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Fully Fuzzy Integer Linear Programming Problems Under Robust Ranking Techniques

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Abstract: The focus of this paper is to find fuzzy optimal solution of fully fuzzy integer linear programming problems (FFILPP) with pentagonal fuzzy numbers under robust ranking technique. A new approach for solving fully fuzzy integer linear programming problems with pentagonal fuzzy number is proposed, based on ranking function. The proposed method is very easy to understand. This is illustrated with relevant numerical examples.

Keywords: Ranking function, Robust ranking function, Pentagonal fuzzy number, Fully fuzzy integer linear programming.

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1. Introduction

The concept linear programming problem is to find out the best solution to the real-world problems where the available informations are not exact or not precise. In that situation linear programming model helps lot. Firstly, the concept Fuzzy linear programming was proposed by Tanaka [13]. It plays a vital role in Fuzzy modeling, which can formulate the uncertainty. Nasseri [9] has proposed a new method for solving the Fuzzy linear programming problems in which he has used the fuzzy ranking method for converting the fuzzy objective function into crisp objective function. Fuzzy linear programming was studied by many researchers [2–4, 6–8, 14]. Sahaya Sudha [11] proposed solving fuzzy linear programming problem using pentagonal fuzzy numbers with robust ranking method. Herrera and Verdegay [5] have proposed three methods for solving three models of Fuzzy integer linear programming. Allahviranloo [1] discussed a model of Fuzzy integer linear programming problem with fuzzy variable and proposed to solve a new method. Pandian and Jayalakshmi [10] have proposed a decomposition method for solving Fuzzy integer linear programming problem with fuzzy variables by using classical integer linear programming. In [12] Stephen Dinagar and Mohamed Jeyavuthin discussed the concept of solving integer linear programming problems with pentagonal fuzzy numbers. In this paper, section 2 contains some basic definitions needed for this work. In section 3, fully fuzzy integer linear programming with fuzzy variables are discussed. In section 4, relevant numerical illustrations are given. Finally, conclusion is included in section 5.

2. Preliminaries

Definition 2.1 (Fuzzy set)**.** *A fuzzy set A in a universe of discourse X is defined as the following set of pairs*

$$A = \{(x, \mu_A(x); x \in X)\}$$

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Here $\mu_A : X \to [0,1]$ is mapping called the degree of membership function of the fuzzy set A and $\mu_A(x)$ is called the membership value of $x \in X$ in the fuzzy set A.

Definition 2.2 (Convex fuzzy set). *A fuzzy set $A = \{(x, \mu_A(x))\} \subseteq X$ is called convex fuzzy set if all A_α are convex set (i.e.) for every element $x_1 \in A_\alpha$ and $x_2 \in A_a$ for every $\alpha \in [0,1]$. $\lambda x_1 + (1-\lambda)x_2 \in A_a$ for all $\lambda \in [0,1]$. Otherwise the fuzzy set is called non-convex fuzzy set.*

Definition 2.3 (Fuzzy number). *A fuzzy set $\tilde{A}$, defined on the set of real number R is said to be fuzzy number if it has the following characteristics*

(1). $\tilde{A}$ is normal

(2). $\tilde{A}$ is convex set

(3). The support of $\tilde{A}$ is closed and bounded.

Definition 2.4 (Pentagonal fuzzy number). *A fuzzy number $\tilde{A}_P$ is pentagonal fuzzy number denoted by $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$, where a_1, a_2, a_3, a_4, a_5 are real numbers and its membership function $\mu_{\tilde{A}_P}(x)$ is given by*

$$\mu_{\tilde{A}_P}(x) = \begin{cases} 0, & x < a_1 \\ \frac{1}{2}\left[\frac{x-a_1}{a_2-a_1}\right], & a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left[\frac{x-a_2}{a_3-a_2}\right], & a_2 \le x \le a_3 \\ 1, & x = a_3 \\ \frac{1}{2} + \frac{1}{2}\left[\frac{a_4-x}{a_4-a_3}\right], & a_3 \le x \le a_4 \\ \frac{1}{2}\left[\frac{a_5-x}{a_5-a_4}\right], & a_4 \le x \le a_5 \\ 0, & x > a_5 \end{cases}$$

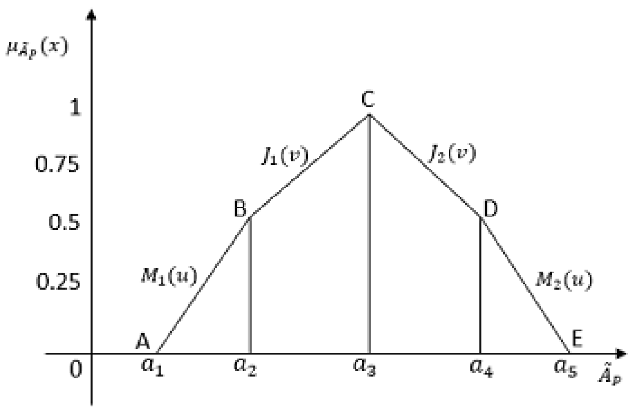

Figure 1. **Graph of a Pentagonal Fuzzy Number**

Definition 2.5. *A pentagonal fuzzy number can be defined as $\tilde{A}_P = (M_1(u), J_1(v), J_2(v), M_2(u))$ for $u \in [0, 0.5]$ and $v \in [0.5, 1]$ where,*

(1). $M_1(u)$ is strictly increasing continuous function on [0,0.5].

(2). $J_1(v)$ is strictly increasing continuous function on [0.5,1].

(3). $J_2(v)$ is strictly decreasing continuous function on [1,0.5].

(4). $M_2(u)$ is strictly decreasing continuous function on [0.5,0].

Remark 2.6. *The pentagonal fuzzy number $\tilde{A}_P$ becomes triangular fuzzy number if $a_3 - a_2 = a_4 - a_3$.*

Definition 2.7. *A linear Programming problem is called fuzzy variable linear programming problem (FVLPP), if some of the parameters are crisp, and variables and right-hand sides are fuzzy numbers. General form of FVLPP as follows:*

$$Maximize\ (or\ Minimize)\ \tilde{z} = \tilde{c}\tilde{x}$$

Subject to,

$$\tilde{A}\tilde{x}\,\{\leq, =, \geq\}\,\tilde{b}, \quad \tilde{x} \geq 0 \ and\ are\ integers,$$

Where $\tilde{c} \in R^n$, $\tilde{A} \in (R)^{m+n}$, $R_i \in (F(R))^m$ and $\tilde{x} \in (F\,(R))^n$.

Definition 2.8. *A ranking function is a map from $F\,(R)$ into real line. Now, we define the orders $F\,(R)$ as follows:*

(1). $\tilde{A}_P \geq \tilde{B}_P$ if and only if $R\left(\tilde{A}_P\right) \geq R(\tilde{B}_P)$.

(2). $\tilde{A}_P \leq \tilde{B}_P$ if and only if $R\left(\tilde{A}_P\right) \leq R(\tilde{B}_P)$.

(3). $\tilde{A}_P = \tilde{B}_P$ if and only if $R\left(\tilde{A}_P\right) = R(\tilde{B}_P)$.

Where $\tilde{A}_P$, $\tilde{B}_P$ are elements of $F(R)$.

3. Fully Fuzzy Integer Linear Programming(FFILPP)

Consider the following fully fuzzy integer linear programming problems:

$$Maximize\ (or\ Minimize)\ \tilde{z} = \tilde{c}^T\tilde{x}$$

Subject to,

$$\tilde{A}\tilde{x}\,\{\leq, =, \geq\}\,\tilde{b}, \quad \tilde{x} \geq 0 \ and\ are\ integers,$$

Where the cost vectors $\tilde{c}^T = (\tilde{c}_j)_{1\times n}$, $\tilde{A} = (\tilde{a}_{ij})_{m\times n}$, $\tilde{x} = (\tilde{x}_j)_{n\times 1}$ and $\tilde{b} = (\tilde{b}_i)_{m\times 1}$ and $\tilde{a}_{ij}$, $\tilde{x}_j$, $\tilde{b}_i$, $\tilde{c}_j \in F\,(R)$, for all $1 \leq j \leq n$ and $1 \leq i \leq m$.

3.1. Ranking Functions

The Robust ranking functions [11] for pentagonal fuzzy number has been utilized to solve the fully fuzzy integer linear programming problems as given below. Let $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ be a pentagonal fuzzy number then the ranking functions are:

(1). $R_1\left(\tilde{A}_P\right) = \int_0^1 \{(a_2 - a_1)\,\alpha + a_1, a_5 - (a_5 - a_4)\,\alpha\}\,(0.5)\,d\alpha$

(2). $R_2\left(\tilde{A}_P\right) = \int_0^1 \left\{\begin{array}{c} (a_2 - a_1)\,\alpha + a_1 + (a_3 - a_2)\,\alpha + a_2 \\ a_4 - (a_4 - a_3)\,\alpha + a_5 + (a_5 - a_4)\,\alpha \end{array}\right\}(0.5)d\alpha$

(3). $R_3\left(\tilde{A}_P\right) = \int_0^1 \left\{\begin{array}{c} (a_4 - a_1)\,\alpha + a_1 + a_2 + a_3, \\ a_3 + a_4 + a_5 + (a_5 - a_2)\,\alpha \end{array}\right\}(0.5)d\alpha$

3.2. Algorithm

Step 1: Formulate the chosen problem into the following fuzzy linear programming problem as

$$Maximize\ \tilde{z} = \sum_{j=i}^{n} \tilde{c}_j \tilde{x}_j$$

Subject to, $\sum_{j=i}^{n} \tilde{a}_{ij}\tilde{x}_j \leq, =, \geq \tilde{b}_i,\ i = 1, 2, 3, \ldots, m;\ \tilde{x}_j \geq 0,\ j = 1, 2, 3, \ldots, n.$

Step 2: Using the Ranking functions, the FFILPP transformed into FVILPP.

Step 3: Optimal solution.

4. Numerical Examples

Example 4.1. *Consider the following fully fuzzy integer linear programing problem*

$$Maximize\ \tilde{Z} = (2, 4, 6, 8, 10)\tilde{x}_1 + (6, 8, 10, 12, 14)\tilde{x}_2$$

Subject to

$$(12, 14, 16, 18, 20)\tilde{x}_1 + (16, 18, 20, 22, 24)\tilde{x}_2 \leq (26, 28, 30, 32, 34);$$
$$(18, 20, 22, 24, 26)\tilde{x}_1 + (22, 24, 26, 28, 30)\tilde{x}_2 \leq (40, 42, 44, 46, 48);$$

$\tilde{x}_1, \tilde{x}_2 \geq 0$ *and integers.*

Now we calculate $R_1\,(2, 4, 6, 8, 10)$ by using Robust's ranking method (i).

$$R_1\,(2, 4, 6, 8, 10) = \int_0^1 (2\alpha + 2, 10 - 2\alpha)\,(0.5)\, d\alpha = 6$$

Similarly,

$$R_1\,(6, 8, 10, 12, 14) = 10,\quad R_1\,(12, 14, 16, 18, 20) = 16,\quad R_1\,(16, 18, 20, 22, 24) = 20,$$
$$R_1\,(26, 28, 30, 32, 34) = 30,\quad R_1\,(18, 20, 22, 24, 26) = 22,\quad R_1\,(22, 24, 26, 28, 30) = 26,$$
$$R_1\,(40, 42, 44, 46, 48) = 44,$$

Using the ranking functions (i) the problem becomes

$$(P_1)\ Maximize\ \tilde{Z} = 6\tilde{x}_1 + 10\tilde{x}_2$$

Subject to

$$16\tilde{x}_1 + 20\tilde{x}_2 \leq 30;$$
$$22\tilde{x}_1 + 26\tilde{x}_2 \leq 44;$$

$\tilde{x}_1,\ \tilde{x}_2 \geq 0$ and are integers.

Now we calculate $R_2\,(2,4,6,8,10)$ by using Robust's ranking method (ii).

$$R_2\,(2,4,6,8,10) = \int_0^1 (2\alpha + 2 + 2\alpha + 4, 8 - 2\alpha + 10 - 2\alpha)\,(0.5)\,d\alpha = 12$$

Similarly,

$$\begin{aligned} &R_2\,(6,8,10,12,14) = 20, \quad R_2\,(12,14,16,18,20) = 32, \quad R_2\,(16,18,20,22,24) = 40,\\ &R_2\,(26,28,30,32,34) = 60, \quad R_2\,(18,20,22,24,26) = 44, \quad R_2\,(22,24,26,28,30) = 52,\\ &R_2\,(40,42,44,46,48) = 88, \end{aligned}$$

Using the ranking functions (ii) the problem becomes

$$(P_2)\ Maximize\ \tilde{Z} = 12\tilde{x}_1 + 20\tilde{x}_2$$

Subject to

$$32\tilde{x}_1 + 40\tilde{x}_2 \le 60;$$

$$44\tilde{x}_1 + 52\tilde{x}_2 \le 88;$$

$\tilde{x}_1,\ \tilde{x}_2 \ge 0$ and are integers.

Now we calculate $\mathrm{R}_3\,(2,4,6,8,10)$ by using Robust's ranking method (iii).

$$R_3\,(2,4,6,8,10) = \int_0^1 (6\alpha + 2 + 4 + 6,\ 6 + 8 + 10 - 6\alpha)\,(0.5)\,d\alpha = 18$$

Similarly,

$$\begin{aligned} &R_3\,(6,8,10,12,14) = 30, \quad R_3\,(12,14,16,18,20) = 48, \quad R_3\,(16,18,20,22,24) = 60,\\ &R_3\,(26,28,30,32,34) = 90, \quad R_3\,(18,20,22,24,26) = 66, \quad R_3\,(22,24,26,28,30) = 78,\\ &R_3\,(40,42,44,46,48) = 132, \end{aligned}$$

Using the ranking functions (iii) the problem becomes

$$(P_3)\ Maximize\ \tilde{Z} = 18\tilde{x}_1 + 30\tilde{x}_2$$

Subject to

$$48\tilde{x}_1 + 60\tilde{x}_2 \le 90;$$

$$66\tilde{x}_1 + 78\tilde{x}_2 \le 132;$$

$\tilde{x}_1,\ \tilde{x}_2 \ge 0$ and are integers.

Using an algorithm for integer linear programming problem,

The solution of the problem (P_1) is $\tilde{x}_1 = 0$, $\tilde{x}_2 = 1$ and $\tilde{Z} = 10$,

The solution of the problem (P_2) is $\tilde{x}_1 = 0$, $\tilde{x}_2 = 1$ and $\tilde{Z} = 20$,

The solution of the problem (P_3) is $\tilde{x}_1 = 0$, $\tilde{x}_2 = 1$ and $\tilde{Z} = 30$.

Ranking Method	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{Z}$
Ranking function (i)	0	1	10
Ranking function (ii)	0	1	20
Ranking function (iii)	0	1	30

Table 1. **Comparison Table**

Example 4.2. *Consider the following fully fuzzy integer linear programing problem*

$$Maximize\ \tilde{Z} = (2,6,8,12,14)\tilde{x}_1 + (4,6,10,12,14)\tilde{x}_2$$

Subject to

$$(6,10,12,14,16)\tilde{x}_1 + (6,8,10,12,16)\tilde{x}_2 \leq (12,14,16,18,20);$$

$$(4,8,10,12,14)\tilde{x}_1 + (6,10,12,14,16)\tilde{x}_2 \leq (14,16,18,22,24);$$

$\tilde{x}_1,\ \tilde{x}_2 \geq 0$ *and are integers.*

Using the ranking functions (i), (ii) and (iii) the corresponding problems are (P_1), (P_2) and (P_3) respectively,

$$(P_1)\ Maximize\ \tilde{Z} = 4.5\tilde{x}_1 + 9\tilde{x}_2$$

Subject to

$$11.5\tilde{x}_1 + 11.5\tilde{x}_2 \leq 16;$$

$$9.5\tilde{x}_1 + 11.5\tilde{x}_2 \leq 19;$$

$\tilde{x}_1, \tilde{x}_2 \geq 0$ and are integers.

$$(P_2)\ Maximize\ \tilde{Z} = 17\tilde{x}_1 + 19.5\tilde{x}_2$$

Subject to

$$23.5\tilde{x}_1 + 20.5\tilde{x}_2 \leq 32;$$

$$19.5\tilde{x}_1 + 23.5\tilde{x}_2 \leq 37.5;$$

$\tilde{x}_1,\ \tilde{x}_2 \geq 0$ and are integers.

$$(P_3)\ Maximize\tilde{Z} = 25.5\tilde{x}_1 + 28\tilde{x}_2$$

Subject to

$$35.5\tilde{x}_1 + 30.5\tilde{x}_2 \leq 48;$$

$$29.5\tilde{x}_1 + 28.5\tilde{x}_2 \leq 56;$$

$\tilde{x}_1,\ \tilde{x}_2 \geq$ and are integers.

Using algorithm for integer linear programming problem,

The solution of the problem (P_1) is $\tilde{x}_1 = 0$, $\tilde{x}_2 = 1$ and $\tilde{Z} = 9$,

The solution of the problem (P_2) is $\tilde{x}_1 = 0$, $\tilde{x}_2 = 1$ and $\tilde{Z} = 19.5$,

The solution of the problem (P_3) is $\tilde{x}_1 = 0$, $\tilde{x}_2 = 1$ and $\tilde{Z} = 28$.

Ranking Method	$\tilde{x}_1$	$\tilde{x}_2$	$\tilde{Z}$
Ranking function (i)	0	1	9
Ranking function (ii)	0	1	19.5
Ranking function (iii)	0	1	28

Table 2. Comparison Table

5. Conclusion

The proposed method provides a solution for fully fuzzy integer linear programming problems with pentagonal fuzzy number. The problem is converted into fuzzy variable integer linear programming problem using new ranking techniques. It is important to note that among three ranking functions, the ranking function (iii) gives the maximum value. This notion can be extended to some other optimization problems in future.

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