

A Note on Study of Game Problem Using Simplex Method

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Abstract: In this paper, optimal strategies of game problem which does not have a saddle point using simplex (Big-M) method calculated.

Keywords: Linear programming problem, Game problem, Optimal strategies, Big-M method.

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1. Introduction

Game theory is the study of the strategic interaction of two or more decision makers, or “players”, who are attentive that their actions affect each other. Game theory describes the situations involving conflict in which the payoff is affected by the actions and counter-actions of clever opponents [1]. Two-person zero-sum games play a essential role in the development of the theory of games. In order to know the theory of game, consider the following game in which player P has two choices from which to select, and player Q has three alternatives for each choice of player Q [1,2]. The payoff matrix M is given below:

		Player Q		
		$j = 1$	$j = 2$	$j = 3$
Player P	$i = 1$	a	b	c
	$i = 2$	d	e	f

In the payoff matrix, the two rows ($i = 1, 2$) represents the two possible strategies that player P can employ, and the three columns ($j = 1, 2, 3$) represent the three possible strategies that player Q can employ. The payoff matrix is oriented to player P, meaning that a positive M_{ij} is a gain for player P and a loss for player Q, and a negative M_{ij} is a gain for player Q and a loss for player P. For example, if player P uses strategy 2 and player Q uses strategy 1, player P receives $M_{12} = d$ units and player Q thus losses d units. Clearly, in our example player Q always loses; however, the objective is to minimize the payoff to player P [1, 2, 3].

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2. Solution of Game Problem Which Does not Have a Saddle Point

Consider a two-person zero-sum game without saddle point, having the payoff matrix for player P as,

		Player Q			
		1	3	-3	7
Player P	1	3	-3	7	
	2	5	4	-6	

Since, Maximin value = -3, Minimaxi value = 2, the payoff matrix does not possess saddle point. Therefore, value of the game lie between -3 and 2. Let the optimal strategies of two players be:

$$S_P = (p_1, p_2), \quad S_Q = (q_1, q_2, q_3),$$

where, $p_1 + p_2 = 1$ and $q_1 + q_2 + q_3 = 1$. Then, the linear programming problem (LPP) can be written as:

$$\left(\begin{array}{l} \text{For Player P :} \\ \text{Minimize } X = x_1 + x_2 = 1/v \\ \text{subject to} \\ \quad x_1 + 2x_2 \geq 1 \\ \quad 3x_1 + 5x_2 \geq 1 \\ \quad -3x_1 + 4x_2 \geq 1 \\ \quad 7x_1 - 6x_2 \geq 1 \\ \text{and } x_1, x_2 \geq 0. \end{array} \right) \left(\begin{array}{l} \text{For Player Q :} \\ \text{Maximize } Y = y_1 + y_2 + y_3 + y_4 = 1/v \\ \text{subject to} \\ \quad y_1 + 3y_2 - 3y_3 + 7y_4 \leq 1 \\ \quad 2y_1 + 5y_2 + 4y_3 - 6y_4 \leq 1 \\ \text{and } y_1, y_2, y_3, y_4 \geq 0. \end{array} \right)$$

Standard form of LPP:

$$\left(\begin{array}{l} \text{For Player P :} \\ \text{Maximize } X^* = -\text{Minimize } X = -(x_1 + x_2) \\ \text{subject to} \\ \quad x_1 + 2x_2 - s_1 = 1, \\ \quad 3x_1 + 5x_2 - s_2 = 1, \\ \quad -3x_1 + 4x_2 - s_3 = 1, \\ \quad 7x_1 - 6x_2 - s_4 = 1, \\ \text{and } x_1, x_2, s_1, s_2, s_3, s_4 \geq 0, \\ \text{where, } s_1, s_2, s_3, s_4 \text{ are surplus variables.} \end{array} \right) \left(\begin{array}{l} \text{For Player Q :} \\ \text{Maximize } Y = y_1 + y_2 + y_3 + y_4 + 0.s_1 + 0.s_2 = 1/v \\ \text{subject to} \\ \quad y_1 + 3y_2 - 3y_3 + 7y_4 + s_1 = 1 \\ \quad 2y_1 + 5y_2 + 4y_3 - 6y_4 + s_2 = 1 \\ \text{and } y_1, y_2, s_1, s_2 \geq 0, \\ \text{where, } s_1, s_2 \text{ are slack variables.} \end{array} \right)$$

By Simplex (Big-M) method, solution of LPP, for Player P is $x_1 = 1/7, x_2 = 0$, and for player Q is $y_1 = 13/20, y_2 = y_3 = y_4 = 1/20$.

Optimal strategies for player P, $p_1 = vx_1 \simeq 2, p_2 = vx_2 = 0$.

Optimal strategies for player Q, $q_1 = vy_1 \simeq 1, q_2 = vy_2 = 0, q_3 = vy_3 = 0, q_4 = vy_4 \simeq 1$.

3. Conclusion

Game problem was successfully solved using simplex (Big-M) method. It has been observed that the optimal strategies for players are same when compared to the solution using other simplex methods.

References

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