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# Decompositions of if $*_g$-continuity

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**Abstract :** The aim of this paper is to give decompositions of a weaker form of intuitionistic fuzzy continuity, namely intuitionistic fuzzy *g-continuity, by providing the concepts of intuitionistic fuzzy *$g_t$-sets, intuitionistic fuzzy *$g_\alpha*$-sets, intuitionistic fuzzy *$g_t$-continuity and intuitionistic fuzzy *$g_\alpha*$-continuity.

**Keywords :** Intuitionistic fuzzy *g-closed set, intuitionistic fuzzy $\hat{\eta}$-closed set, intuitionistic fuzzy *$g_t$-set, intuitionistic fuzzy *$g_\alpha*$-set, intuitionistic fuzzy *$g_t$-continuity, intuitionistic fuzzy *$g_\alpha*$-continuity.

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## 1 Introduction

Levine [10], Mashhour et. al. [11] and Njastad [12] introduced semi-open sets, preopen sets and $\alpha$-open sets respectively. In 1961, Levine [9] obtained a decomposition of continuity which was later improved by Rose [16]. Tong [22] decomposed continuity into $\alpha$-continuity and A-continuity and showed that his decomposition is independent of Levine's. Hatir et. al. [8] also obtained a decomposition of continuity. The concept of $\omega$-closed sets was introduced and studied by Sheik John and Sundaram [18]. Veerakumar [23] and Abd El-Monsef et. al. [1] introduced $\hat{g}$-closed sets and $\alpha\hat{g}$-closed sets in topological spaces respectively. It is known that $\omega$-closed sets and $\hat{g}$-closed sets are same. Veerakumar [24] introduced and studied $\alpha^*$g-closed sets in topological spaces. Benchalli et. al. [4] introduced and studied the notion of $\omega\alpha$-closed sets. It is known that $\omega\alpha$-closed sets, $\alpha^*$g-closed sets and $\alpha\hat{g}$-closed sets are all same. Palaniappan et. al. [13] introduced and studied $\hat{\eta}$-closed sets in topological spaces.

In 1965, Zadeh [25] introduced fuzzy sets and in 1968, Chang [5] introduced fuzzy topology. After the introduction of fuzzy sets and fuzzy topology, several researches were worked on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced

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by Atanassov [3] as a generalization of fuzzy sets. In 1997, Coker [6] introduced the concept of intuitionistic fuzzy topological spaces. The decomposition of intuitionistic fuzzy continuity is one of the many problems in intuitionistic fuzzy topology. Recently, Ramesh and Thirumalaiswamy [14] have obtained decompositions of a weaker form of intuitionistic fuzzy continuity.

In this paper we introduce $^*g_t$-continuity and $^*g_{\alpha}*$-continuity to obtain decompositions of $^*$g-continuity in intuitionistic fuzzy topological spaces.

## 2 Preliminaries

Throughout this paper, (X, $\tau$) and (Y, $\sigma$) (simply, X and Y) denote intuitionistic fuzzy topological spaces (briefly, IFTSs) on which no separation axioms are assumed. Let A be a subset of a space X. The closure of A and the interior of A are denoted by cl(A) and int(A), respectively.

The following definitions are useful in the sequel.

**Definition 2.1.** *A subset A of an intuitionistic fuzzy topological space (X, $\tau$) is said to be intuitionistic fuzzy semi-open [7] (resp. intuitionistic fuzzy preopen [7], intuitionistic fuzzy $\alpha$-open [7]) if $A \subseteq cl(int(A))$ (resp. $A \subseteq int(cl(A))$, $A \subseteq int(cl(int(A)))$). The complement of intuitionistic fuzzy semi-open (resp. intuitionistic fuzzy preopen, intuitionistic fuzzy $\alpha$-open) set is called intuitionistic fuzzy semi-closed (resp. intuitionistic fuzzy preclosed, intuitionistic fuzzy $\alpha$-closed (briefly, IF $\alpha$-closed)) set.*

**Definition 2.2.** *A subset A of an intuitionistic fuzzy topological space (X, $\tau$) is said to be*

*(i) an intuitionistic fuzzy t-set [2] if $int(A) = int(cl(A))$.*

*(ii) an intuitionistic fuzzy $\alpha$*-set [14] if $int(A) = int(cl(int(A)))$.*

**Remark 2.3.**

*(i) Every intuitionistic fuzzy t-set is an intuitionistic fuzzy $\alpha$*-set, but not conversely [14].*

*(ii) An intuitionistic fuzzy open set need not be an intuitionistic fuzzy $\alpha$*-set [15].*

*(iii) The union of two intuitionistic fuzzy $\alpha$*-sets need not be an intuitionistic fuzzy $\alpha$*-set [15].*

*(iv) Arbitrary intersection of intuitionistic fuzzy $\alpha$*-sets is an intuitionistic fuzzy $\alpha$*-set [15].*

**Definition 2.4.** *A subset A of an intuitionistic fuzzy topological space (X, $\tau$) is called*

*(i) an intuitionistic fuzzy g-closed [20] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is intuitionistic fuzzy open in X.*

*(ii) an intuitionistic fuzzy $\hat{g}$-closed [19] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is intuitionistic fuzzy semi-open in X.*

*The complement of an intuitionistic fuzzy g-closed (resp. intuitionistic fuzzy $\hat{g}$-closed) set is called intuitionistic fuzzy g-open (resp. intuitionistic fuzzy $\hat{g}$-open).*

*For a subset A of an intuitionistic fuzzy topological space X, the intuitionistic fuzzy $\alpha$-closure (resp. intuitionistic fuzzy semi-closure, intuitionistic fuzzy pre-closure) of A, denoted by $\alpha cl(A)$ (resp. $scl(A)$,* *pcl(A)), is the intersection of all intuitionistic fuzzy $\alpha$-closed (resp. intuitionistic fuzzy semi-closed, intuitionistic fuzzy preclosed) subsets of X containing A. Dually, the intuitionistic fuzzy $\alpha$-interior (resp. intuitionistic fuzzy semi-interior, intuitionistic fuzzy pre-interior) of A, denoted by $\alpha$int(A) (resp. sint(A), pint(A)), is the union of all intuitionistic fuzzy $\alpha$-open (resp. intuitionistic fuzzy semi-open, intuitionistic fuzzy preopen) subsets of X contained in A.*

**Proposition 2.5** ([14]). *Let A and B be intuitionistic fuzzy subsets of an intuitionistic fuzzy topological space X. If B is an intuitionistic fuzzy $\alpha$*-set, then $\alpha int(A \cap B) = \alpha int(A) \cap int(B)$.*

**Definition 2.6.** *A subset A of an intuitionistic fuzzy topological space (X, $\tau$) is called*

*(i) an intuitionistic fuzzy *g-closed (briefly, IF *g-closed) [15] if $cl(A) \subseteq U$, whenever $A \subseteq U$ and U is intuitionistic fuzzy $\hat{g}$-open in (X, $\tau$).*

*(ii) an intuitionistic fuzzy $\alpha\hat{g}$-closed (briefly, IF $\alpha\hat{g}$-closed) [15] if $\alpha cl(A) \subseteq U$, whenever $A \subseteq U$ and U is intuitionistic fuzzy $\hat{g}$-open in (X, $\tau$).*

*(iii) an intuitionistic fuzzy $\hat{\eta}$-closed (briefly, IF $\hat{\eta}$-closed) [15] if $pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is intuitionistic fuzzy $\hat{g}$-open in (X, $\tau$).*

*The complement of intuitionistic fuzzy *g-closed set (resp. intuitionistic fuzzy $\alpha\hat{g}$-closed set, intuitionistic fuzzy $\hat{\eta}$-closed set) is intuitionistic fuzzy *g-open (resp. intuitionistic fuzzy $\alpha\hat{g}$-open, intuitionistic fuzzy $\hat{\eta}$-open).*

**Remark 2.7.** *The following hold in any intuitionistic fuzzy topological spaces:*

*(i) Every intuitionistic fuzzy $\alpha$-closed set is intuitionistic fuzzy $\alpha\hat{g}$-closed, but not conversely [15]*

*(ii) Every intuitionistic fuzzy $\alpha\hat{g}$-closed set is intuitionistic fuzzy $\hat{\eta}$-closed, but not conversely [15].*

*(iii) Every intuitionistic fuzzy *g-closed set is intuitionistic fuzzy $\alpha\hat{g}$-closed, but not conversely [15].*

*(iv) Every intuitionistic fuzzy closed set is (briefly, IF closed) intuitionistic fuzzy $\alpha$-closed, but not conversely [7].*

*(v) Every intuitionistic fuzzy closed set is intuitionistic fuzzy *g-closed, but not conversely [15].*

**Definition 2.8.** *A subset S of an intuitionistic fuzzy topological space (X, $\tau$) is said to be*

*(i) intuitionistic fuzzy $\hat{g}lc$*-set [15] if $S = U \cap F$, where U is intuitionistic fuzzy $\hat{g}$-open and F is intuitionistic fuzzy closed in (X, $\tau$).*

*(ii) intuitionistic fuzzy $D\eta$*-set [15] if $S = U \cap F$, where U is intuitionistic fuzzy $\hat{g}$-open and F is intuitionistic fuzzy $\alpha$-closed in (X, $\tau$).*

*(iii) intuitionistic fuzzy $D\eta$**-set [15] if $S = U \cap F$, where U is intuitionistic fuzzy $\alpha\hat{g}$-open and F is intuitionistic fuzzy t-set in (X, $\tau$).*

**Definition 2.9.** *An intuitionistic fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is said to be* *(i)* *intuitionistic fuzzy $\alpha$-continuous [17] if for each $V \in \sigma$, $f^{-1}(V)$ is an intuitionistic fuzzy $\alpha$-open set in $(X, \tau)$.*

*(ii)* *intuitionistic fuzzy $\alpha\hat{g}$-continuous (briefly, IF $\alpha\hat{g}$-continuous) [15] if for each $V \in \sigma$, $f^{-1}(V)$ is an intuitionistic fuzzy $\alpha\hat{g}$-open set in $(X, \tau)$.*

*(iii)* *intuitionistic fuzzy $\hat{\eta}$-continuous (briefly, IF $\hat{\eta}$-continuous) [15] if for each $V \in \sigma$, $f^{-1}(V)$ is an intuitionistic fuzzy $\hat{\eta}$-open set in $(X, \tau)$.*

*(iv)* *intuitionistic fuzzy $D\eta^*$-continuous [15] if for each $V \in \sigma$, $f^{-1}(V)$ is an intuitionistic fuzzy $D\eta^*$-set in $(X, \tau)$.*

*(v)* *intuitionistic fuzzy $D\eta^{**}$-continuous [15] if for each $V \in \sigma$, $f^{-1}(V)$ is an intuitionistic fuzzy $D\eta^{**}$-set in $(X, \tau)$.*

*(vi)* *intuitionistic fuzzy $D^*\eta^*$-continuous [15] if for each $V^c \in \sigma$, $f^{-1}(V)$ is an intuitionistic fuzzy $D\eta^*$-set in $(X, \tau)$.*

*(vii)* *intuitionistic fuzzy $^*g$-continuous (briefly, IF $^*g$-continuous) [15] if for each $V^c \in \sigma$, $f^{-1}(V)$ is an intuitionistic fuzzy $^*g$-closed set in $(X, \tau)$.*

*(viii)* *intuitionistic fuzzy $\hat{G}LC^*$-continuous [15] if for each $V^c \in \sigma$, $f^{-1}(V)$ is an intuitionistic fuzzy $\hat{g}lc^*$-set in $(X, \tau)$.*

**Remark 2.10** ([15])**.** *The following hold in any IFTSs.*

*(i)* *Every IF $^*g$-continuous function is IF $\alpha\hat{g}$-continuous but not conversely.*

*(ii)* *Every IF $^*g$-continuous function is IF $\hat{\eta}$-continuous but not conversely.*

Recently, the following decompositions have been established in [15].

**Theorem 2.11.** *An intuitionistic fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy $\alpha$-continuous if and only if it is both intuitionistic fuzzy $\alpha\hat{g}$-continuous and intuitionistic fuzzy $D^*\eta^*$-continuous.*

**Theorem 2.12.** *An intuitionistic fuzzy function $f : (X, \tau) \to (Y, \sigma)$ is intuitionistic fuzzy $\alpha\hat{g}$-continuous if and only if it is both intuitionistic fuzzy $\hat{\eta}$-continuous and intuitionistic fuzzy $D\eta^{**}$-continuous.*

## 3 On if *$g_t$-sets and if *$g_\alpha*$-sets

**Definition 3.1.** *A subset $S$ of an intuitionistic fuzzy topological space $(X, \tau)$ is called*

*(i)* *intuitionistic fuzzy $^*g_t$-set (briefly, IF $^*g_t$-set) if $S = U \cap F$, where $U$ is intuitionistic fuzzy $^*g$-open in $X$ and $F$ is an intuitionistic fuzzy t-set in $X$,*

*(ii)* *intuitionistic fuzzy $^*g_\alpha*$-set (briefly, IF $^*g_\alpha*$-set) if $S = U \cap F$, where $U$ is intuitionistic fuzzy $^*g$-open in $X$ and $F$ is an intuitionistic fuzzy $\alpha^*$-set in $X$.*

*The family of all intuitionistic fuzzy $^*g_t$-sets (resp. intuitionistic fuzzy $^*g_\alpha*$-sets) in an intuitionistic fuzzy topological space $(X, \tau)$ is denoted by IF$^*g_t(X, \tau)$(resp. IF$^*g_\alpha*(X, \tau)$).*

**Proposition 3.2.** *Let S be a subset of an intuitionistic fuzzy topological space (X, $\tau$).*

*(i) If S is an intuitionistic fuzzy t-set, then* $S \in IF^{*}g_{t}(X, \tau)$.

*(ii) If S is an intuitionistic fuzzy* $\alpha^{*}$*-set, then* $S \in IF^{*}g_{\alpha *}(X, \tau)$.

*(iii) If S is an intuitionistic fuzzy *g-open set in X, then* $S \in IF^{*}g_{t}(X, \tau)$ *and* $S \in IF^{*}g_{\alpha *}(X, \tau)$.

**Proposition 3.3.** *In an intuitionistic fuzzy topological space X, every intuitionistic fuzzy* $^{*}g_{t}$*-set is intuitionistic fuzzy* $^{*}g_{\alpha *}$*-set but not conversely.*

**Example 3.4.** *Let* $X = \{a, b\}$ *and* $\tau = \{0_{\sim}, G, 1_{\sim}\}$ *be an intuitionistic fuzzy topology (briefly, IFT) on X, where* $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. *Consider the intuitionistic fuzzy set (briefly, IFS)* $S = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$. *Then S is intuitionistic fuzzy* $^{*}g_{\alpha *}$*-set but it is not intuitionistic fuzzy* $^{*}g_{t}$*-set.*

**Remark 3.5.** *The following examples show that*

*(i) the converse of Proposition 3.2 need not be true.*

*(ii) the concepts of intuitionistic fuzzy* $^{*}g_{t}$*-sets and intuitionistic fuzzy* $\hat{\eta}$*-open sets are independent.*

*(iii) the concepts of intuitionistic fuzzy* $^{*}g_{\alpha *}$*-sets and intuitionistic fuzzy* $\alpha\hat{g}$*-open sets are independent.*

**Example 3.6.** *Let* $X = \{a, b\}$ *and* $\tau = \{0_{\sim}, G, 1_{\sim}\}$ *be an IFT on X, where* $G = \langle x, (0.7, 0.5), (0.2, 0.4)\rangle$. *In (X,* $\tau$*), the set* $S = \langle x, (0.8, 0.6), (0.1, 0.3)\rangle$ *is intuitionistic fuzzy* $^{*}g_{t}$*-set but not an intuitionistic fuzzy t-set.*

**Example 3.7.** *Let* $X = \{a, b\}$ *and* $\tau = \{0_{\sim}, G, 1_{\sim}\}$ *be an IFT on X, where* $G = \langle x, (0.5, 0.7), (0.4, 0.2)\rangle$. *In (X,* $\tau$*), the set* $S = \langle x, (0.6, 0.8), (0.3, 0.1)\rangle$ *is intuitionistic fuzzy* $^{*}g_{\alpha *}$*-set but not an intuitionistic fuzzy* $\alpha^{*}$*-set.*

**Example 3.8.** *Let* $X = \{a, b\}$ *and* $\tau = \{0_{\sim}, G, 1_{\sim}\}$ *be an IFT on X, where* $G = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$. *In (X,* $\tau$*), the set* $S = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$ *is both intuitionistic fuzzy* $^{*}g_{t}$*-set and intuitionistic fuzzy* $^{*}g_{\alpha *}$*-set, but it is not an intuitionistic fuzzy *g-open set.*

**Example 3.9.** *Let* $X = \{a, b\}$ *and* $\tau = \{0_{\sim}, G, 1_{\sim}\}$ *be an IFT on X, where* $G = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$. *In (X,* $\tau$*), the set* $S = \langle x, (0.2, 0.1), (0.7, 0.8)\rangle$ *is intuitionistic fuzzy* $^{*}g_{t}$*-set but not an intuitionistic fuzzy* $\hat{\eta}$*-open set and the set* $S = \langle x, (0.45, 0.4), (0.45, 0.5)\rangle$ *is an intuitionistic fuzzy* $\hat{\eta}$*-open set but not intuitionistic fuzzy* $^{*}g_{t}$*-set.*

**Example 3.10.** *Let* $X = \{a, b\}$ *and* $\tau = \{0_{\sim}, G, 1_{\sim}\}$ *be an IFT on X, where* $G = \langle x, (0.4, 0.3), (0.5, 0.6)\rangle$. *In (X,* $\tau$*), the set* $S = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$ *is intuitionistic fuzzy* $^{*}g_{\alpha *}$*-set but not an intuitionistic fuzzy* $\alpha\hat{g}$*-open set.*

**Example 3.11.** *Let* $X = \{a, b\}$ *and* $\tau = \{0_{\sim}, G, 1_{\sim}\}$ *be an IFT on X, where* $G = \langle x, (0.3, 0.7), (0.6, 0.2)\rangle$. *In (X,* $\tau$*), the set* $S = \langle x, (0.7, 0.8), (0.2, 0.1)\rangle$ *is an intuitionistic fuzzy* $\alpha\hat{g}$*-open set but not intuitionistic fuzzy* $^{*}g_{\alpha *}$*-set.*

**Remark 3.12.**

*(i) The union of two intuitionistic fuzzy* $^{*}g_{t}$*-sets need not be intuitionistic fuzzy* $^{*}g_{t}$*-set.*

*(ii) The union of two intuitionistic fuzzy $^*g_\alpha*$-sets need not be intuitionistic fuzzy $^*g_\alpha*$-set.*

**Example 3.13.** *Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.7), (0.6, 0.2)\rangle$. In $(X, \tau)$, the set $A = \langle x, (0.5, 0.8), (0.2, 0.1)\rangle$ and $B = \langle x, (0.6, 0.8), (0.2, 0.1)\rangle$ are both intuitionistic fuzzy $^*g_t$-sets and intuitionistic fuzzy $^*g_\alpha*$-sets. But $A\cup B = \langle x, (0.6, 0.8), (0.2, 0.1)\rangle$ is neither intuitionistic fuzzy $^*g_t$-set nor intuitionistic fuzzy $^*g_\alpha*$-set.*

**Lemma 3.14.**

*(i) A subset S of X is intuitionistic fuzzy *g-open [15] if and only if $F \subseteq int(S)$ whenever $F \subseteq S$ and F is intuitionistic fuzzy $\hat{g}$-closed in X.*

*(ii) A subset S of X is intuitionistic fuzzy $\alpha\hat{g}$-open [15] if and only if $F \subseteq \alpha int(S)$ whenever $F \subseteq S$ and F is intuitionistic fuzzy $\hat{g}$-closed in X.*

*(iii) A subset S of X is intuitionistic fuzzy $\hat{\eta}$-open [15] if and only if $F \subseteq pint(S)$ whenever $F \subseteq S$ and F is intuitionistic fuzzy $\hat{g}$-closed in X.*

**Theorem 3.15.** *A subset S is intuitionistic fuzzy *g-open in $(X, \tau)$ if and only if it is both intuitionistic fuzzy $\alpha\hat{g}$-open and intuitionistic fuzzy $^*g_\alpha*$-set in $(X, \tau)$.*

*Proof.* Necessity. The proof is obvious.

Sufficiency. Let S be an intuitionistic fuzzy $\alpha\hat{g}$-open set and intuitionistic fuzzy $^*g_\alpha*$-set. Since S is intuitionistic fuzzy $^*g_\alpha*$-set, $S = A\cap B$, where A is intuitionistic fuzzy *g-open and B is an intuitionistic fuzzy $\alpha^*$-set. Assume that $F \subseteq S$, where F is intuitionistic fuzzy $\hat{g}$-closed in X. Since A is intuitionistic fuzzy *g-open, by Lemma 3.14(1), $F \subseteq int(A)$. Since S is intuitionistic fuzzy $\alpha\hat{g}$-open in X, by Lemma 3.14(2),

$F \subseteq \alpha int(S) = S \cap int(cl(int(S))) = (A\cap B)\cap int(cl(int(A\cap B)))$

$\subseteq A\cap B\cap int(cl(int(A)))\cap int(cl(int(B))) = A\cap B\cap int(cl(int(A)))\cap int(B) \subseteq int(B)$.

Therefore, we obtain $F \subseteq int(B)$ and hence $F \subseteq int(A)\cap int(B) = int(S)$. Hence S is intuitionistic fuzzy *g-open. □

**Theorem 3.16.** *A subset S is intuitionistic fuzzy *g-open in $(X, \tau)$ if and only if it is both intuitionistic fuzzy $\hat{\eta}$-open and intuitionistic fuzzy $^*g_t$-set in $(X, \tau)$.*

*Proof.* Similar to Theorem 3.15. □

**Remark 3.17.** *We obtain the following diagram by the above discussions:*

$$\begin{array}{ccccc}
\text{IF closed} & \longrightarrow & \text{IF } {}^*g\text{-closed} & \longrightarrow & \text{IF } {}^*g_t\text{-set} \\
 & & \downarrow & & \\
\downarrow & & \text{IF } {}^*g_\alpha*\text{-set} & & \not\updownarrow \\
 & & \not\updownarrow & & \\
\text{IF } \alpha\text{-closed} & \longrightarrow & \text{IF } \alpha\hat{g}\text{-closed} & \longrightarrow & \text{IF } \hat{\eta}\text{-closed}
\end{array}$$

*None of the above implications is reversible as shown by the following Examples.*

**Example 3.18.** *Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.6), (0.7, 0.3)\rangle$. In $(X, \tau)$, the set $A = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$ is intuitionistic fuzzy $\alpha$-closed but it is neither intuitionistic fuzzy *g-closed set nor intuitionistic fuzzy closed set.*

**Example 3.19.** *Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. In $(X, \tau)$, the set $A = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ is an intuitionistic fuzzy $\alpha\hat{g}$-closed but not an intuitionistic fuzzy $\alpha$-closed set.*

**Example 3.20.** *Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.4), (0.7, 0.5)\rangle$. In $(X, \tau)$, the set $A = \langle x, (0.3, 0.3), (0.6, 0.5)\rangle$ is an intuitionistic fuzzy g-closed set but not an intuitionistic fuzzy $\alpha\hat{g}$-closed set.*

**Example 3.21.**

*(i) Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.6), (0.7, 0.3)\rangle$. In $(X, \tau)$, the set $A = \langle x, (0.1, 0.2), (0.8, 0.7)\rangle$ is an intuitionistic fuzzy $\alpha\hat{g}$-closed set but not an intuitionistic fuzzy g-closed set.*

*(ii) Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$. In $(X, \tau)$, the set $A = G$ is intuitionistic fuzzy $*g_\alpha*$-set but it is neither intuitionistic fuzzy *g-closed set nor intuitionistic fuzzy $\alpha\hat{g}$-closed set.*

**Example 3.22.**

*(i) Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.3, 0.7), (0.6, 0.2)\rangle$. In $(X, \tau)$, the set $A = \langle x, (0.6, 0.8), (0.2, 0.1)\rangle$ is an intuitionistic fuzzy $\alpha\hat{g}$-closed set but not intuitionistic fuzzy $*g_\alpha*$-set.*

*(ii) Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$. In $(X, \tau)$, the set $A = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ is an intuitionistic fuzzy $\hat{\eta}$-closed set but not intuitionistic fuzzy $\alpha\hat{g}$-closed set.*

**Example 3.23.** *Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.2, 0.3), (0.7, 0.6)\rangle$. In $(X, \tau)$, the set $A = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ is intuitionistic fuzzy *g-closed, but it is neither intuitionistic fuzzy $\alpha$-closed nor intuitionistic fuzzy closed set.*

**Example 3.24.** *Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.5, 0.7), (0.4, 0.2)\rangle$. In $(X, \tau)$, the set (1) $A = \langle x, (0.6, 0.8), (0.3, 0.1)\rangle$ is intuitionistic fuzzy $*g_t$-set but not intuitionistic fuzzy *g-closed set, (2) $A = \langle x, (0.5, 0.3), (0.4, 0.6)\rangle$ is intuitionistic fuzzy $\hat{\eta}$-closed set but not intuitionistic fuzzy $*g_t$-set, (3) $A= \langle x, (0.6, 0.8), (0.3, 0.1)\rangle$ is intuitionistic fuzzy $*g_t$-set but not intuitionistic fuzzy $\hat{\eta}$-closed set.*

**Remark 3.25.** *The concepts of intuitionistic fuzzy g-closed sets and intuitionistic fuzzy $\alpha\hat{g}$-closed sets are independent by the Examples 3.20 and 3.21.*

**Remark 3.26.** *The concepts of intuitionistic fuzzy *g-closed sets and intuitionistic fuzzy $\alpha$-closed sets are independent by the Examples 3.18 and 3.23.*

**Proposition 3.27.** *Let (X, $\tau$) be an intuitionistic fuzzy topological space. Then a subset A of X is intuitionistic fuzzy closed if and only if it is both intuitionistic fuzzy *g-closed and intuitionistic fuzzy $\hat{g}lc$*-set.*

*Proof.* Necessity is trivial. To prove the sufficiency, assume that A is both intuitionistic fuzzy *g-closed and intuitionistic fuzzy $\hat{g}$lc*-set. Then A= U∩V, where U is intuitionistic fuzzy $\hat{g}$-open and V is intuitionistic fuzzy closed in X. Therefore A ⊆ U and A ⊆ V and so by hypothesis, cl(A) ⊆ U and cl(A) ⊆ V, thus cl(A) ⊆ U∩V= A and hence cl(A)= A. Therefore A is intuitionistic fuzzy closed in X. □

**Remark 3.28.** *The following Examples show that the concepts of intuitionistic fuzzy *g-closed sets and intuitionistic fuzzy $\hat{g}lc$*-sets are independent.*

**Example 3.29.** *Let X = {a, b} and $\tau$ = {$0_\sim$, G, $1_\sim$} be an IFT on X, where G = ⟨ x, (0.4, 0.3), (0.5, 0.6)⟩. In (X, $\tau$), the set A = ⟨ x, (0.6, 0.7), (0.3, 0.2)⟩ is intuitionistic fuzzy *g-closed but not an intuitionistic fuzzy $\hat{g}lc$*-set.*

**Example 3.30.** *Let X = {a, b} and $\tau$ = {$0_\sim$, G, $1_\sim$} be an IFT on X, where G = ⟨ x, (0.4, 0.3), (0.5, 0.6)⟩. In (X, $\tau$), the set A = ⟨ x, (0.3, 0.2), (0.6, 0.7)⟩ is intuitionistic fuzzy $\hat{g}lc$*-set but not an intuitionistic fuzzy *g-closed set.*

# 4 Decompositions of if *g-continuity

**Definition 4.1.** *An intuitionistic fuzzy function f : (X, $\tau$) → (Y, $\sigma$) is said to be*

*(i) intuitionistic fuzzy *$g_t$-continuous (briefly, IF *$g_t$-continuous) if for each V∈$\sigma$, $f^{-1}$(V)∈IF*$g_t$(X, $\tau$).*

*(ii) intuitionistic fuzzy *$g_\alpha$∗-continuous (briefly, IF *$g_\alpha$∗-continuous) if for each V∈$\sigma$, $f^{-1}$(V)∈IF*$g_\alpha$∗(X, $\tau$).*

**Proposition 4.2.** *For an intuitionistic fuzzy function f : (X, $\tau$) → (Y, $\sigma$), the following implications hold:*

*(i) intuitionistic fuzzy *g-continuity ⇒ intuitionistic fuzzy *$g_t$-continuity;*

*(ii) intuitionistic fuzzy *g-continuity ⇒ intuitionistic fuzzy *$g_\alpha$∗-continuity;*

*(iii) intuitionistic fuzzy *g-continuity ⇒ intuitionistic fuzzy $\alpha\hat{g}$-continuity ⇒ intuitionistic fuzzy $\hat{\eta}$-continuity.*

*The reverse implications in Proposition 4.2 are not true as shown in the following Examples.*

**Example 4.3.** *Let X = Y = {a, b}, $\tau$ = {$0_\sim$, $G_1$, $1_\sim$}, and $\sigma$ = {$0_\sim$, $G_2$, $1_\sim$} be IFTs on X and Y respectively, where $G_1$ = ⟨ x, (0.3, 0.2), (0.6, 0.7)⟩ and $G_2$ = ⟨ y, (0.6, 0.7), (0.3, 0.2)⟩. Let f : (X, $\tau$) → (Y, $\sigma$) be the identity intuitionistic fuzzy function. Then f is intuitionistic fuzzy *$g_t$-continuous function. However, f is neither intuitionistic fuzzy *g-continuous nor intuitionistic fuzzy $\hat{\eta}$-continuous.*

**Example 4.4.** *Let $X = Y = \{a, b\}$, $\tau = \{0_\sim, G_1, 1_\sim\}$, and $\sigma = \{0_\sim, G_2, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$ and $G_2 = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity intuitionistic fuzzy function. Then f is intuitionistic fuzzy $^*g_\alpha*$-continuous function. However, f is neither intuitionistic fuzzy *g-continuous nor intuitionistic fuzzy $\alpha\hat{g}$-continuous.*

The following Example (4.5) and Example (4.6) show that the concepts of intuitionistic fuzzy $^*g_\alpha*$-continuity and intuitionistic fuzzy $\alpha\hat{g}$-continuity are independent.

**Example 4.5.** *Let $X = Y = \{a, b\}$, $\tau = \{0_\sim, G_1, 1_\sim\}$, and $\sigma = \{0_\sim, G_2, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.3, 0.2), (0.6, 0.7)\rangle$ and $G_2 = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity intuitionistic fuzzy function. Then f is intuitionistic fuzzy $^*g_\alpha*$-continuous function but it is not intuitionistic fuzzy $\alpha\hat{g}$-continuous.*

**Example 4.6.** *Let $X = Y = \{a, b\}$, $\tau = \{0_\sim, G_1, 1_\sim\}$, and $\sigma = \{0_\sim, G_2, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.3, 0.7), (0.6, 0.2)\rangle$ and $G_2 = \langle y, (0.7, 0.8), (0.2, 0.1)\rangle$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity intuitionistic fuzzy function. Then f is intuitionistic fuzzy $\alpha\hat{g}$-continuous function but it is not intuitionistic fuzzy $^*g_\alpha*$-continuous.*

The following Example (4.7) and Example (4.8) show that intuitionistic fuzzy $^*g_t$-continuity and intuitionistic fuzzy $\hat{\eta}$-continuity are independent.

**Example 4.7.** *Let $X = Y = \{a, b\}$, $\tau = \{0_\sim, G_1, 1_\sim\}$, and $\sigma = \{0_\sim, G_2, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$ and $G_2 = \langle y, (0.2, 0.1), (0.7, 0.8)\rangle$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity intuitionistic fuzzy function. Then f is intuitionistic fuzzy $^*g_t$-continuous function but it is not intuitionistic fuzzy $\hat{\eta}$-continuous.*

**Example 4.8.** *Let $X = Y = \{a, b\}$, $\tau = \{0_\sim, G_1, 1_\sim\}$, and $\sigma = \{0_\sim, G_2, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.6, 0.7), (0.3, 0.2)\rangle$ and $G_2 = \langle y, (0.45, 0.4), (0.45, 0.5)\rangle$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity intuitionistic fuzzy function. Then f is intuitionistic fuzzy $\hat{\eta}$-continuous function but it is not intuitionistic fuzzy $^*g_t$-continuous.*

**Example 4.9.** *Let $X = Y = \{a, b\}$, $\tau = \{0_\sim, G_1, 1_\sim\}$, and $\sigma = \{0_\sim, G_2, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.7, 0.6), (0.2, 0.3)\rangle$ and $G_2 = \langle y, (0.6, 0.5), (0.3, 0.4)\rangle$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity intuitionistic fuzzy function. Then f is intuitionistic fuzzy $\hat{\eta}$-continuous function but not intuitionistic fuzzy $\alpha\hat{g}$-continuous.*

**Example 4.10.** *Let $X = Y = \{a, b\}$, $\tau = \{0_\sim, G_1, 1_\sim\}$, and $\sigma = \{0_\sim, G_2, 1_\sim\}$ be IFTs on X and Y respectively, where $G_1 = \langle x, (0.3, 0.4), (0.6, 0.5)\rangle$ and $G_2 = \langle y, (0.7, 0.6), (0.2, 0.3)\rangle$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity intuitionistic fuzzy function. Then f is intuitionistic fuzzy $^*g_\alpha*$-continuous function but not intuitionistic fuzzy $^*g_t$-continuous.*

**Remark 4.11.** *By the above discussions, we obtain the following diagram.*

$$
\begin{array}{ccccc}
\text{IF } \alpha\hat{g}\text{-continuity} & & \longrightarrow & & \text{IF } \hat{\eta}\text{-continuity} \\
 & \nwarrow & & \nearrow & \\
\not\updownarrow & & \text{IF } {}^{*}g\text{-continuity} & & \not\updownarrow \\
 & \swarrow & & \searrow & \\
\text{IF } {}^{*}g_{\alpha}{*}\text{-continuity} & & \longleftarrow & & \text{IF } {}^{*}g_{t}\text{-continuity}
\end{array}
$$

*None of the above implications is reversible.*

**Theorem 4.12.** *An intuitionistic fuzzy function* $f : (X, \tau) \rightarrow (Y, \sigma)$ *is intuitionistic fuzzy* $^{*}g$*-continuous if and only if it is both intuitionistic fuzzy* $\alpha\hat{g}$*-continuous and intuitionistic fuzzy* $^{*}g_{\alpha}{*}$*-continuous.*

*Proof.* The proof follows immediately from Theorem 3.15. □

**Theorem 4.13.** *An intuitionistic fuzzy function* $f : (X, \tau) \rightarrow (Y, \sigma)$ *is intuitionistic fuzzy* $^{*}g$*-continuous if and only if it is both intuitionistic fuzzy* $\hat{\eta}$*-continuous and intuitionistic fuzzy* $^{*}g_{t}$*-continuous.*

*Proof.* From Theorem 3.16, the proof is immediate. □

**Corollary 4.1.** *An intuitionistic fuzzy function* $f : (X, \tau) \rightarrow (Y, \sigma)$ *is intuitionistic fuzzy* $^{*}g$*-continuous if and only if it is intuitionistic fuzzy* $\hat{\eta}$*-continuous, intuitionistic fuzzy* $D\eta^{**}$*-continuous and intuitionistic fuzzy* $^{*}g_{\alpha}{*}$*-continuous.*

*Proof.* It follows from Theorem 2.12. □

**Remark 4.14.** *Intuitionistic fuzzy* $^{*}g$*-continuity and intuitionistic fuzzy* $\hat{G}LC^{*}$*-continuity are independent of each other.*

**Example 4.15.** *Let* $X = Y = \{a, b\}$, $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$, *and* $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ *be IFTs on X and Y respectively, where* $G_1 = \langle x, (0.4, 0.3), (0.5, 0.6)\rangle$ *and* $G_2 = \langle y, (0.3, 0.2), (0.6, 0.7)\rangle$. *Let* $f : (X, \tau) \rightarrow (Y, \sigma)$ *be the identity intuitionistic fuzzy function. Then f is intuitionistic fuzzy* $^{*}g$*-continuous function but not intuitionistic fuzzy* $\hat{G}LC^{*}$*-continuous.*

**Example 4.16.** *Let* $X = Y = \{a, b\}$, $\tau = \{0_{\sim}, G_1, 1_{\sim}\}$, *and* $\sigma = \{0_{\sim}, G_2, 1_{\sim}\}$ *be IFTs on X and Y respectively, where* $G_1 = \langle x, (0.4, 0.3), (0.5, 0.6)\rangle$ *and* $G_2 = \langle y, (0.6, 0.7), (0.3, 0.2)\rangle$. *Let* $f : (X, \tau) \rightarrow (Y, \sigma)$ *be the identity intuitionistic fuzzy function. Then f is intuitionistic fuzzy* $\hat{G}LC^{*}$*-continuous function but not intuitionistic fuzzy* $^{*}g$*-continuous.*

**Theorem 4.17.** *An intuitionistic fuzzy function* $f : (X, \tau) \rightarrow (Y, \sigma)$ *is intuitionistic fuzzy continuous if and only if it is both intuitionistic fuzzy* $^{*}g$*-continuous and intuitionistic fuzzy* $\hat{G}LC^{*}$*-continuous.*

*Proof.* It follows from Proposition 3.27. □

# 5 Conclusion

In 1965, Zadeh [25] introduced fuzzy sets and in 1968, Chang [5] introduced fuzzy topology. After the introduction of fuzzy sets and fuzzy topology, several researches were worked on the generalizations of the notions of fuzzy sets and fuzzy topology. The concept of intuitionistic fuzzy sets was introduced by

Atanassov [3] as a generalization of fuzzy sets. In 1997, Coker [6] introduced the concept of intuitionistic fuzzy topological spaces. The decomposition of intuitionistic fuzzy continuity is one of the many problems in intuitionistic fuzzy topology. Recently, Ramesh and Thirumalaiswamy [14] have obtained decompositions of a weaker form of intuitionistic fuzzy continuity. Such an attempt has been taken by the authors of this paper.

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