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Decompositions of rg-continuity via Idealization

Research Article

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Abstract: In this paper, we introduce the notions of $r\alpha g$-$\mathcal{I}$-open sets, gpr-$\mathcal{I}$-open sets, C_r-$\mathcal{I}$-sets and C_r^*-$\mathcal{I}$-sets in ideal topological spaces and investigate some of their properties and using these notions we obtain three decompositions of rg-continuity.

MSC: 54C08.

Keywords: $r\alpha g$-$\mathcal{I}$-continuity, gpr-$\mathcal{I}$-continuity, C_r-$\mathcal{I}$-continuity, C_r^*-$\mathcal{I}$-continuity and rg-continuity.

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1. Introduction and Preliminaries

In 1970, Levine [11] initiated the study of so called g-closed sets in topological spaces. The concept of g-continuity was introduced and studied by Balachandran et.al in 1991 [2]. In 1993, Palaniappan and Rao [17] introduced the notions of regular generalized closed (rg-closed) sets and rg-continuity in topological spaces. In 2000, Sundaram and Rajamani [20] obtained three different decompositions of rg-continuity by providing two types of weaker forms of continuity, namely C_r-continuity and C_r^*-continuity. Recently, Noiri et. al. [15] introduced the notions of αg-$\mathcal{I}$-open sets, gp-$\mathcal{I}$-open sets, gs-$\mathcal{I}$-open sets, $C(S)$-$\mathcal{I}$-sets, C^*-$\mathcal{I}$-sets and S^*-$\mathcal{I}$-sets to obtain three different decompositions of g-continuity via idealization. Recently Ravi et. al. [18] obtained three different decompositions of rg-continuity. In this paper, we introduce the notions of $r\alpha g$-$\mathcal{I}$-open sets, gpr-$\mathcal{I}$-open sets, C_r-$\mathcal{I}$-sets and C_r^*-$\mathcal{I}$-sets to obtain further decompositions of rg-continuity.

Let (X, τ) be a topological space. An ideal is defined as a nonempty collection $\mathcal{I}$ of subsets of X satisfying the following two conditions:

(i) If $A \in \mathcal{I}$ and $B \subseteq A$, then $B \in \mathcal{I}$

(ii) If $A \in \mathcal{I}$ and $B \in \mathcal{I}$, then $A \cup B \in \mathcal{I}$.

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For a subset $A \subseteq X$, $A^*(\mathcal{I}) = \{x \in X/U \cap A \notin \mathcal{I}$ for each neighborhood U of $x\}$ is called the local function of A with respect to $\mathcal{I}$ and τ [9]. We simply write A^* instead of $A^*(\mathcal{I})$ in case there is no chance for confusion. X^* is often a proper subset of X. For every ideal topological space $(X,\ \tau,\ \mathcal{I})$ there exists a topology $\tau^*(\mathcal{I})$, finer than τ, generated by $\beta(\mathcal{I},\tau)$ $= \{U \setminus I : U \in \tau$ and $I \in \mathcal{I}\}$, but in general $\beta(\mathcal{I},\tau)$ is not always a topology [22]. Also, $\mathrm{cl}^*(A) = A \cup A^*$ defines a Kuratowski closure operator for $\tau^*(\mathcal{I})$ [22]. Additionally, $\mathrm{cl}^*(A) \subseteq \mathrm{cl}\ (A)$ for any subset A of X [7]. Throughout this paper, X denotes the ideal topological space $(X,\tau,\mathcal{I})$ and also $\mathrm{cl}(A)$ and $\mathrm{int}(A)$ denote the closure of A and the interior of A in (X,τ), respectively.

Definition 1.1. A subset A of (X,τ) is said to be

(1) α-open [13] if $A \subseteq int(cl(int(A)))$,

(2) preopen [12] if $A \subseteq int(cl(A))$,

(3) regular open [19] if $A = int(cl(A))$,

(4) rg-open [17] iff $F \subseteq int(A)$ whenever $F \subseteq A$ and F is regular closed in $(X,\ \tau)$,

(5) gpr-open [4] iff $F \subseteq pint(A)$ whenever $F \subseteq A$ and F is regular closed in $(X,\ \tau)$,

(6) $r\alpha g$-open [18] iff $F \subseteq \alpha int(A)$ whenever $F \subseteq A$ and F is regular closed in $(X,\ \tau)$,

(7) a t-set [21] if $int(A) = int(cl(A))$,

(8) an α^*-set [6] if $int(A) = int(cl(int(A)))$,

(9) a C_r-set [18] if $A = U \cap V$, where U is rg-open and V is a t-set in $(X,\ \tau)$,

(10) a C_r^*-set [18] if $A = U \cap V$, where U is rg-open and V is an α^*-set in $(X,\ \tau)$.

The complements of the above mentioned open sets are called their respective closed sets. The preinterior $pint(A)$ (resp. α-interior, $\alpha int(A)$) of A is the union of all preopen sets (resp. α-open sets) contained in A. The α-closure $\alpha cl(A)$ of A is the intersection of all α-closed sets containing A.

Lemma 1.1 ([1])**.** *If A is a subset of X, then*

(1) $pint(A) = A \cap int(cl(A))$,

(2) $\alpha int(A) = A \cap int(cl(int(A)))$ and $\alpha cl(A) = A \cup cl(int(cl(A)))$.

Definition 1.2. A subset A of an ideal topological space $(X,\tau,\mathcal{I})$ is said to be

(1) pre-$\mathcal{I}$-open [3] if $A \subseteq int(cl^*(A))$,

(2) α-$\mathcal{I}$-open [5] if $A \subseteq int(cl^*(int(A)))$,

(3) a t-$\mathcal{I}$-set [5] if $int(cl^*(A)) = int(A)$,

(4) an α^*-$\mathcal{I}$-set [5] if $int(cl^*(int(A))) = int(A)$.

Also, we have α-$\mathcal{I}$-int(A) = $A \cap int(cl^*(int(A)))$ [15] and p-$\mathcal{I}$-int(A) = $A \cap int(cl^*(A))$ [15], where α-$\mathcal{I}$-int(A) denotes the α-$\mathcal{I}$ interior of A in (X, τ, $\mathcal{I}$) which is the union of all α-$\mathcal{I}$-open sets of (X, τ, $\mathcal{I}$) contained in A. p-$\mathcal{I}$-int(A) has similar meaning.

Remark 1.1. *The following hold in a topological space.*

(1) Every rg-open set is gpr-open but not conversely [4].

(2) Every rg-open set is $r\alpha g$-open but not conversely [18].

2. $r\alpha g$-$\mathcal{I}$-open Sets and gpr-$\mathcal{I}$-open Sets

Definition 2.1. A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called

(1) $r\alpha g$-$\mathcal{I}$-open if $F \subseteq \alpha$-$\mathcal{I}$-$int(A)$ whenever $F \subseteq A$ and F is regular closed in X.

(2) gpr-$\mathcal{I}$-open if $F \subseteq p$-$\mathcal{I}$-$int(A)$ whenever $F \subseteq A$ and F is regular closed in X.

Proposition 2.1. *For a subset of an ideal topological space, the following hold:*

(1) Every $r\alpha g$-$\mathcal{I}$-open set is $r\alpha g$-open.

(2) Every gpr-$\mathcal{I}$-open set is gpr-open.

(3) Every $r\alpha g$-open set is gpr-open.

Proof.

(1) Let A be an $r\alpha g$-$\mathcal{I}$-open set. Let $F \subseteq A$ and F is regular closed in X. Then, $F \subseteq \alpha$-$\mathcal{I}$-int$(A) = A \cap (\text{int}(\text{cl}^*(\text{int}(A))))$ $\subseteq A \cap \text{int}(\text{cl}(\text{int}(A))) = \alpha\text{int}(A)$. This shows that A is $r\alpha g$-open.

(2) Let A be gpr-$\mathcal{I}$-open set. Let $F \subseteq A$ and F is regular closed in X. Then, $F \subseteq p$-$\mathcal{I}$-int$(A) = A \cap \text{int}(\text{cl}^*(A)) \subseteq A \cap \text{int}(\text{cl}(A)) = p\text{int}(A)$. This shows that A is gpr-open.

(3) It follows from the definitions.

□

Remark 2.1. *The converses of Proposition 2.1 are not true, in general.*

Example 2.1. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{b\}\}$. Then $\{a, b, d\}$ is $r\alpha g$-open but not an $r\alpha g$-$\mathcal{I}$-open set.*

Example 2.2. *In Example 2.1, $\{a, b, d\}$ is gpr-open but not a gpr-$\mathcal{I}$-open set.*

Example 2.3. *Let X= $\{a, b, c, d\}$ and $\tau = \{\emptyset, \{d\}, \{a, c\}, \{a, c, d\}, X\}$. Then $\{b, c, d\}$ is gpr-open set but not an $r\alpha g$-open.*

Proposition 2.2. *For a subset of an ideal topological space, the following hold:*

(1) Every $r\alpha g$-$\mathcal{I}$-open set is gpr-$\mathcal{I}$-open.

(2) Every rg-open set is gpr-$\mathcal{I}$-open.

(3) Every rg-open set is $r\alpha g$-$\mathcal{I}$-open.

Proof.

(1) Let A be $r\alpha g$-$\mathcal{I}$-open. Then, for any regular closed set F with F $\subseteq A$, we have F $\subseteq \alpha$-$\mathcal{I}$-$\mathrm{int}(A) = A \cap \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(A))) \subseteq A \cap \mathrm{int}(\mathrm{cl}^*(A)) = p$-$\mathcal{I}$-$\mathrm{int}(A)$ which implies that A is *gpr*-$\mathcal{I}$-open.

(2) Let A be an rg-open set. Then, for any regular closed set F with $F \subseteq A$, we have $F \subseteq \mathrm{int}(A) \subseteq \mathrm{int}((\mathrm{int}(A))^*) \cup \mathrm{int}(A) = \mathrm{int}((\mathrm{int}(A))^*) \cup \mathrm{int}(\mathrm{int}(A)) \subseteq \mathrm{int}((\mathrm{int}(A))^* \cup \mathrm{int}(A)) = \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(A)))$. That is, $F \subseteq A \cap \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(A))) = \alpha$-$\mathcal{I}$-$\mathrm{int}(A)$ = A$\cap$int(cl*(int(A))) $\subseteq$ A$\cap$int(cl*(A)) = p-$\mathcal{I}$-int(A) which implies that A is *gpr*-$\mathcal{I}$-open.

(3) Let A be an rg-open set. Then, for any regular closed set F with $F \subseteq A$, we have $F \subseteq \mathrm{int}(A) \subseteq \mathrm{int}((\mathrm{int}(A))^*) \cup \mathrm{int}(A) = \mathrm{int}((\mathrm{int}(A))^*) \cup \mathrm{int}(\mathrm{int}(A)) \subseteq \mathrm{int}((\mathrm{int}(A))^* \cup \mathrm{int}(A)) = \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(A)))$. That is, $F \subseteq A \cap \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(A))) = \alpha$-$\mathcal{I}$-$\mathrm{int}(A)$ which implies that A is $r\alpha g$-$\mathcal{I}$-open.

□

Remark 2.2. *The converses of Proposition 2.2 are not true, in general.*

Example 2.4. *Let $X = \{a, b, c, d, e\}$, $\tau = \{\emptyset, X, \{a\}, \{e\}, \{a, e\}, \{c, d\}, \{a, c, d\}, \{c, d, e\}, \{a, c, d, e\}, \{b, c, d, e\}\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{e\}, \{a, e\}\}$. Then $\{a, b, d, e\}$ is gpr-$\mathcal{I}$-open but not an $r\alpha g$-$\mathcal{I}$-open set.*

Example 2.5. *Let $X = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ and $\mathcal{I} = \{\emptyset, \{d\}\}$. Then $\{a, b, d\}$ is gpr-$\mathcal{I}$-open but not a rg-open set.*

Example 2.6. *In Example 2.5, $\{a, b, d\}$ is $r\alpha g$-$\mathcal{I}$-open but not a rg-open set.*

Remark 2.3. *By Remark 1.1, Propositions 2.1 and 2.2, we have the following diagram. In this diagram, there is no implication which is reversible as shown by examples above.*

$$\begin{tikzcd}
 & r\alpha g\text{-}\mathcal{I}\text{-open} \arrow[r] \arrow[d] & r\alpha g\text{-open} \arrow[d] \\
rg\text{-open} \arrow[ur] \arrow[dr] & & \\
 & gpr\text{-}\mathcal{I}\text{-open} \arrow[r] & gpr\text{-open}
\end{tikzcd}$$

3. C_r-$\mathcal{I}$-sets and C_r^*-$\mathcal{I}$-sets

Definition 3.1. *A subset A of an ideal topological space $(X, \tau, \mathcal{I})$ is called*

(1) a C_r-$\mathcal{I}$-set if $A = U \cap V$, where U is rg-open and V is a t-$\mathcal{I}$-set,

(2) a C_r^*-$\mathcal{I}$-set if $A = U \cap V$, where U is rg-open and V is an α^*-$\mathcal{I}$-set.

We have the following proposition:

Proposition 3.1. *For a subset of an ideal topological space, the following hold:*

(1) Every t-$\mathcal{I}$-set is an α^*-$\mathcal{I}$-set [5] and a C_r-$\mathcal{I}$-set.

(2) Every α^*-$\mathcal{I}$-set is a C_r^*-$\mathcal{I}$-set.

(3) Every C_r-$\mathcal{I}$-set is a C_r^*-$\mathcal{I}$-set.

(4) Every rg-open set is a C_r-$\mathcal{I}$-set.

(5) Every C_r-set is a C_r-$\mathcal{I}$-set and a C_r^*-$\mathcal{I}$-set.

(6) Every C_r^*-set is a C_r^*-$\mathcal{I}$-set.

From Proposition 3.1, We have the following diagram.

$$\begin{tikzcd}
rg\text{-open set} \arrow[r] & C_r\text{-set} \arrow[r] \arrow[d] & C_r\text{-}\mathcal{I}\text{-set} \arrow[d] & t\text{-}\mathcal{I}\text{-set} \arrow[l] \arrow[d] \\
 & C_r^*\text{-set} \arrow[r] & C_r^*\text{-}\mathcal{I}\text{-set} & \alpha^*\text{-}\mathcal{I}\text{-set} \arrow[l]
\end{tikzcd}$$

Remark 3.1. *The converses of implications in Diagram II need not be true as the following examples show.*

Example 3.1. *In Example 2.1, $\{a,\ b,\ d\}$ is C_r-$\mathcal{I}$-set but not a C_r-set.*

Example 3.2. *In Example 2.1, $\{a,\ b,\ c\}$ is C_r-$\mathcal{I}$-set but not a t-$\mathcal{I}$-set.*

Example 3.3. *In Example 2.1, $\{a,\ d\}$ is C_r-set but not a rg-open set.*

Example 3.4. *In Example 2.3, $\{a,\ b,\ d\}$ is C_r^*-set but not a C_r-set.*

Example 3.5. *In Example 2.4, $\{a,\ b,\ c,\ e\}$ is C_r^*-$\mathcal{I}$-set but not a C_r-$\mathcal{I}$-set.*

Example 3.6. *In Example 2.1, $\{a,\ b,\ d\}$ is C_r^*-$\mathcal{I}$-set but not a C_r^*-set.*

Example 3.7. *In Example 2.1, $\{a,\ b,\ c\}$ is C_r^*-$\mathcal{I}$-set but not an α^*-$\mathcal{I}$-set.*

Example 3.8. *Let $X = \{a,\ b,\ c\}$, $\tau = \{\emptyset,\ X,\ \{a,\ b\}\}$ and $\mathcal{I} = \{\emptyset,\ \{a\}\}$. Then $\{b\}$ is α^*-$\mathcal{I}$-set but not a t-$\mathcal{I}$-set.*

Remark 3.2. *Examples 3.9 and 3.10 show that C_r-$\mathcal{I}$-sets and C_r^*-sets are independent of each other.*

Example 3.9. *In Example 2.4, $\{a,\ c,\ d\}$ is C_r^*-set but not a C_r-$\mathcal{I}$-set.*

Example 3.10. *In Example 2.1, $\{a,\ b,\ d\}$ is C_r-$\mathcal{I}$-set but not a C_r^*-set.*

Proposition 3.2. *A subset A of X is rg-open if and only if it is both gpr-$\mathcal{I}$-open and a C_r-$\mathcal{I}$-set in X.*

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is gpr-$\mathcal{I}$-open and a C_r-$\mathcal{I}$-set in X. Let $F \subseteq A$ and F is regular closed in X. Since A is a C_r-$\mathcal{I}$-set in X, $A = U \cap V$, where U is rg-open and V is a t-$\mathcal{I}$-set. Since A is gpr-$\mathcal{I}$-open, $F \subseteq p$-$\mathcal{I}$-$\mathrm{int}(A) = A \cap \mathrm{int}(\mathrm{cl}^*(A)) = (U \cap V) \cap \mathrm{int}(\mathrm{cl}^*(U \cap V)) \subseteq (U \cap V) \cap \mathrm{int}(\mathrm{cl}^*(U) \cap \mathrm{cl}^*(V)) = (U \cap V) \cap \mathrm{int}(\mathrm{cl}^*(U)) \cap \mathrm{int}(\mathrm{cl}^*(V))$. This implies $F \subseteq \mathrm{int}(\mathrm{cl}^*(V)) = \mathrm{int}(V)$ since V is a t-$\mathcal{I}$-set. Since F is regular closed, U is rg-open and $F \subseteq U$, we have $F \subseteq \mathrm{int}(U)$. Therefore, $F \subseteq \mathrm{int}(U) \cap \mathrm{int}(V) = \mathrm{int}(U \cap V) = \mathrm{int}(A)$. Hence A is rg-open in X. $\square$

Corollary 3.1. *A subset A of X is rg-open if and only if it is both $r\alpha g$-$\mathcal{I}$-open and a C_r-$\mathcal{I}$-set in X.*

Proof. This is an immediate consequence of Proposition 3.2. $\square$

Proposition 3.3. *A subset A of X is rg-open if and only if it is both $r\alpha g$-$\mathcal{I}$-open and a C_r^*-$\mathcal{I}$-set in X.*

Proof. Necessity is trivial. We prove the sufficiency. Assume that A is $r\alpha g$-$\mathcal{I}$-open and a C_r^*-$\mathcal{I}$-set in X. Let $F \subseteq A$ and F is regular closed in X. Since A is a C_r^*-$\mathcal{I}$-set in X, $A = U \cap V$, where U is rg-open and V is an α^*-$\mathcal{I}$-set. Now since F is regular closed, $F \subseteq U$ and U is rg-open, $F \subseteq$ int(U). Since A is $r\alpha g$-$\mathcal{I}$-open, $F \subseteq \alpha$-$\mathcal{I}$-$\mathrm{int}(A) = A \cap \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(A))) = (U \cap V) \cap \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(U \cap V))) = (U \cap V) \cap \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(U) \cap \mathrm{int}(V))) \subseteq (U \cap V) \cap \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(U)) \cap \mathrm{cl}^*(\mathrm{int}(V))) = (U \cap V) \cap \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(U))) \cap \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(V))) = (U \cap V) \cap \mathrm{int}(\mathrm{cl}^*(\mathrm{int}(U))) \cap \mathrm{int}(V)$, since V is an α^*-$\mathcal{I}$-set. This implies $F \subseteq \mathrm{int}(V)$. Therefore, $F \subseteq \mathrm{int}(U) \cap \mathrm{int}(V) = \mathrm{int}(U \cap V) = \mathrm{int}(A)$. Hence A is rg-open in X. $\square$

Remark 3.3.

(1) The concepts of gpr-$\mathcal{I}$-open sets and C_r-$\mathcal{I}$-sets are independent of each other.

(2) The concepts of $r\alpha g$-$\mathcal{I}$-open sets and C_r-$\mathcal{I}$-sets are independent of each other.

(3) The concepts of $r\alpha g$-$\mathcal{I}$-open sets and C_r^*-$\mathcal{I}$-sets are independent of each other.

Example 3.11. *In Example 2.5,*

(1) $\{b,\ c,\ d\}$ is C_r-$\mathcal{I}$-set but not a gpr-$\mathcal{I}$-open.

(2) $\{a,\ b,\ d\}$ is gpr-$\mathcal{I}$-open but not a C_r-$\mathcal{I}$-set.

Example 3.12. *In Example 2.5,*

(i) $\{b,\ c,\ d\}$ is C_r-$\mathcal{I}$-set but not an $r\alpha g$-$\mathcal{I}$-open set.

(ii) $\{a,\ b,\ d\}$ is $r\alpha g$-$\mathcal{I}$-open set but not a C_r-$\mathcal{I}$-set.

Example 3.13. *In Example 2.5,*

(1) $\{b,\ c,\ d\}$ is C_r^*-$\mathcal{I}$-set but not an $r\alpha g$-$\mathcal{I}$-open set.

(2) $\{a,\ b,\ d\}$ is $r\alpha g$-$\mathcal{I}$-open set but not a C_r^*-$\mathcal{I}$-set.

4. Decompositions of rg-continuity

Definition 4.1. *A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be rg-continuous [17] (resp. gpr-continuous [4], $r\alpha g$-continuous [18], C_r-continuous [18] and C_r^*-continuous [18]) if $f^{-1}(V)$ is rg-open (resp. gpr-open, $r\alpha g$-open, C_r-set and C_r^*-set) in (X, τ) for every open set V in (Y, σ).*

Definition 4.2. *A mapping $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be $r\alpha g$-$\mathcal{I}$-continuous (resp. gpr-$\mathcal{I}$-continuous, C_r-$\mathcal{I}$-continuous and C_r^*-$\mathcal{I}$-continuous) if for every $V \in \sigma$, $f^{-1}(V)$ is $r\alpha g$-$\mathcal{I}$-open (resp. gpr-$\mathcal{I}$-open, a C_r-$\mathcal{I}$-set and a C_r^*-$\mathcal{I}$-set) in $(X, \tau, \mathcal{I})$.*

From Propositions 3.2 and 3.3 and Corollary 3.1 we have the following decompositions of rg-continuity.

Theorem 4.1. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space. For a mapping $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is rg-continuous;

(2) f is gpr-$\mathcal{I}$-continuous and C_r-$\mathcal{I}$-continuous;

(3) f is $r\alpha g$-$\mathcal{I}$-continuous and C_r-$\mathcal{I}$-continuous;

(4) f is $r\alpha g$-$\mathcal{I}$-continuous and C_r^*-$\mathcal{I}$-continuous.

Remark 4.1.

(1) The concepts of gpr-$\mathcal{I}$-continuity and C_r-$\mathcal{I}$-continuity are independent of each other.

(2) The concepts of $r\alpha g$-$\mathcal{I}$-continuity and C_r-$\mathcal{I}$-continuity are independent of each other.

(3) The concepts of $r\alpha g$-$\mathcal{I}$-continuity and C_r^*-$\mathcal{I}$-continuity are independent of each other.

Example 4.1.

(1) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$, $\mathcal{I} = \{\emptyset, \{b\}\}$ and $\sigma = \{\emptyset, Y, \{a, d\}\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be the identity function. Then f is C_r-$\mathcal{I}$-continuous but not gpr-$\mathcal{I}$-continuous.

(2) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{d\}, \{a, c\}, \{a, c, d\}\}$, $\mathcal{I} = \{\emptyset, \{a\}, \{d\}, \{a, d\}\}$ and $\sigma = \{\emptyset, Y, \{b, c, d\}\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be the identity function. Then f is gpr-$\mathcal{I}$-continuous but not C_r-$\mathcal{I}$-continuous.

Example 4.2.

(1) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$, $\mathcal{I} = \{\emptyset, \{c\}\}$ and $\sigma = \{\emptyset, Y, \{b, c, d\}\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be the identity function. Then f is C_r-$\mathcal{I}$-continuous but not $r\alpha g$-$\mathcal{I}$-continuous.

(2) Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$, $\mathcal{I} = \{\emptyset, \{d\}\}$ and $\sigma = \{\emptyset, Y, \{a, b, d\}\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be the identity function. Then f is $r\alpha g$-$\mathcal{I}$-continuous but not C_r-$\mathcal{I}$-continuous.

Example 4.3. *Let X, Y, τ, $\mathcal{I}$ and f be as in Example 4.2 (2). Let $\sigma = \{\emptyset, Y, \{b, c, d\}\}$. Then f is C_r^*-$\mathcal{I}$-continuous but not $r\alpha g$-$\mathcal{I}$-continuous. In Example 4.2 (2), f is $r\alpha g$-$\mathcal{I}$-continuous but not C_r^*-$\mathcal{I}$-continuous.*

Corollary 4.1 ([18])**.** *Let $(X, \tau, \mathcal{I})$ be an ideal topological space and $\mathcal{I} = \{\emptyset\}$. For a mapping $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, the following properties are equivalent:*

(1) f is rg-continuous;

(2) f is gpr-continuous and C_r-continuous;

(3) f is $r\alpha g$-continuous and C_r-continuous;

(4) f is $r\alpha g$-continuous and C_r^*-continuous.

Proof. Since $\mathcal{I}= \{\emptyset\}$, we have $A^* = \mathrm{cl}(A)$ and $\mathrm{cl}^*(A) = A^* \cup A = \mathrm{cl}(A)$ for any subset A of X [[5], Proposition 2.4(a)]. Therefore, we obtain (1) A is $r\alpha g$-$\mathcal{I}$-open (resp. gpr-$\mathcal{I}$-open) if and only if it is $r\alpha g$-open (resp. gpr-open) and (2) A is a C_r-$\mathcal{I}$-set (resp. a C_r^*-$\mathcal{I}$-set) if and only if it is a C_r-set (resp. a C_r^*-set). The proof follows from Theorem 4.1 immediately. □

References

[1] D.Andrijevic, *Semi-preopen sets*, Mat. Vesnik, 38(1)(1986), 24-32.

[2] K.Balachandran, P.Sundaram and H.Maki, *On generalized continuous maps in topological spaces*, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 12(1991), 5-13.

[3] J.Dontchev, *On pre-I-open sets and a decomposition of $\mathcal{I}$-continuity*, Banyan Math. J., 2(1996).

[4] Y.Gnanambal, *On generalized preregular closed sets in topological spaces*, Indian J. Pure Appl. Math., 28(3)(1997), 351-360.

[5] E.Hatir and T.Noiri, *On decompositions of continuity via idealization*, Acta Math. Hungar., 96(4)(2002), 341-349.

[6] E.Hatir, T.Noiri and S.Yuksel, *A decomposition of continuity*, Acta Math. Hungar., 70(1996), 145-150.

[7] E.Hayashi, *Topologies defined by local properties*, Math. Ann., 156(1964), 205-215.

[8] D.Jankovic and T.R.Hamlett, *New topologies from old via ideals*, Amer. Math. Monthly, 97(4)(1990), 295-310.

[9] K.Kuratowski, *Topology*, Vol. 1, Academic Press, New York (1966).

[10] N.Levine, *A decomposition of continuity in topological spaces*, Amer. Math. Monthly, 68(1961), 44-46.

[11] N.Levine, *Generalized closed sets in topology*, Rend. Circ. Mat. Palermo, 19(2)(1970), 89-96.

[12] A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb, *On precontinuous and weak precontinuous mappings*, Proc. Math. Phys. Soc. Egypt, 53(1982), 47-53.

[13] O.Njastad, *On some classes of nearly open sets*, Pacific J. Math., 15(1965), 961-970.

[14] T.Noiri, *Almost αg-closed functions and separation axioms*, Acta Math. Hungar., 82(3)(1999), 193-205.

[15] T.Noiri, M.Rajamani and V.Inthumathi, *On decompositions of g-continuity via idealization*, Bull. Cal. Math. Soc., 99(4)(2007), 415-424.

[16] T.Noiri, M.Rajamani and V.Inthumathi, *Some decompositions of regular generalized continuity via idealization*, Kochi J. Math., 5(2010), 87-95.

[17] N.Palaniappan and K.C.Rao, *Regular generalized closed sets*, Kyungpook Math. J., 33(1993), 211-219.

[18] O.Ravi, A.Pandi, I.Rajasekaran and A.Hamari Choudhi, *Further decompositions of rg-continuity*, submitted.

[19] M.H.Stone, *Applications of the theory of Boolean rings to general topology*, Trans. Amer. Math. Soc., 41(1937), 375-481.

[20] P.Sundaram and M.Rajamani, *Some decompositions of regular generalized continuous maps in topological spaces*, Far East J. Math. Sci., special volume, Part II, (2000), 179-188.

[21] J.Tong, *On decomposition of continuity in topological spaces*, Acta Math. Hungar., 54(1989), 51-55.

[22] R.Vaidyanathaswamy, *The localization theory in set topology*, Proc. Indian Acad. Sci., Sect A, 20(1944), 51-61.