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A Study on Dusty Couple Stress Fluid Heated From Below in the Presence of Horizontal Magnetic Field with Hall Currents

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Abstract: An investigation is made on the effect of Hall currents and uniform horizontal Magnetic field on the thermal stability of dusty couple-stress fluid is considered. The analysis is carried out within the limitation of framework of linear stability theory and normal mode technique. A dispersion relation governing the effect of dust particles, Hall currents, magnetic field and couple stress are derived. For the case of stationary convection, dust particles and Hall currents are found destabilizing effect whereas couple-stress has stabilizing effect on the system. Magnetic field has a stabilizing or destabilizing effect on the thermal convection under the restrictions. It has been observed that oscillatory modes are introduced due to the presence of magnetic field and Hall currents which were non-existent in their absence. Graphs have been plotted by giving numerical values to the parameters to depict the stability characteristics.

Keywords: Thermal convection, couple-stress fluid, magnetic field, Hall currents, Dust particles.

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1. Introduction

An investigation is made on the effect of Hall currents and uniform horizontal Magnetic field on the thermal stability of dusty couple-stress fluid is considered. The analysis is carried out within the limitation of framework of linear stability theory and normal mode technique. A dispersion relation governing the effect of dust particles, Hall currents, magnetic field and couple stress are derived. For the case of stationary convection, dust particles and Hall currents are found destabilizing effect whereas couplestress has stabilizing effect on the system. Magnetic field has a stabilizing or destabilizing effect on the thermal convection under the restrictions. It has been observed that oscillatory modes are introduced due to the presence of magnetic field and Hall currents which were non-existent in their absence. Graphs have been plotted by giving numerical values to the parameters to depict the stability characteristics.

Applications of couple-stress fluid occur in the attention of the study of the mechanism of lubrication of synovial joints, that has become the object of scientific research. A human joint is a dynamically loaded bearing that has articular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze-film action is capable of providing significant protection to the cartilage surface. The shoulder, hip, knee and ankle joints are the loaded-bearing synovial joints of the human body and these joints have a low friction coefficient and negligible wear. Normal synovial fluid is a viscous, non-Newtonian fluid and is generally clear or yellowish. The synovial fluid has been modeled as a couple-stress

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fluid in human joints by Walicki and Walicka [22]. Sharma [16] has studied thermal instability in a viscoelastic uid in the hydromagnetics. Oldroyd [9] investigated non-Newtonian effects in study motion of some idealized elasticviscousliquids. Kent [4] studied instability of laminar ow of a magnetouid. Veronis [20] analysed connective instability in a compressible atmosphere. Sharma [17] investigated the thermal instability of compressible fluid in the presence of rotation and magnetic field. Kumar [5] have studied the effect of magnetic field on thermal instability of a rotating Rivlin-Ericksen visco-elatic fluid. Kumar [6] have studied the problem of thermalsolutal instability of couple-stress rotating fluid in the presence of magnetic field and found that magnetic field has both stabilizing and destabilizing effects on the system under certain conditions whereas rotation has a stabilizing effect on the system.

Raptis and Ram [13] have studied the effect of rotation and Hall currents on free convection and mass transfer flow. Sharma and Rani [18] investigated the Hall effects on thermosolutal instability of plasma. Sunil [19] analyzed the effect of Hall currents on thermosolutal instability of compressible Rivlin Ericksen fluid. Shivakumara [19] has investigated the effect of various non-uniform basic temperature gradients on the onset of convection in a couple-stress fluid saturated porous layer. Rani and Tomar [11, 12] investigated thermal and thermosolutal convection problem of micropolar fluid subjected to Hall current. Kumar [7] examined the effect of Hall currents on thermal instability of compressible dusty viscoelastic fluid saturated in a porous medium subjected to vertical magnetic field.

In this study, since there is growing importance of non-Newtonian uids, convection in uid layer heated from below under magnetic eld, our objective is to investigate the effect of Hall current on thermal instability of a dusty couple-stress fluid in the presence of horizontal magnetic field. Here well-know governing partial differential equations are reduced to the ordinary differential equations. Numerical solution of the problem is obtained using Newton-Raphson method. These numerical results for various physical parameters concerned within the problem are demonstrated graphically.

2. Formulation of the Problem

Let p, ρ, T, α, v, μ^1, k_r and $\overrightarrow{q}(u, v, w)$ denote respectively pressure, density, temperature, thermal coefficient of expansion, kinematic viscosity, couple-stress viscosity, thermal diffusivity and velocity of the fluid. $\overrightarrow{q_d}(\overline{x}, t)$ and $N(\overline{x}, t)$ denote the velocity and number density of particles, respectively. $K = 6\pi\mu\eta$, where η is radius of the particle, is a constant and $\overline{x} = (x, y, z)$. Then equation of motion, continuity and heat conduction of couple-stress (Stokes, 1966 and Joseph, 1976) in hydromagnetics are

$$\frac{\partial \overrightarrow{q}}{\partial t} = -\frac{1}{\rho_0}\nabla p + \overrightarrow{g}\alpha\theta + \left(v - \frac{\mu^1}{\rho_0}\nabla^2\right)\overrightarrow{q} + \frac{KN_0}{\rho_0}(\overrightarrow{q_d} - \overrightarrow{q}) + \frac{\mu_e}{4\pi\rho_0}[(\nabla \times \overrightarrow{h}) \times \overrightarrow{H}] \tag{1}$$

$$\nabla \cdot \overrightarrow{q} = 0 \tag{2}$$

$$\frac{\partial \overrightarrow{h}}{\partial t} = \nabla \times (\overrightarrow{q} \times \overrightarrow{H}) + \eta\nabla^2\overrightarrow{H} - \frac{c}{4\pi Ne}\nabla \times [(\nabla \times \overrightarrow{h}) \times \overrightarrow{H}] \tag{3}$$

and

$$\nabla \cdot \overrightarrow{h} = 0 \tag{4}$$

The equation of state for the fluid is

$$\rho = \rho_0[1 - \alpha(T - To)] \tag{5}$$

Where α is coefficient of thermal expansion and the suffix zero refers to value at the reference level $z = 0$. Assume uniform particle size, spherical shape and small relative velocities between the fluid and particles. The presence of particles add an extra force term, proportional to the velocity difference between particles and fluid, appears in equation of motion (1). Since the force exerted by the fluid on the particles is equal and opposite to the exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equation of motion for the particles. The distance between particles are assumed to be quite large compared with their diameter that inter-particle reactions are not considered for. The effect of pressure, gravity and magnetic field on suspended particles, assuming large distances apart, are negligibly small and therefore ignored. The equations of motion continuity for the particle, under the above approximation, are

$$mN_0\frac{\partial \overrightarrow{q_d}}{\partial t} = KN_0(\overrightarrow{q} - \overrightarrow{q_d}) \tag{6}$$

and

$$\frac{\partial N}{\partial t} + \nabla\cdot(N\cdot\overrightarrow{q_d}) = 0 \tag{7}$$

Here mN is represent the mass of the particles per unit volume. Let c_v, c_{pt} denote the heat capacity of the fluid at constant volume and the heat capacity of the particles. Assuming that the particles and fluids are in thermal equilibrium, then the equation of heat conduction given by

$$\frac{\partial T}{\partial t} + \frac{mNC_{pt}}{\rho_0 C_v}\left(\frac{\partial}{\partial t} + \overrightarrow{q_d}\cdot\nabla\right)T = K_T\nabla^2 T \tag{8}$$

where v is kinematic viscosity, μ' is couple-stress viscosity, k_T is thermal diffusivity and α is coefficient of thermal expansion which are assumed to be constants.

3. Basic State of the Problem

The basic motionless solution state is described by $\overrightarrow{q} = (0,0,0)$, $\overrightarrow{q_d} = (0,0,0)$, $\overrightarrow{H} = (0,0,H)$, $T = T_0 - \beta z$, $N = N_0 = constant$, where β may be either positive or negative and

$$\rho = \rho(z), \quad p = p(z), \quad T = T(z) \;\; and \;\; \rho = \rho_0[1 + \alpha\beta z] \tag{9}$$

4. Perturbation Equations and Normal Mode Analysis

Let $\overrightarrow{q}(u,v,w)$, $\overrightarrow{q_d}(l,r,s)$, $\overrightarrow{h}(h_x,h_y,h_z)$, θ, $\delta\rho$, δp denote respectively the perturbations in fluid velocity $q = (0,0,0)$, dust particles velocity $\overrightarrow{q_d} = (0,0,0)$, magnetic field $\overrightarrow{H}(0,0,H)$, temperature T, density ρ and pressure p. After linearizing the perturbation and analyzing the perturbation into normal modes, we assume that the perturbation quantities are of the form

$$[w,\theta,h_z,\zeta,\xi] = [W(z),\theta(z),K(z),Z(z),X(z)].\exp\{ik_x x + ik_y y + nt\}. \tag{10}$$

Where k_x and k_y are the wave number in x and y directions respectively and $k = \sqrt{K_x^2 + K_y^2}$ is the resultant wave number of propagation and n is the growth rate which is, in general, a complex constant and, $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial t}$ and $\xi = \frac{\partial hy}{\partial x} - \frac{\delta hx}{\delta x}$ are the z-components of the vorticity and current density respectively. The linearized hydromagnetics perturbation equations for couple-stress fluid become

$$\frac{\partial\overrightarrow{q}}{\partial t} = -\frac{1}{\rho_0}\nabla\delta p + \overrightarrow{g}\alpha\theta + \left(v - \frac{\mu^1}{\rho_0}\nabla^2\right)\overrightarrow{q} + \frac{KN_0}{\rho_0}(\overrightarrow{q_d} - \overrightarrow{q}) + \frac{\mu_e}{4\pi\rho_0}[(\nabla\times\overrightarrow{h})\times\overrightarrow{H}] \tag{11}$$

$$\nabla\cdot\overrightarrow{q} = 0 \tag{12}$$

$$mN_0\frac{\partial \overrightarrow{q_d}}{\partial t} = KN_0(\overrightarrow{q} - \overrightarrow{q_d}) \tag{13}$$

$$(1+h_1)\frac{\partial \theta}{\partial t} = \beta(\omega + h_1 s) + K_T \nabla^2\theta \tag{14}$$

$$\frac{\partial \overrightarrow{h}}{\partial t} = \nabla \times (\overrightarrow{q} \times \overrightarrow{H}) + \eta\nabla^2\overrightarrow{H} - \frac{c}{4\pi Ne}\nabla \times [(\nabla \times \overrightarrow{h}) \times \overrightarrow{H}] \tag{15}$$

$$\nabla \cdot \overrightarrow{h} = 0 \tag{16}$$

The change in density $\delta\rho$ caused by the perturbation θ in temperature is given by

$$\delta\rho = -\rho_0\alpha\theta \tag{17}$$

For the considered form of the perturbations in equation (10), equations (11) to (17), after eliminating the physical quantities using the non-demensional parameters $a = kd$, $\sigma = \frac{nd^2}{v}$, $p_1 = \frac{v}{k_T}$, $p_2 = \frac{v}{\eta}$, $D = \frac{D^*}{d}$, $D^* = dDF = \frac{\mu'}{v\rho_0 d^2}$, $\sigma' = \frac{n'd^2}{v}$, $H_1 = 1 + h_1$ and $n' = n\left(1 + \frac{\frac{mN_0K}{\rho_0}}{mn+K}\right)$ dropping (*) for convenience, give

$$[\sigma' + F(D^2 - a^2) - 1](D^2 - a^2)W = -\frac{ga^2d^2\alpha}{v}\Theta + \frac{\mu_e Hd}{4\pi\rho_0 v}(D^2 - a^2)DK \tag{18}$$

$$[\sigma' - d^2[1 - F(D^2 - a^2)]]Z = \frac{\mu_e Hd}{4\pi\rho_0 v}^{DX} \tag{19}$$

$$(D^2 - a^2 - B\sigma p_1)\Theta = \frac{\beta d^2}{k_T}(B + \frac{\tau_1 v}{K_T}\sigma) \tag{20}$$

$$(D^2 - a^2 - \sigma p_2)K = -\frac{Hd}{\eta}DW + \frac{cHd}{4\pi N'e\eta}DX \tag{21}$$

$$(D^2 - a^2 - \sigma p_2)X = -\frac{Hd}{\eta}DZ + \frac{cH}{4\pi N'e\eta d}(D^2 - a^2)DK \tag{22}$$

where $F = \frac{\mu'}{\rho_0 d^2}$ is the couple stress parameter. After eliminating various physical parameters like Θ, Z, X, K from equations (18) to (22), we obtain the final stability governing equation as

$$\begin{aligned} &\Big\{[\sigma' - d^2[1 - F(D^2 - a^2)](D^2 - a^2)]W + \frac{Ra^2}{(D^2 - a^2 - H_1\sigma p_1)}\left(B + \frac{\tau_1 v}{K_T}\right)W \\ &+ Q\left[\frac{(D^2 - a^2 - \sigma p_2)[\sigma' - d^2[1 - F(D^2 - a^2)]] + QD^2}{(D^2 - a^2 - \sigma p_2)^2[\sigma' - d^2[1 - F(D^2 - a^2)]] + Q(D^2 - a^2 - \sigma p_2)D^2} \right.\\ &\left. -M[\sigma' - d^2[1 - F(D^2 - a^2)]]\Big\}(D^2 - a^2)D^2\right]DW = 0 \end{aligned} \tag{23}$$

Where $R = \frac{g\alpha\beta d^4}{VK_T}$ is the Rayleigh number, $Q = \frac{\mu_0 H^2 d^2}{4\pi\rho v\eta}$ is the Chandrasekhar number and $M = (\frac{cH}{4\pi Ne\eta})^2$ is the non-dimensional number accounting for Hall currents. We now consider the case where both the boundaries are free as well as perfect conductors of heat. The appropriate boundary conditions for the equation (23) are

$$W = 0, \quad Z = 0 \quad and \quad D^2W = 0, \quad D^4W = 0 \quad at \quad z = 0 \quad and \quad z = 1 \tag{24}$$

From equation (24), it is clear that all the even order derivatives of W must vanish for $z = 0$ and 1. Therefore, the proper solution of equation (23) characterizing the lowest mode is

$$W = W_0 sin\pi z \tag{25}$$

Where W_0 is a constant. Substituting the proper solution. We obtain

$$\begin{aligned} R_1 = &\left[\frac{(1 + x + iBP_1\sigma_1)}{x(B + \frac{\tau_1 v}{K_T}\sigma)}\right] Q_1 \left[\frac{(1 + x + iP_2\sigma_1)\{i\sigma' + [F_1(1 + x)]\} + Q_1}{(1 + x + iP_2\sigma)^2\{i\sigma' + [1 + F_1(1 + x)]\} + Q_1(1 + x + iP_2\sigma_1)}\right. \\ &\left. - M\{i\sigma' + [1 + F_1(1 + x)]\}(1 + x) - \{i\sigma' + [1 + F_1(1 + x)]\}(1 + x)\right] \end{aligned} \tag{26}$$

Where $k_x = k\cos\theta$ [Chandrasekhar (1981)], $R_1 = \frac{R}{\pi^4}$, $i\sigma_1 = \frac{\sigma}{\pi^2}$, $x = \frac{a^2}{\pi^2}$, $F_1 = \pi^2 F$ and $Q_1 = \frac{Q}{\pi^2}$. Equation (26) is the required dispersion relation including the parameters characterizing the Hall currents, magnetic field dust particles and couple-stress.

5. Analytical Discussion

5.1. Stationary Convection

At stationary convection, when the instability sets, the marginal state will be characterized by $\sigma = 0$. Thus the instability sets in as stationary convection putting $\sigma = 0$ in the equation (26), reduces to

$$R_1 = \frac{(1+x)Q_1}{xB}\left[\frac{(1+x)[1+F_1(1+x)]+Q_1}{(1+x)[1+F_1(1+x)]+Q_1-M[1+F_1(1+x)]}\right]+\frac{(1+x)^2[1+F_1(1+x)]}{xB} \tag{27}$$

The above expression is the modified Rayleigh number R_1 as a function of the parameters H_1, M, Q_1, F_1 and dimensionless wave number x . To study the effect of Hall currents, magnetic field and couple-stress, we examine the nature of $\frac{dR_1}{dB}$, $\frac{dR_1}{dM}$, $\frac{dR_1}{dQ_1}$, $\frac{dR_1}{dF_1}$ analytically. From equation (27), we have

$$\frac{dR_1}{dB} = -\frac{Q_1}{xB^2}\left[\frac{(1+x)\left[1+F_1\left(1+x\right)\right]+Q_1}{(1+x)\left[1+F_1\left(1+x\right)\right]+Q_1-M\left[1+F_1\left(1+x\right)\right]}\right]-\frac{(1+x)^2\left[1+F_1\left(1+x\right)\right]}{xB^2} \tag{28}$$

Which clearly confirms that dust particles have destabilizing effect on a couple-stress fluid on the thermal convection. From equation (27), we have

$$\frac{dR_1}{dM} = -\frac{Q_1}{xB}\left[\frac{(1+x)\left[1+F_1\left(1+x\right)\right]+Q_1}{\left[(1+x)\left[1+F_1\left(1+x\right)\right]+Q_1-M\left[1+F_1\left(1+x\right)\right]\right]^2}\right] \tag{29}$$

Which confirms that Hall currents have a destabilizing effect on a couple-stress fluid on the thermal convection. This result is same as observed by Singh and Dixit.

$$\begin{aligned}\frac{dR_1}{dQ_1} &= \frac{1}{xB}\left[\frac{(1+x)\left[1+F_1\left(1+x\right)\right]+Q_1}{(1+x)\left[1+F_1\left(1+x\right)\right]+Q_1-M\left[1+F_1\left(1+x\right)\right]}\right]\\ &-\frac{1}{xB}\left[\frac{\left[1+F_1\left(1+x\right)\right]Q_1 M}{\left[(1+x)\left[1+F_1\left(1+x\right)\right]+Q_1-M\left[1+F_1\left(1+x\right)\right]\right]^2}\right]\end{aligned} \tag{30}$$

Which shows that magnetic field has a stabilizing/destabilizing effect on a couple-stress dusty fluid on thermal convection the condition

$$(1+x)\left[1+F_1\left(1+x\right)\right]+Q_1 < \ or\ > M\left[1+F_1\left(1+x\right)\right].$$

But, for the permissible values of various parameters, the above effect is stabilizing only if

$$(1+x)\left[1+F_1\left(1+x\right)\right]+Q_1 > M\left[1+F_1\left(1+x\right)\right]$$

From equation (27), we have

$$\begin{aligned}\frac{dR_1}{dF_1} &= \frac{(1+x)\,Q_1}{xB}\Big\{(1+x)[(1+x)[1+F_1(1+x)]-Q_1-M[1+F_1(1+x)]]\\ &-\frac{[(1+x)\left[1+F_1\left(1+x\right)\right]+Q_1]\left[1+x-M\right]}{[(1+x)[1+F_1(1+x)]-Q_1-M[1+F_1(1+x)]^2]}+1\Big\}\end{aligned} \tag{31}$$

Which clears that couple-stress has a stabilizing effect on a couple-stress dusty fluid on thermal convection system.

5.2. Stability of the system and Oscillatory Modes

Multiply (18) with W^* (complex conjugate of W) and integrate over the range of z using equations (19) to (20) with the boundary condition (24), we get the conditions for the principle of exchange of stabilities (PES) is satisfied (i.e σ is real and the marginal states are charterized by $\sigma = 0$) and oscillations enter into play and it is given by

$$\begin{aligned}(1-\sigma')I_1 - FI_2 + \frac{g_0\alpha K_T a^2}{V\beta}\left(B + \frac{\tau_1 v}{K_T}\sigma^*\right)^{-1}(I_3 + BP_1\sigma^* I_4) \\ - \frac{\mu_e\eta}{4\pi\rho_0 v}[I_5 + \sigma^* p_2 I_6] + \frac{\mu_e\eta d^2}{4\pi\rho_0 v}[I_7 + \sigma^* p_2 I_8] + d^2[(\sigma'-1)I_9 - FI_{10}] = 0\end{aligned} \tag{32}$$

Where

$$\begin{aligned}
I_1 &= \int (|DW|^2 + a^2|W|^2)dz, & I_2 &= \int (|D^2W|^2 + 2a^2|DW|^2 + a^4|W|^2)dz,\\
I_3 &= \int (|D\theta|^2 + a^2|\theta|^2)dz, & I_4 &= \int (|\theta|^2)dz\\
I_5 &= \int (|D^2K|^2 + 2a^2|DK|^2 + a^4|K|^2)dz, & I_6 &= \int (|DK|^2 + a^2|K|^2)dz\\
I_7 &= \int (|DX|^2 + a^2|X|^2)dz, & I_8 &= \int (|X|^2)dz\\
I_9 &= \int (|Z|^2)dz, & I_{10} &= \int (|DZ|^2 + a^2|Z|^2)dz
\end{aligned}$$

Where σ^* is the complex conjugate of σ. All the integrals I_1 to I_{10} are positive definite, putting $\sigma = i\sigma$, in equation (32) and equating the imaginary parts, we obtain

$$\sigma_i\left[I_1 + \frac{g_0\alpha k_T^2 a^2}{\beta\tau_1 v^2}I_3 + \frac{g_0\alpha k_T a^2}{\beta v}P_1 I_4 - \frac{\mu_e\eta}{4\pi\rho_0 v}p_2 I_6 + \frac{\mu_e\eta d^2}{4\pi\rho_0 v}p_2 I_8 - d^2 I_9\right] = 0 \tag{33}$$

From (33), it is clear that σ_i (growth rate parameter) may be zero or nonzero, which gives the modes may be nonoscillatory or oscillatory. In the absence of stable magnetic field (hence Hall currents) and dust particles, equation (33) becomes

$$\sigma_i\left[I_1 + \frac{g_0\alpha k_T^2 a^2}{\beta\tau_1 v^2}I_3 + \frac{g_0\alpha k_T a^2}{\beta v}P_1 I_4 + \frac{\mu_e\eta d^2}{4\pi\rho_0 v}P_2 I_8\right] = 0 \tag{34}$$

It may be inferred from equation (33) that σ_i may be positive or negative which means that the system may be stable or unstable while equation (34) predicts that $\sigma_i = 0$ necessarily because all the terms in the bracket are positive definite. which implies that oscillatory modes are not allowed in the system and Principle of Exchange of Stabilities (PES) is satisfied in the absence of magnetic field (hence Hall currents) and dust particles.

6. Numerical Computations

Now, the critical thermal Rayleigh number for the onset of instability is determined for critical wave number obtained by using Newton-Raphson method, by means of the condition $\frac{dR_1}{dx} = 0$. The numerical values of critical thermal Rayleigh number R_1 and critical wave number x determined for various values of dust particles B, Hall Currents M, magnetic field Q_1 and couple-stress F_1. Graphs have been potted between critical Rayleigh number R_1 and Parameters B, M, Q_1, and F_1, and by substituting some numerical values to them.

In Figure 1, the critical Rayleigh number R_1 decreases with increase in dust particles parameter B which shows that have dust particles have destabilizing effect on the system that indicates when the critical Rayleigh number R_1 is plotted against dust particles B for fixed value of $Q_1 = 1$, $F_1 = 1$ and $M = 10$. In Figure 2, the critical Rayleigh number R_1 decreases with increase in Hall currents parameter M which indicates that Hall currents have a destabilizing effect on the system when critical Rayleigh number R_1 is plotted against Hall currents parameter M for fixed value of $Q_1 = 3000$, $B = 1$, $F_1 = 1$. In Figure 3, the critical Rayleigh number R_1 decreases to certain values of Q_1 and gradually which shows that magnetic field has both stabilizing and destabilizing effect on the system whenever Critical Rayleigh number R_1 is plotted against magnetic field parameter Q_1 for fixed value of $H_1 = 0.1$, $M = 6$ and $F_1 = 0.1$. In Figure 4, the critical Rayleigh number R_1 increases with increase in couple-stress parameter F_1 which shows that couple-stress has a stabilizing effect on the system when critical Rayleigh number R_1 is plotted against couple-stress parameter F_1 for fixed value of $B = 1$, $M = 1$ and $Q_1 = 5$.

7. Conclusion

In the present paper, the combine effect of Hall currents on an electrically conducting couple-stress fluid layer heated from below in the presence of horizontal magnetic field is considered. Dispersion relation governing the effects of dust particles, Hall currents, magnetic field and couple-stresses has been investigated analytically as well as graphically. The main results from the analysis are summarized as follows:

(1). For the case of stationary convection, dust particles have a destabilizing effect on the system as can be seen from equation (28), and graphically from Figure 1.

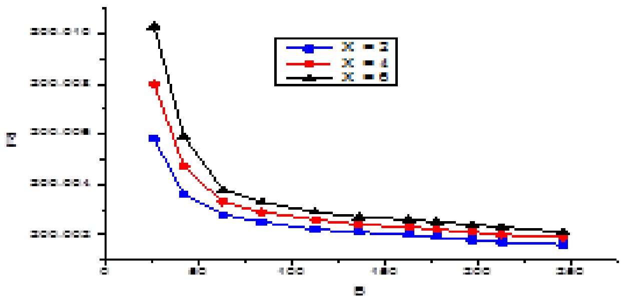

Figure 1. Variation of critical Rayleigh number R_1 with dust particles B for fixed value of $Q_1 = 1$, $F_1 = 1$, $M = 10$ and $x = 2, 4, 6$.

(2). Hall currents have a destabilizing effect on the system which can be seen from equation (29) and graphically from Figure 2.

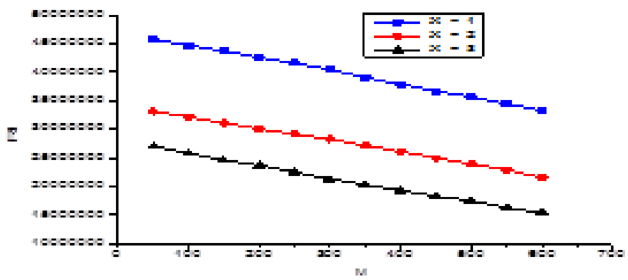

Figure 2. Variation of critical Rayleigh number R_1 with dust particles B for fixed value of $Q_1 = 3000$, $B = 1$, $F_1 = 1$ and $x = 1, 2, 3$.

(3). Magnetic field has a stabilizing or destabilizing effect on the thermal convection as can be seen from equation (30) and graphically from Figure 3.

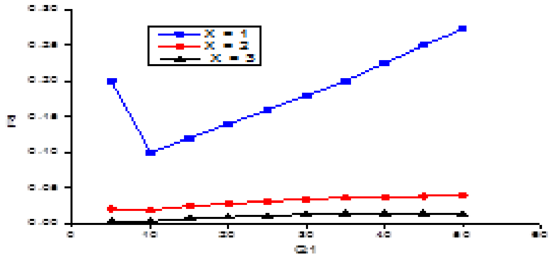

Figure 3. Variation of critical Rayleigh number R_1 with dust particles Q_1 for fixed value of $H_1 = 0.1$, $M = 6$, $F_1 = 0.1$ and $x = 1, 2, 3$.

(4). Couple-stress has stabilizing effect on the system for the permissible values of various parameters which can be expressed from equation (31).

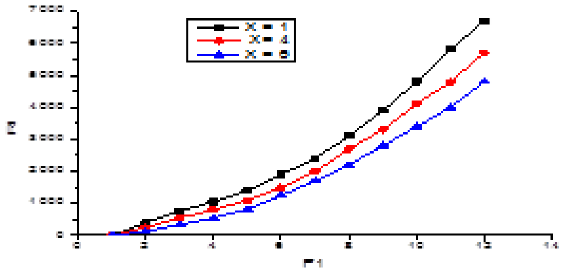

Figure 4. Variation of critical Rayleigh number R_1 with dust particles F_1 for fixed value of $B = 1$, $M = 1$, $Q_1 = 5$ and $x = 1, 4, 6$.

(5). The principle of exchange of stabilities is satisfied in the absence of magnetic field (hence Hall currents) and dust particles.

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Nomenclature

c	: Speed of light
d	: Depth of layer
e	: Charge of an electron
F	: Couple stress parameter $\left(\frac{\mu^1}{\rho_0 d^2}\right)$
$g(0,0,-g)$	: Acceleration due to gravity field
$\overrightarrow{H}(H,0,0)$	: Uniform magnetic field
$\overrightarrow{h}(h_x,h_y,h_z)$	: Perturbations in magnetic field
k_x	: Wave number in x-direction
k_y	: Wave number in y-direction
k	: Resultant wave number $k=\sqrt{K_x^2+K_y^2}$
k_T	: Thermal diffusivity
M	: Hall current parameter $=\left(\frac{cH}{4\pi Ne\eta}\right)^2$
N	: Electron number density
n	: Growth rate

p	: Fluid pressure
P_1	: Prandtl number $= \left(\frac{V}{K_T}\right)$
P_2	: Magnetic Prandtl number $= \left(\frac{V}{\eta}\right)$
$\overrightarrow{q}(u, v, w)$	: Component of velocity after perturbation
$\overrightarrow{q_d}(l, r, s)$	: Component of particles velocity after perturbation
Q	: Chandrasekhar number $= \left(\frac{\mu_e H^2 d^2}{4\pi\rho_n v\eta}\right)$
R	: Rayleigh number $= \left(\frac{g\alpha\beta d^4}{VK_T}\right)$
R_1	: Critical Rayleigh number
t	: Time coordinate
T	: Temperature
x	: Dimensionless wave number
$\overrightarrow{x}(x, y, z)$	: Space coordinates

Greek Symbols

α	: Coefficient of thermal expansion
β	: Uniform temperature gradient
η	: Electrical resistivity
η'	: Suspended particle radius
θ	: Perturbation in temperature
δp	: Perturbation in pressure p
ρ	: Fluid density
$\delta\rho$	: Perturbation in density ρ
v	: Kinematic viscosity
v'	: Couple stress viscoelasticity
μ'	: Couple stress viscosity
μ_e	: Magnetic permeability
∇, ∂, D	: Del operator, curly operator and Derivative with respect to $z(= d/dz)$