a \times b + c$.

4.3.3 Finding the value of polynomial (power of each unknown is one) (*).

Q. If $a = 2$, $b = 5$ and $c = 7$, find value of $ab + bc + ca$.

4.3.4 Finding the value of rational expression (power of each unknown is one) (***).

Q. If $a = 3$, $b = 1$, $x = 2$, $y = 4$, then find the value of $\frac{2a-3b}{x+y}$.

4.3.5 Finding the value of expression using radical sign (power of each unknown is one) (****).

Q. If $a = 3$, $b = 2$, $x = 6$, find the value of $\sqrt{\frac{7x}{5ab}}$.

The third level sub-concepts of sub-concept no. 4.5: Expression of binomial, trinomial etc:

4.5.1 Simplification of expression (****).

Q. Simplify:

$x^2(x + 1) - x(x + 1) + 3(x + 1)$

4.5.2 Expressing an expression in terms of binomial or trinomial (****).

Q. Write this expression as expression of $(x + 1)$.

$x^3 + 2x + 3$.

II. Identification the elementary concepts of algebraic expression among thirty most usable elementary concepts of algebra in respect of their use in public examinations of three Boards combined: The total

number of used elementary concepts of algebraic expression with their frequency of use in the West Bengal, C.I.S.C.E. and Visva-Bharati final examinations are shown in the following Table-3.

1	2	3	4	3	7	8	9
		West Bengal		C.I.S.C.E.		Visva-Bharati	
		Number of concept used	Frequency of used	Number of concept used	Frequency of used	Number of concept used	Frequency of used
Algebraic expressions	312	48	503	32	189	30	383

Table 3: The total number of used elementary concepts of Algebraic expression with their frequency of use in the West Bengal, C.I.S.C.E. and Visva-Bharati final examinations [33].

Total 48 elementary concepts of algebraic expression used to solve the arithmetical and algebraic problems of mathematics question papers of Madhyamik Pariksha (M.P.) (1999, 2000) of W.B.B.S.E. Their total number of frequency of used is 503. Total 32 elementary concepts of algebraic expression used to solve the arithmetical and algebraic problems of mathematics question papers of Indian Certificate of Secondary Education (I.C.S.E.) Examination of (2005, 2006) of C.I.S.C.E. Their total number of frequency of used is 189.

Total 30 elementary concepts of algebraic expression used to solve the arithmetical and algebraic problems of mathematics question papers of School Certificate Examination (S.C.E.) (1999, 2000) of Visva-Bharati (V.B.). Their total number of frequency of used is 383. A list of elementary concepts of algebraic expression among the top thirty elementary concepts of algebra three Boards combined according to frequency of use is given in the following Table-4.

1	2	3
S. No.	Name of the elementary concepts	Frequency of use
1.	Multiplication of positive integer by positive integer.	156
2.	Division of integer: if both the dividend and divisor are of positive sign, then the quotient is positive.	147
3.	For all integers a and b, $a + b = b + a$	136
4.	Subtraction of two positive integers (when minuend = subtrahend)	60
5.	Multiplying positive integer by negative integer or negative integer by positive integer.	49
6.	Addition of two positive integers.	46
7.	Integers as rational numbers.	46
8.	Expression using multiplication: writing by using sign and symbols.	39
9.	Multiplying any integer by zero.	36
10.	Subtraction of two positive integers when minuend > subtrahend.	32
11.	Finding the value of an algebraic expression when the unknowns are other algebraic expressions.	30

Table 4: A list of elementary concepts of algebraic expression among the top thirty elementary concepts of algebra three Boards combined according to frequency of use [33].

III. Identification the elementary concepts of algebraic expression among thirty most usable elementary concepts of algebra in respect of their use in mathematics examinations in M.P. under W.B.B.S.E.: The elementary concepts of algebraic expression among the top thirty elementary concepts of algebra in MP according to frequency of use have been identified and sequenced according to frequency of use are shown in columns- 2 & 3 of Table-5.

IV. To identify the order of difficulty of the elementary concepts of algebraic expression among thirty most usable elementary concepts of algebra in respect of their use in mathematics examinations in MP under W.B.B.S.E.: The list of identified 12 elementary concepts and their frequency of used in MP, rank in respect of frequency of use, correct response, percentage of correct responses and order of difficulty are shown in Table-5.

1	2	3	4	5	6	7
S. No.	Name of the elementary concept	frequency	Rank in respect of frequency of use R1	Correct Response	Percentage of Correct Responses	Order of Difficulty
1.	Division of integer: if both the dividend and divisor are of positive sign, then the quotient is positive.	71	1	176	90.7	11
2.	Multiplication of positive integer by positive integer.	69	2	177	91.2	12
3.	For all integers a and b, $a + b = b + a$	52	3	147	75.8	7
4.	Multiplying positive integer by negative integer or negative integer by positive integer.	27	4	105	54.1	2
5.	Integers as rational numbers.	21	5	42	21.6	1
6.	Subtraction of two positive integers (when minuend = subtrahend).	19	7	148	76.3	8
7.	Addition of two positive integers.	19	7	169	87.1	10
8.	Expression using multiplication: writing by using sign and symbols.	19	7	129	66.5	4
9.	Multiplying any integer by zero.	17	9	123	63.4	3
10.	Subtraction of two positive integers (when minuend > subtrahend).	16	10.5	138	71.1	5
11.	Addition of integer and zero.	16	10.5	154	79.4	9
12.	Concept of opposite numbers.	14	12	144	74.2	6

Table 5: List of elementary concepts and their frequency of used in MP, rank in respect of frequency of use, correct response, percentage of correct responses and order of difficulty [33].

8. Conclusion

(i). Total 312 elementary concepts of the major concept: 'Algebraic expression' have been identified and sequenced. In total, 110 elementary concepts out of these 312 elementary concepts have been considered in the prescribed text books of W.B.B.S.E. Similarly, N.C.E.R.T. text books have also considered only 181 elementary concepts out of 312.So, it is a major gap of our text books of mathematics. In this paper, 25 elementary concepts of the first level sub-concept: 4 Algebraic expression and its value using whole number only has been presented sequentially. 9 elementary concepts out of 25 have taken into consideration in the prescribed text books of both W.B.B.S.E.& N.C.E.R.T.

(ii). The relationship among the number of elementary concepts of algebraic expression used to solve the arithmetical and algebraical problems of mathematics question papers of the mentioned years of three Boards is Visva-Bharati < C.I.S.C.E. < W.B.B.S.E. The relationship among the total number of frequencies of the elementary concepts of algebraic expression used to solve the arithmetical and algebraical problems of mathematics question papers of the mentioned years of three Boards is C.I.S.C.E. < Visva-Bharati < W.B.B.S.E.

(iii). It is clear from Table-4 that total 11 elementary concepts of algebraic expression among the top thirty elementary concepts of algebra three Boards combined according to frequency of use have been identified. These 11 concepts have sequentially been presented according to frequency of use.

(iv). Table-5 reveals that total 12 elementary concepts of algebraic expression among top thirty elementary concepts of algebra of M.P. under W.B.B.S.E. according to frequency of use have been identified. These concepts are presented sequentially according to frequency of use.

(v). The students were able to response about 53 % correctly on the total test. The most difficult elementary concept for students was expression of polynomial as rational expression (only 4.1% students responded the item correctly). It has been assigned difficulty rank value 1. The easiest concept is multiplication of positive integer by positive integer (91.2%) having difficulty rank value 30. The interpretation of the other difficulty rank values of other concepts will

follow from the same line of argument. The column-4 & the column-7 of Table-5 reveal that rank-values according to frequency of use and that of according to order of difficulty of maximum elementary concept are not related. It indicates that frequency of response of students to a concept is not predictable from number of correct use of the concept and vice-versa. It is an important draw back in our mathematics teaching learning attainment strategy. So, we may conclude that we have to deeply think to get rid of this unwanted situation. Arrangement for remedial teaching classes and activity based sequential presentation of concepts in the classroom may be considered as possible solution of the problem of low achievement in mathematics.

After all, this study indicates that the low achievements of the students are responsible for the poor performance in mathematics at M.P. under W.B.B.S.E.

9. Further Research

(i). This study may be extended to the other branches of mathematics at secondary level, namely, arithmetic, geometry, mensuration and trigonometry at secondary level.

(ii). This study may also be extended to mathematics examination of different Board or University.

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