International Journal of Mathematics And its Applications
Volume 3, Issue 2 (2015), 25–34.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

On Weakly (1,2)*-$g^{\#}$-closed Sets

Research Article

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Abstract: In this paper, we introduce weakly (1,2)*-$g^{\#}$-closed sets and investigate the relationships among the related (1,2)*-generalized closed sets.

MSC: 54E55.

Keywords: (1,2)*-$g^{\#}$-closed set, (1,2)*-g-closed set, (1,2)*-αg-closed set, (1,2)*-$g^{\#}$-irresolute function.

1. Introduction

Rajan [2] studied and investigated the properties of the notion of (1,2)*-$g^{\#}$-closed sets. In this paper, we introduce a new class of (1,2)*-generalized closed sets called weakly (1,2)*-$g^{\#}$-closed sets which contains the above mentioned class. Also, we investigate the relationships among the related (1,2)*-generalized closed sets.

2. Preliminaries

Throughout the paper, X, Y and Z denote bitopological spaces (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) respectively.

Definition 2.1. *Let A be a subset of a bitopological space X. Then A is called $\tau_{1,2}$-open [1] if $A = P \cup Q$, for some $P \in \tau_1$ and $Q \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed. The family of all $\tau_{1,2}$-open (resp. $\tau_{1,2}$-closed) sets of X is denoted by $(1,2)^\star$-O(X) (resp. $(1,2)^\star$-C(X)).*

Definition 2.2 ([1])**.** *Let A be a subset of a bitopological space X. Then*

(1) the $\tau_{1,2}$-interior of A, denoted by $\tau_{1,2}$-int(A), is defined by $\cup$ { $U : U \subseteq A$ and U is $\tau_{1,2}$-open};

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(2) the $\tau_{1,2}$-closure of A, denoted by $\tau_{1,2}$-cl(A), is defined by $\cap$ { $U : A \subseteq U$ and U is $\tau_{1,2}$-closed}.

Remark 2.3 ([1]). *Notice that $\tau_{1,2}$-open subsets of X need not necessarily form a topology.*

Definition 2.4 ([4]). *A subset A of a bitopological space X is said to be $(1,2)^\star$-nowhere dense in X if $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)) = ϕ.*

Definition 2.5 ([1]). *Let A be a subset of a bitopological space X. Then A is called*

(1) regular $(1,2)^\star$-open set if $A = \tau_{1,2}$-int($\tau_{1,2}$-cl(A)). The complement of regular $(1,2)^\star$-open set is regular $(1,2)^\star$-closed.

(2) $(1,2)^\star$-π-open if the finite union of regular $(1,2)^\star$-open sets.

(3) $(1,2)^\star$-semi-open if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A)). The complement of $(1,2)^\star$-semi-open set is $(1,2)^\star$-semi-closed.

(4) $(1,2)^\star$-α-open if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))). The complement of $(1,2)^\star$-α-open set is $(1,2)^\star$-α-closed. The $(1,2)^\star$-α-closure of a subset A of X, denoted by $(1,2)^\star$-$\alpha cl(A)$, is defined to be the intersection of all $(1,2)^\star$-α-closed sets of X containing A.

Definition 2.6. *Let A be a subset of a bitopological space X. Then A is called*

(1) a $(1,2)^\star$-generalized closed (briefly, $(1,2)^\star$-g-closed) set [4] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^\star$-g-closed set is called $(1,2)^\star$-g-open set.

(2) an $(1,2)^\star$-α-generalized closed (briefly, $(1,2)^\star$-αg-closed) set [4] if $(1,2)^\star$-$\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of $(1,2)^\star$-αg-closed set is called $(1,2)^\star$-αg-open set.

(3) $(1,2)^\star$-$g^\#$-closed set [2] if $\tau_{1,2}$-cl(A) $\subseteq U$ whenever $A \subseteq U$ and U is $(1,2)^\star$-αg-open in X. The complement of $(1,2)^\star$-$g^\#$-closed set is called $(1,2)^\star$-$g^\#$-open set.

Definition 2.7 ([1]). *A function $f : X \to Y$ is called:*

(1) $(1,2)^\star$-open if $f(V)$ is $\sigma_{1,2}$-open in Y for every $\tau_{1,2}$-open set V of X.

(2) $(1,2)^\star$-closed if $f(V)$ is $\sigma_{1,2}$-closed in Y for every $\tau_{1,2}$-closed set V of X.

Definition 2.8 ([2]). *A function $f : X \to Y$ is called:*

(1) $(1,2)^\star$-αg-irresolute if the inverse image of every $(1,2)^\star$-αg-closed (resp. $(1,2)^\star$-αg-open) set in Y is $(1,2)^\star$-αg-closed (resp. $(1,2)^\star$-αg-open) in X.

(2) $(1,2)^\star$-$g^\#$-irresolute if the inverse image of every $(1,2)^\star$-$g^\#$-closed set in Y is $(1,2)^\star$-$g^\#$-closed in X.

Definition 2.9 ([3]). *Let X and Y be two bitopological spaces. A function $f : X \to Y$ is called:*

(1) (1,2)*-R-map if $f^{-1}(V)$ is regular (1,2)*-open in X for each regular (1,2)*-open set V of Y.

(2) perfectly (1,2)*-continuous if $f^{-1}(V)$ is $\tau_{1,2}$-clopen in X for each $\sigma_{1,2}$-open set V of Y.

Definition 2.10 ([4]). *A subset A of a bitopological space X is called:*

(1) *a weakly (1,2)*-g-closed (briefly, (1,2)*-wg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_{1,2}$-open in X.*

(2) *a weakly (1,2)*-πg-closed (briefly, (1,2)*-wπg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-π-open in X.*

(3) *a regular weakly (1,2)*-generalized closed (briefly, (1,2)*-rwg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is regular (1,2)*-open in X.*

Definition 2.11. *A bitopological space X is said to be almost (1,2)*-connected [4] (resp. (1,2)*-g$^\#$-connected [2], (1,2)*-connected [2]) if X cannot be written as a disjoint union of two non-empty regular (1,2)*-open (resp. (1,2)*-g$^\#$-open, $\tau_{1,2}$-open) sets.*

Remark 2.12 ([4]). *For a subset of a bitopological space, we have following implications: regular (1,2)*-open $\rightarrow$ (1,2)*-π-open $\rightarrow$ $\tau_{1,2}$-open $\rightarrow$ (1,2)*-αg-open*

The reverses of the above implications are not true.

3. Weakly (1,2)*-$g^\#$-closed Sets

We introduce the definition of weakly (1,2)*-$g^\#$-closed sets in bitopological spaces and study the relationships of such sets.

Definition 3.1. *A subset A of a bitopological space X is called a weakly (1,2)*-g$^\#$-closed (briefly, (1,2)*-wg$^\#$-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U whenever A $\subseteq$ U and U is (1,2)*-αg-open in X.*

Theorem 3.2. *Every (1,2)*-g$^\#$-closed set is (1,2)*-wg$^\#$-closed but not conversely.*

Example 3.3. *Let X = $\{a, b, c\}$, τ_1 = $\{\phi, X\}$ and τ_2 = $\{\phi, \{a, b\}, X\}$. Then the sets in $\{\phi, \{a, b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{c\}, X\}$ are called $\tau_{1,2}$-closed. Then the set $\{a\}$ is (1,2)*-wg$^\#$-closed set but it is not a (1,2)*-g$^\#$-closed in X.*

Theorem 3.4. *Every (1,2)*-wg$^\#$-closed set is (1,2)*-wg-closed but not conversely.*

Proof. Let A be any (1,2)*-w$g^\#$-closed set and U be any $\tau_{1,2}$-open set containing A. Then U is a (1,2)*-αg-open set containing A. We have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U. Thus, A is (1,2)*-wg-closed. □

Example 3.5. *Let X = $\{a, b, c\}$, τ_1 = $\{\phi, X\}$ and τ_2 = $\{\phi, \{a\}, X\}$. Then the sets in $\{\phi, \{a\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Then the set $\{a, b\}$ is (1,2)*-wg-closed but it is not a (1,2)*-wg$^\#$-closed.*

Theorem 3.6. *Every (1,2)*-wg$^\#$-closed set is (1,2)*-wπg-closed but not conversely.*

Proof. Let A be any (1,2)*-w$g^\#$-closed set and U be any (1,2)*-π-open set containing A. Then U is a (1,2)*-αg-open set containing A. We have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U. Thus, A is (1,2)*-wπg-closed. □

Example 3.7. *In Example 3.5, the set $\{a, c\}$ is (1,2)*-wπg-closed but it is not a (1,2)*-wg$^\#$-closed.*

Theorem 3.8. *Every (1,2)*-wg$^\#$-closed set is (1,2)*-rwg-closed but not conversely.*

Proof. Let A be any (1,2)*-w$g^\#$-closed set and U be any regular (1,2)*-open set containing A. Then U is a (1,2)*-αg-open set containing A. We have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U. Thus, A is (1,2)*-rw$g$-closed. □

Example 3.9. *In Example 3.5, the set $\{a\}$ is (1,2)*-rwg-closed but it is not a (1,2)*-wg$^\#$-closed.*

Theorem 3.10. *If a subset A of a bitopological space X is both $\tau_{1,2}$-closed and (1,2)*-g-closed, then it is (1,2)*-wg$^\#$-closed in X.*

Proof. Let A be a (1,2)*-g-closed set in X and U be any $\tau_{1,2}$-open set containing A. Then U $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))). Since A is $\tau_{1,2}$-closed, U $\supseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) and hence (1,2)*-w$g^\#$-closed in X. □

Theorem 3.11. *If a subset A of a bitopological space X is both $\tau_{1,2}$-open and (1,2)*-wg$^\#$-closed, then it is $\tau_{1,2}$-closed.*

Proof. Since A is both $\tau_{1,2}$-open and (1,2)*-w$g^\#$-closed, A $\supseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) = $\tau_{1,2}$-cl(A) and hence A is $\tau_{1,2}$-closed in X. □

Corollary 3.12. *If a subset A of a bitopological space X is both $\tau_{1,2}$-open and (1,2)*-wg$^\#$-closed, then it is both regular (1,2)*-open and regular (1,2)*-closed in X.*

Theorem 3.13. *Let X be a bitopological space and A $\subseteq$ X be $\tau_{1,2}$-open. Then, A is (1,2)*-wg$^\#$-closed if and only if A is (1,2)*-g$^\#$-closed.*

Proof. Let A be (1,2)*-$g^\#$-closed. By Theorem 3.2, it is (1,2)*-w$g^\#$-closed. Conversely, let A be (1,2)*-w$g^\#$-closed. Since A is $\tau_{1,2}$-open, by Theorem 3.11, A is $\tau_{1,2}$-closed. Hence A is (1,2)*-$g^\#$-closed. □

Theorem 3.14. *If a set A of X is (1,2)*-wg$^\#$-closed, then $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $-$ A contains no non-empty (1,2)*-αg-closed set.*

Proof. Let F be a (1,2)*-αg-closed set such that F $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $-$ A. Since F^c is (1,2)*-αg-open and A $\subseteq$ F^c, from the definition of (1,2)*-w$g^\#$-closedness it follows that $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ F^c. i.e., F $\subseteq$ ($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)))^c. This implies that F $\subseteq$ ($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))) $\cap$ ($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)))^c = ϕ. □

Theorem 3.15. *If a subset A of a bitopological space X is (1,2)*-nowhere dense, then it is (1,2)*-wg$^\#$-closed.*

Proof. Since $\tau_{1,2}$-int(A) $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)) and A is (1,2)*-nowhere dense, $\tau_{1,2}$-int(A) = ϕ. Therefore $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) = ϕ and hence A is (1,2)*-w$g^\#$-closed in X. □

The converse of Theorem 3.15 need not be true as seen in the following example.

Example 3.16. *Let X = $\{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Then the set $\{a\}$ is (1,2)*-wg$^\#$-closed set but not (1,2)*-nowhere dense in X.*

Remark 3.17. *The following examples show that (1,2)*-wg$^\#$-closedness and (1,2)*-semi-closedness are independent.*

Example 3.18. *In Example 3.3, we have the set $\{a, c\}$ is (1,2)*-wg$^\#$-closed set but not (1,2)*-semi-closed in X.*

Example 3.19. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Then the set $\{a\}$ is (1,2)*-semi-closed set but not (1,2)*-wg$^\#$-closed in X.*

Remark 3.20. *From the above discussions and known results, we obtain the following diagram, where $A \to B$ represents A implies B but not conversely.*

$$\tau_{1,2}\text{-closed} \to (1,2)^\star\text{-}wg^\#\text{-closed} \to (1,2)^\star\text{-}wg\text{-closed} \to (1,2)^\star\text{-}w\pi g\text{-closed} \to (1,2)^\star\text{-}rwg\text{-closed}$$

Definition 3.21. *A subset A of a bitopological space X is called (1,2)*-wg$^\#$-open if A^c is (1,2)*-wg$^\#$-closed in X.*

Proposition 3.22.

(1) Every (1,2)*-g$^\#$-open set is (1,2)*-wg$^\#$-open but not conversely.

(2) Every (1,2)*-g-open set is (1,2)*-wg$^\#$-open but not conversely.

Theorem 3.23. *A subset A of a bitopological space X is (1,2)*-wg$^\#$-open if $G \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)) whenever $G \subseteq A$ and G is (1,2)*-αg-closed.*

Proof. Let A be any (1,2)*-w$g^\#$-open. Then A^c is (1,2)*-w$g^\#$-closed. Let G be an (1,2)*-αg-closed set contained in A. Then G^c is an (1,2)*-αg-open set containing A^c. Since A^c is (1,2)*-wg$^\#$-closed, we have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A^c)) $\subseteq G^c$. Therefore $G \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)).
Conversely, we suppose that $G \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)) whenever $G \subseteq A$ and G is (1,2)*-αg-closed. Then G^c is an (1,2)*-αg-open set containing A^c and $G^c \supseteq (\tau_{1,2}\text{-int}(\tau_{1,2}\text{-cl}(A)))^c$. It follows that $G^c \supseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A^c)). Hence A^c is (1,2)*-wg$^\#$-closed and so A is (1,2)*-wg$^\#$-open. □

Definition 3.24. *Let $f : X \to Y$ be a function. Then f is said to be*

(1) contra (1,2)*-g$^\#$-continuous [2] (resp. (1,2)*-g$^\#$-continuous [5]) if the inverse image of every $\sigma_{1,2}$-open (resp. $\sigma_{1,2}$-closed) set in Y is (1,2)*-g$^\#$-closed set in X.

(2) (1,2)*-continuous [5] if the inverse image of every $\sigma_{1,2}$-open set in Y is $\tau_{1,2}$-open set in X.

Theorem 3.25. *The following are equivalent for a function $f : X \to Y$:*

(1) f is contra (1,2)*-g$^\#$-continuous.

(2) the inverse image of every $\sigma_{1,2}$-closed set of Y is (1,2)*-g$^\#$-open in X.

Proof. Let U be any $\sigma_{1,2}$-closed set of Y. Since Y \U is $\sigma_{1,2}$-open, then by (1), it follows that $f^{-1}(Y \setminus U) = X \setminus f^{-1}(U)$ is (1,2)*-$g^\#$-closed. This shows that $f^{-1}(U)$ is (1,2)*-$g^\#$-open in X.
Converse is similar. □

4. Weakly (1,2)*-$g^\#$-continuous Functions

Definition 4.1. *Let X and Y be two bitopological spaces. A function $f: X \to Y$ is called weakly (1,2)*-$g^\#$-continuous (briefly, (1,2)*-w$g^\#$-continuous) if $f^{-1}(U)$ is a (1,2)*-w$g^\#$-open set in X for each $\sigma_{1,2}$-open set U of Y.*

Example 4.2. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a\}, Y\}$. Then the sets in $\{\phi, \{a\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-closed. The function $f: X \to Y$ defined by f(a) = b, f(b) = c and f(c) = a is (1,2)*-w$g^\#$-continuous, because every subset of Y is (1,2)*-w$g^\#$-closed in X.*

Theorem 4.3. *Every (1,2)*-$g^\#$-continuous function is (1,2)*-w$g^\#$-continuous.*

Proof. It follows from Theorem 3.2. □

The converse of Theorem 4.3 need not be true as seen in the following example.

Example 4.4. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{b\}, Y\}$. Then the sets in $\{\phi, \{b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a, c\}, Y\}$ are called $\sigma_{1,2}$-closed. Let $f: X \to Y$ be the identity function. Then f is (1,2)*-w$g^\#$-continuous but not (1,2)*-$g^\#$-continuous.*

Theorem 4.5. *A function $f: X \to Y$ is (1,2)*-w$g^\#$-continuous if and only if $f^{-1}(U)$ is a (1,2)*-w$g^\#$-closed set in X for each $\sigma_{1,2}$-closed set U of Y.*

Proof. Let U be any $\sigma_{1,2}$-closed set of Y. According to the assumption $f^{-1}(U^c) = X \setminus f^{-1}(U)$ is (1,2)*-w$g^\#$-open in X, so $f^{-1}(U)$ is (1,2)*-w$g^\#$-closed in X.
The converse can be proved in a similar manner. □

Definition 4.6. *A bitopological space X is said to be locally (1,2)*-$g^\#$-indiscrete if every (1,2)*-$g^\#$-open set of X is $\tau_{1,2}$-closed in X.*

Theorem 4.7. *Let $f: X \to Y$ be a function. If f is contra (1,2)*-$g^\#$-continuous and X is locally (1,2)*-$g^\#$-indiscrete, then f is (1,2)*-continuous.*

Proof. Let V be a $\sigma_{1,2}$-closed in Y. Since f is contra (1,2)*-$g^\#$-continuous, $f^{-1}(V)$ is (1,2)*-$g^\#$-open in X. Since X is locally (1,2)*-$g^\#$-indiscrete, $f^{-1}(V)$ is $\tau_{1,2}$-closed in X. Hence f is (1,2)*-continuous. □

Theorem 4.8. *Let $f: X \to Y$ be a function. If f is contra (1,2)*-$g^\#$-continuous and X is locally (1,2)*-$g^\#$-indiscrete, then f is (1,2)*-w$g^\#$-continuous.*

Proof. Let $f: X \to Y$ be contra (1,2)*-$g^\#$-continuous and X is locally (1,2)*-$g^\#$-indiscrete. By Theorem 4.7, f is (1,2)*-continuous, then f is (1,2)*-w$g^\#$-continuous. □

Proposition 4.9. *If $f: X \to Y$ is perfectly (1,2)*-continuous and (1,2)*-w$g^\#$-continuous, then it is (1,2)*-R-map.*

Proof. Let V be any regular (1,2)*-open subset of Y. According to the assumption, $f^{-1}(V)$ is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed in X. Since $f^{-1}(V)$ is $\tau_{1,2}$-closed, it is (1,2)*-wg$^{\#}$-closed. We have $f^{-1}(V)$ is both $\tau_{1,2}$-open and (1,2)*-wg$^{\#}$-closed. Hence, by Corollary 3.12, it is regular (1,2)*-open in X, so f is (1,2)*-R-map. □

Definition 4.10. *A bitopological space X is called (1,2)*-g$^{\#}$-compact (resp. (1,2)*-compact) if every cover of X by (1,2)*-g$^{\#}$-open (resp. $\tau_{1,2}$-open) sets has finite subcover.*

Definition 4.11. *A bitopological space X is weakly (1,2)*-g$^{\#}$-compact (briefly, (1,2)*-wg$^{\#}$-compact) if every (1,2)*-wg$^{\#}$-open cover of X has a finite subcover.*

Remark 4.12. *Every (1,2)*-wg$^{\#}$-compact space is (1,2)*-g$^{\#}$-compact.*

Theorem 4.13. *Let $f : X \to Y$ be surjective (1,2)*-wg$^{\#}$-continuous function. If X is (1,2)*-wg$^{\#}$-compact, then Y is (1,2)*-compact.*

Proof. Let $\{A_i : i \in I\}$ be an $\sigma_{1,2}$-open cover of Y. Then $\{ f^{-1}(A_i) : i \in I\}$ is a (1,2)*-wg$^{\#}$-open cover in X. Since X is (1,2)*-wg$^{\#}$-compact, it has a finite subcover, say $\{ f^{-1}(A_1), f^{-1}(A_2),...., f^{-1}(A_n)\}$. Since f is surjective $\{A_1, A_2,...., A_n\}$ is a finite subcover of Y and hence Y is (1,2)*-compact. □

Definition 4.14. *A bitopological space X is weakly (1,2)*-g$^{\#}$-connected (briefly, (1,2)*-wg$^{\#}$-connected) if X cannot be written as the disjoint union of two non-empty (1,2)*-wg$^{\#}$-open sets.*

Theorem 4.15. *If a bitopological space X is (1,2)*-wg$^{\#}$-connected, then X is almost (1,2)*-connected (resp. (1,2)*-g$^{\#}$-connected).*

Proof. It follows from the fact that each regular (1,2)*-open set (resp. (1,2)*-g$^{\#}$-open set) is (1,2)*-wg$^{\#}$-open. □

Theorem 4.16. *For a bitopological space X, the following statements are equivalent:*

(1) X is (1,2)*-wg$^{\#}$-connected.

(2) The empty set ϕ and X are only subsets which are both (1,2)*-wg$^{\#}$-open and (1,2)*-wg$^{\#}$-closed.

(3) Each (1,2)*-wg$^{\#}$-continuous function from X into a discrete space Y which has at least two points is a constant function.

Proof. (1) $\Rightarrow$ (2). Let $S \subseteq X$ be any proper subset, which is both (1,2)*-wg$^{\#}$-open and (1,2)*-wg$^{\#}$-closed. Its complement $X \setminus S$ is also (1,2)*-wg$^{\#}$-open and (1,2)*-wg$^{\#}$-closed. Then $X = S \cup (X \setminus S)$ is a disjoint union of two non-empty (1,2)*-wg$^{\#}$-open sets which is a contradiction with the fact that X is (1,2)*-wg$^{\#}$-connected. Hence, $S = \phi$ or X.

(2) $\Rightarrow$ (1). Let $X = A \cup B$ where $A \cap B = \phi$, $A \neq \phi$, $B \neq \phi$ and A, B are (1,2)*-wg$^{\#}$-open. Since $A = X \setminus B$, A is (1,2)*-wg$^{\#}$-closed. According to the assumption $A = \phi$, which is a contradiction.

(2) $\Rightarrow$ (3). Let $f : X \to Y$ be a (1,2)*-wg$^{\#}$-continuous function where Y is a discrete bitopological space with at least two points. Then $f^{-1}(\{y\})$ is (1,2)*-wg$^{\#}$-closed and (1,2)*-wg$^{\#}$-open for each $y \in Y$ and $X = \cup \{ f^{-1}(\{y\}) : y \in Y\}$. According to the assumption, $f^{-1}(\{y\}) = \phi$ or $f^{-1}(\{y\}) = X$. If $f^{-1}(\{y\}) = \phi$ for all $y \in Y$, f will not be a function. Also there is no exist more than one $y \in Y$ such that $f^{-1}(\{y\}) = X$. Hence, there exists only one $y \in Y$ such that $f^{-1}(\{y\}) = X$ and $f^{-1}(\{y_1\}) = \phi$ where $y \neq y_1 \in Y$. This shows that f is a constant function.

(3) $\Rightarrow$ (2). Let $S \neq \phi$ be both (1,2)*-wg$^{\#}$-open and (1,2)*-wg$^{\#}$-closed in X. Let $f : X \to Y$ be a (1,2)*-wg$^{\#}$-continuous function defined by $f(S) = \{a\}$ and $f(X \setminus S) = \{b\}$ where $a \neq b$. Since f is constant function we get $S = X$. □

Theorem 4.17. *Let $f : X \to Y$ be a (1,2)*-wg$^\#$-continuous surjective function. If X is (1,2)*-wg$^\#$-connected, then Y is (1,2)*-connected.*

Proof. We suppose that Y is not (1,2)*-connected. Then Y = A ∪ B where A ∩ B = ϕ, A $\neq \phi$, B $\neq \phi$ and A, B are $\sigma_{1,2}$-open sets in Y. Since f is (1,2)*-w$g^\#$-continuous surjective function, X = $f^{-1}(A) \cup f^{-1}(B)$ are disjoint union of two non-empty (1,2)*-w$g^\#$-open subsets. This is contradiction with the fact that X is (1,2)*-w$g^\#$-connected. □

5. Weakly (1,2)*-$g^\#$-open Functions and Weakly (1,2)*-$g^\#$-closed Functions

Definition 5.1. *Let X and Y be bitopological spaces. A function $f : X \to Y$ is called weakly (1,2)*-g$^\#$-open (briefly, (1,2)*-wg$^\#$-open) if f(V) is a (1,2)*-wg$^\#$-open set in Y for each $\tau_{1,2}$-open set V of X.*

Remark 5.2. *Every (1,2)*-g$^\#$-open function is (1,2)*-wg$^\#$-open but not conversely.*

Example 5.3. *Let X = {a, b, c, d}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ, {a, b, d}, X}. Then the sets in {ϕ, {a}, {a, b, d}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, {c}, {b, c, d}, X} are called $\tau_{1,2}$-closed. Let Y = {a, b, c, d}, σ_1 = {ϕ, {a}, Y} and σ_2 = {ϕ, {b, c}, {a, b, c}, Y}. Then the sets in {ϕ, {a}, {b, c}, {a, b, c}, Y} are called $\sigma_{1,2}$-open and the sets in {ϕ, {d}, {a, d}, {b, c, d}, Y} are called $\sigma_{1,2}$-closed. Let $f : X \to Y$ be the identity function. Then f is (1,2)*-wg$^\#$-open but not (1,2)*-g$^\#$-open.*

Definition 5.4. *Let X and Y be bitopological spaces. A function $f : X \to Y$ is called weakly (1,2)*-g$^\#$-closed (briefly, (1,2)*-wg$^\#$-closed) if f(V) is a (1,2)*-wg$^\#$-closed set in Y for each $\tau_{1,2}$-closed set V of X. It is clear that an (1,2)*-open function is (1,2)*-wg$^\#$-open and a (1,2)*-closed function is (1,2)*-wg$^\#$-closed.*

Theorem 5.5. *Let X and Y be bitopological spaces. A function $f : X \to Y$ is (1,2)*-wg$^\#$-closed if and only if for each subset B of Y and for each $\tau_{1,2}$-open set G containing $f^{-1}(B)$ there exists a (1,2)*-wg$^\#$-open set F of Y such that $B \subseteq F$ and $f^{-1}(F) \subseteq G$.*

Proof. Let B be any subset of Y and let G be an $\tau_{1,2}$-open subset of X such that $f^{-1}(B) \subseteq G$. Then F = Y \f(X \G) is (1,2)*-w$g^\#$-open set containing B and $f^{-1}(F) \subseteq G$.

Conversely, let U be any $\tau_{1,2}$-closed subset of X. Then f^{-1}(Y \f(U)) ⊆ X \U and X \U is $\tau_{1,2}$-open. According to the assumption, there exists a (1,2)*-w$g^\#$-open set F of Y such that Y \f(U) ⊆ F and $f^{-1}(F)$ ⊆ X \U. Then U ⊆ X \$f^{-1}(F)$. From Y \F ⊆ f(U) ⊆ f(X \$f^{-1}(F)$) ⊆ Y \F it follows that f(U) = Y \F, so f(U) is (1,2)*-w$g^\#$-closed in Y. Therefore f is a (1,2)*-w$g^\#$-closed function. □

Remark 5.6. *The composition of two (1,2)*-wg$^\#$-closed functions need not be a (1,2)*-wg$^\#$-closed as we can see from the following example.*

Example 5.7. *Let X = {a, b, c}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ, {a, b}, X}. Then the sets in {ϕ, {a}, {a, b}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, {c}, {b, c}, X} are called $\tau_{1,2}$-closed. Let Y = {a, b, c}, σ_1 = {ϕ, {a}, Y} and σ_2 = {ϕ, {b, c}, Y}. Then the sets in {ϕ, {a}, {b, c}, Y} are called $\sigma_{1,2}$-open and the sets in {ϕ, {a}, {b, c}, Y} are called $\sigma_{1,2}$-closed. Let Z = {a, b, c}, η_1 = {ϕ, Z} and η_2 = {ϕ, {a, b}, Z}. Then the sets in {ϕ, {a, b}, Z} are called $\eta_{1,2}$-open* *and the sets in* $\{\phi, \{c\}, Z\}$ *are called* $\eta_{1,2}$*-closed. We define* $f : X \to Y$ *by* $f(a) = c$, $f(b) = b$ *and* $f(c) = a$ *and let* $g : Y \to Z$ *be the identity function. Hence both f and g are* $(1,2)^\star$*-wg*$^\#$*-closed functions. For a* $\tau_{1,2}$*-closed set* $U = \{b, c\}$, $(g \circ f)(U) = g(f(U)) = g(\{a, b\}) = \{a, b\}$ *which is not* $(1,2)^\star$*-wg*$^\#$*-closed in Z. Hence the composition of two* $(1,2)^\star$*-wg*$^\#$*-closed functions need not be a* $(1,2)^\star$*-wg*$^\#$*-closed.*

Theorem 5.8. *Let X, Y and Z be bitopological spaces. If* $f : X \to Y$ *is a* $(1,2)^\star$*-closed function and* $g : Y \to Z$ *is a* $(1,2)^\star$*-wg*$^\#$*-closed function, then* $g \circ f : X \to Z$ *is a* $(1,2)^\star$*-wg*$^\#$*-closed function.*

Definition 5.9. *A function* $f : X \to Y$ *is called a weakly* $(1,2)^\star$*-g*$^\#$*-irresolute (briefly,* $(1,2)^\star$*-wg*$^\#$*-irresolute) if* $f^{-1}(U)$ *is a* $(1,2)^\star$*-wg*$^\#$*-open set in X for each* $(1,2)^\star$*-wg*$^\#$*-open set U of Y.*

Example 5.10. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, X\}$ *and* $\tau_2 = \{\phi, \{a, c\}, X\}$. *Then the sets in* $\{\phi, \{b\}, \{a, c\}, X\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\phi, \{b\}, \{a, c\}, X\}$ *are called* $\tau_{1,2}$*-closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ *and* $\sigma_2 = \{\phi, \{b\}, Y\}$. *Then the sets in* $\{\phi, \{b\}, Y\}$ *are called* $\sigma_{1,2}$*-open and the sets in* $\{\phi, \{a, c\}, Y\}$ *are called* $\sigma_{1,2}$*-closed. Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^\star$*-wg*$^\#$*-irresolute.*

Remark 5.11. *The following examples show that* $(1,2)^\star$*-αg-irresoluteness and* $(1,2)^\star$*-wg*$^\#$*-irresoluteness are independent of each other.*

Example 5.12. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ *and* $\tau_2 = \{\phi, \{a, b\}, X\}$. *Then the sets in* $\{\phi, \{a, b\}, X\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\phi, \{c\}, X\}$ *are called* $\tau_{1,2}$*-closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ *and* $\sigma_2 = \{\phi, \{a\}, Y\}$. *Then the sets in* $\{\phi, \{a\}, Y\}$ *are called* $\sigma_{1,2}$*-open and the sets in* $\{\phi, \{b, c\}, Y\}$ *are called* $\sigma_{1,2}$*-closed. Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^\star$*-wg*$^\#$*-irresolute but not* $(1,2)^\star$*-αg-irresolute.*

Example 5.13. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ *and* $\tau_2 = \{\phi, \{b\}, X\}$. *Then the sets in* $\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ *are called* $\tau_{1,2}$*-closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ *and* $\sigma_2 = \{\phi, \{a, b\}, Y\}$. *Then the sets in* $\{\phi, \{a, b\}, Y\}$ *are called* $\sigma_{1,2}$*-open and the sets in* $\{\phi, \{c\}, Y\}$ *are called* $\sigma_{1,2}$*-closed. Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^\star$*-αg-irresolute but not* $(1,2)^\star$*-wg*$^\#$*-irresolute.*

Remark 5.14. *Every* $(1,2)^\star$*-g*$^\#$*-irresolute function is* $(1,2)^\star$*-wg*$^\#$*-continuous but not conversely. Also, the concepts of* $(1,2)^\star$*-g*$^\#$*-irresoluteness and* $(1,2)^\star$*-wg*$^\#$*-irresoluteness are independent of each other.*

Example 5.15. *Let* $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, \{a\}, X\}$ *and* $\tau_2 = \{\phi, \{b, c\}, \{a, b, c\}, X\}$. *Then the sets in* $\{\phi, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\phi, \{d\}, \{a, d\}, \{b, c, d\}, X\}$ *are called* $\tau_{1,2}$*-closed. Let* $Y = \{a, b, c, d\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ *and* $\sigma_2 = \{\phi, \{a, b, d\}, Y\}$. *Then the sets in* $\{\phi, \{a\}, \{a, b, d\}, Y\}$ *are called* $\sigma_{1,2}$*-open and the sets in* $\{\phi, \{c\}, \{b, c, d\}, Y\}$ *are called* $\sigma_{1,2}$*-closed. Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^\star$*-wg*$^\#$*-continuous but not* $(1,2)^\star$*-g*$^\#$*-irresolute.*

Example 5.16. *Let* $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ *and* $\tau_2 = \{\phi, \{b, c\}, X\}$. *Then the sets in* $\{\phi, \{a\}, \{b, c\}, X\}$ *are called* $\tau_{1,2}$*-open and the sets in* $\{\phi, \{a\}, \{b, c\}, X\}$ *are called* $\tau_{1,2}$*-closed. Let* $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ *and* $\sigma_2 = \{\phi, \{a, b\}, Y\}$. *Then the sets in* $\{\phi, \{a\}, \{a, b\}, Y\}$ *are called* $\sigma_{1,2}$*-open and the sets in* $\{\phi, \{c\}, \{b, c\}, Y\}$ *are called* $\sigma_{1,2}$*-closed. Let* $f : X \to Y$ *be the identity function. Then f is* $(1,2)^\star$*-wg*$^\#$*-irresolute but not* $(1,2)^\star$*-g*$^\#$*-irresolute.*

Example 5.17. *In Example 5.13, f is* $(1,2)^\star$*-g*$^\#$*-irresolute but not* $(1,2)^\star$*-wg*$^\#$*-irresolute.*

Theorem 5.18. *The composition of two (1,2)*-wg$^\#$-irresolute functions is also (1,2)*-wg$^\#$-irresolute.*

Theorem 5.19. *Let $f : X \to Y$ and $g : Y \to Z$ be functions such that $g \circ f : X \to Z$ is (1,2)*-wg$^\#$-closed function. Then the following statements hold:*

(1) if f is (1,2)*-continuous and injective, then g is (1,2)*-wg$^\#$-closed.

(2) if g is (1,2)*-wg$^\#$-irresolute and surjective, then f is (1,2)*-wg$^\#$-closed.

Proof.

(1) Let F be a $\sigma_{1,2}$-closed set of Y. Since $f^{-1}(F)$ is $\tau_{1,2}$-closed in X, we can conclude that $(g \circ f)(f^{-1}(F))$ is (1,2)*-w$g^\#$-closed in Z. Hence g(F) is (1,2)*-w$g^\#$-closed in Z. Thus g is a (1,2)*-w$g^\#$-closed function.

(2) It can be proved in a similar manner as (1).

□

Theorem 5.20. *If $f : X \to Y$ is an (1,2)*-wg$^\#$-irresolute function, then it is (1,2)*-wg$^\#$-continuous.*

Remark 5.21. *The converse of the above theorem need not be true in general. The function $f : X \to Y$ in the Example 5.13 is (1,2)*-wg$^\#$-continuous but not (1,2)*-wg$^\#$-irresolute.*

Theorem 5.22. *If $f : X \to Y$ is surjective (1,2)*-wg$^\#$-irresolute function and X is (1,2)*-wg$^\#$-compact, then Y is (1,2)*-wg$^\#$-compact.*

Theorem 5.23. *If $f : X \to Y$ is surjective (1,2)*-wg$^\#$-irresolute function and X is (1,2)*-wg$^\#$-connected, then Y is (1,2)*-wg$^\#$-connected.*

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