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An Algorithm to Balance an Unbalanced Transportation Problem to Achieve Improved Initial Solution

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Abstract: This paper suggests an algorithm to balance an unbalanced transportation problem in order to obtain improved initial feasible solution of unbalanced transportation problem proceed with Vogel's Approximation Method, in comparison to other existing algorithms to balance the transportation problem. The algorithm is supported by numerical examples.

Keywords: Unbalanced Transportation Problem, dummy row/column, Transportation Problem, Vogel's Approximation Method.
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1. Introduction

In 1941, Hitchcock [4] originally developed the basic transportation problem. In 1953, Charnes [1] developed the stepping stone method which provided an alternative way of determining the simplex method information. In 1963, Dantzig [3] used the simplex method to the transportation problems as the primal simplex transportation method. Till date, several researchers studied extensively to solve cost minimizing transportation problem in various ways. A transportation problem is unbalanced if the sum of all available quantities is not equal to the sum of requirements or vice-versa. In regular approach, unbalanced transportation problem is balanced by introducing either a dummy row or dummy column to the cost matrix. If total availability is more than the total requirement, a dummy column (destination) is introduced with the requirement to overcome the difference between total availability and total requirement. Similar, if total requirement is more than the total availabilities. Cost for dummy row / column cells are set to zero. Further, problem is usually solved by Vogel's Approximation Method to find an initial solution. This paper suggests an algorithm to balance the unbalanced problem which gives improved initial feasible solution on proceeding with Vogel's Approximation Method.

2. Analysis

Goyal [7] suggested that to assume the largest unit cost of transportation to and from a dummy row or column, present in the given cost matrix rather than assuming to be zero as usual in Vogel's Approximation method. He claimed that by this modification, the allocation of units to dummy row or column is automatically given least priority and in addition to this the row or column penalty costs are considered for each interaction. He justified his suggestion by comparing the solution of a numerical problem with VAM. Ramakrishnan [2] discussed some improvement to Goyal's Modified VAM for Unbalanced

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TP. While Shimshak [8] suggested to ignore the penalty cost involved with the dummy row or column. So that to give least priority to the allocation of units in dummy row or column. With this suggestion Shimshak [8] obtained initial solution by Vogel's Approximation Method. N. Girmay and T. Sharma [5] suggested a heuristic approach of balancing the unbalanced transportation problem stating "no dummy row or column is required to introduce in the cost matrix to find an initial solution to the unbalanced transportation problem".

3. Suggestion

Present study suggests an algorithm to find the exact destination/origin where one should decrease the availability / requirement whichever is greater in order to balance the transportation problem and get improved initial solution for unbalanced transportation problem in comparison to dummy row/column method. The suggested algorithm is less laborious, economical and sometimes gives more feasible initial solution on proceeding with Vogel's Approximation Method.

4. Formulation of Transportation Problem

Let the transportation problem consist of m origins and n destinations, where

x_{ij} = the amount of goods transported from the i^{th} origin to the j^{th} destination.

c_{ij} = the cost involved in transporting per unit product from the i^{th} origin to the j^{th} destination.

a_i = the number of units available at the i^{th} origin.

b_j = the number of units required at the j^{th} destination.

Consider the linear transportation problem as:

$$Minimize\ Z = \sum \sum c_{ij} x_{ij}$$

Subject to the constraints

$$\begin{aligned} a_{ij} &= \sum x_{ij}; \quad \text{for all } i \in I = (1, 2, \ldots, m) \\ b_{ij} &= \sum x_{ij}; \quad \text{for all } j \in J = (1, 2, \ldots, n) \\ and\ \ x_{ij} &\geq 0; \qquad \text{for all } (i, j) \in I \times J. \end{aligned}$$

For unbalanced T.P. either $a_{ij} < b_{ij}$ or $a_{ij} > b_{ij}$.

5. Proposed Algorithm

Step 1: Find the sum of all the requirements and the availabilities.

Step 2: Choose the one which is lesser.

Case (i): When the sum of availability is less.

Check each row and mark (✓) the maximum cost cell and the corresponding columns. Which column gets more entries, deduct the difference from the requirement of that column in order to balance the given unbalanced transportation problem.

Case (ii): When the sum of requirements is less.

Check each column and mark (✓) the maximum cost cell and the corresponding rows. Which row gets more entries, deduct the difference from the availability of that row in order to balance the given unbalanced transportation problem.

Remarks. If there is a tie to chosen the maximum cost cell than consider two or more corresponding rows/columns to deduct equal amount from each such that the sum of the deducted amount is equal to the difference between total availability and requirement.

Step 3: Apply regular Vogel's Approximation method to find initial solution of the given problem.

Let us consider the following examples to support the algorithm.

6. Numerical Illustrations

Example 6.1. *Consider the following cost matrix of transportation problem.*

	D_1	D_2	D_3	Capacity
O_1	4	8	8	76
O_2	16	24	16	82
O_3	8	16	24	77
Requirement	72	102	41	

Solution. By traditional method of balancing using dummy row/column. Here $a_i = 235$ and $b_j = 215$, therefore $a_i > b_j$. Balanced T.P. by introducing dummy column with zero costs.

	D_1	D_2	D_3	Dummy	Capacity
O_1	4	8	8	0	76
O_2	16	24	16	0	82
O_3	8	16	24	0	77
Requirement	72	102	41	4	

Initial Feasible Solution by Vogel's Approximation Method

	D_1	D_2	D_3	Dummy	Capacity
O_1	4	**35** 8	**41** 8	0	76
O_2	16	**62** 24	16	**20** 0	82
O_3	**72** 8	**5** 16	24	0	77
Requirement	72	102	41	4	

Allocations: $x_{12} = 35$, $x_{13} = 41$, $x_{22} = 62$, $x_{31} = 72$, $x_{32} = 5$ and $x_{24} = 20$. Initial Transportation Cost is 2752/-. The optimum transportation cost is 2424/-.

Initial Solution by the Proposed Algorithm

Step 1: Find sum of all requirements and availabilities. Here $a_i = 235$ and $b_j = 215$ i.e., Requirement $<$ Availability. Reduce availability by 20.

Step 2: Case (i): when the sum of requirement is less.

Check each column to maximum cost and the corresponding row which gets more entries choose to deduct the requirement in order to balance the given unbalanced transportation problem. Here, in 1^{st}, 2^{nd}, and 3^{rd} column are having maximum costs in 2, 2, and 3 rows respectively. That is; cell $(2,1)$, $(2,2)$, and $(3,3)$. Now 2^{nd} row has more marked entries so choose 2^{nd} row to make reduction. Therefore, the availability of 2^{nd} row will be reduced by 20 units.

Balanced T.P. by proposed algorithm:

	D_1	D_2	D_3	Capacity
O_1	4	8	8	76
O_2	**16**✓	**24**✓	**16**	$82 - 20 = 62$
O_3	8	16	24✓	77
Requirement	72	102	41	

Step 3: Initial feasible solution by Vogel's Approximation method

	D_1	D_2	D_3	Capacity
O_1	4	**76** 8	8	76
O_2	16	**21** 24	**41** 16	62
O_3	**72** 8	**5** 16	24	77
Requirement	72	102	41	

Total Initial Cost $= 76(8) + 21(24) + 41(16) + 72(8) + 16(5) = 2524/-$.

The initial feasible cost obtained by the proposed algorithm is lesser than the initial feasible cost obtained by regular dummy row approach of balancing the Transportation Problem.

Example 6.2. *Consider the following Cost matrix for the T.P.*

	I	II	III	IV	V	Available
1	5	8	6	6	3	800
2	4	7	7	6	5	500
3	8	4	6	6	4	900
Requirement	400	400	500	400	800	

Here $a_i = 2200$ and $b_j = 2500$ so $a_i < b_j$. Balanced T.P. by introducing dummy column with zero costs.

	I	II	III	IV	V	Available
1	5	8	6	6	3	800
2	4	7	7	6	5	500
3	8	4	6	6	4	900
Dummy	0	0	0	0	0	300
Requirement	400	400	500	400	800	

Initial Feasible Solution by Vogel's Approximation Method

	I	II	III	IV	V	Available
1	5	8	**200** 6	**300** 6	**300** 3	800
2	**400** 4	7	7	**100** 6	5	500
3	8	**400** 4	6	6	**500** 4	900
Dummy	0	0	**300** 0	0	0	300
Requirement	400	400	500	400	800	

Allocations: $x_{13} = 200$, $x_{14} = 300$, $x_{15} = 300$, $x_{21} = 400$, $x_{24} = 100$, and $x_{32} = 300$. Initial transportation cost = 9700/-.

Initial solution by proposed algorithm

Step 1: Find sum of all requirements and availabilities. Whichever is more, choose to reduce. Here $a_i = 2200$ and $b_j = 2500$ i.e., Requirement > Availability. Reduce the requirement by 300 units.

Step 2: Here the sum of availability is less so check each row to maximum cost and the column which gets more entries choose to deduct the requirement in order to balance the given unbalanced transportation problem. Here cell $(1, 2)$, $(2, 2)$, $(2, 3)$ and $(1, 3)$ are having maximum cost in 1st, 2nd, and 3rd row. Now column II has more marked entries so we choose column II to make reduction. Also we observe that 2nd row has maximum cost 7 in 2nd and 3rd cells, there is a tie so choose column III also for the reduction. Therefore, the requirements of column II and III to will be reduced by 150 from each. Balanced T.P. by present approach

	I	II	III	IV	V	Available
1	5	**8**✓	6	6	3	800
2	4	**7**✓	**7**✓	6	5	500
3	**8**✓	4	6	6	4	900
Requirement	400	400 − 150 = 250	500 − 150 = 350	400	800	

Now $a_i = b_j = 2200$.

Step 3: Apply Vogel's Approximation method to find initial solution of the given problem.

Allocations: $x_{15} = 800$, $x_{21} = 400$, $x_{24} = 100$, $x_{32} = 250$, $x_{33} = 350$ and $x_{34} = 300$. Initial feasible Cost = 9500/-. The optimal transportation cost = 9200/-.

	I	II	III	IV	V	Available
1	5	8	6	6	**800** 3	800
2	**400** 4	7	7	**100** 6	5	500
3	8	**250** 4	**350** 6	**300** 6	4	900
Requirement	400	100	500	400	800	

7. Conclusion

The initial feasible cost obtained using the proposed algorithm is lesser and nearer to optimal solution than the initial feasible cost calculated by regular dummy row/column approach of balancing the TP. Also this approach reduces the steps of SS/MODI method.

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