



Contra Regular Generalized Semipre Continuous Mapping in Intuitionistic Fuzzy Topological Spaces

Research Article

V.Vaishnavy^{1*} and D.Jayanthi²,

1 MSc Mathematics, Avinashilingam University, Coimbatore, Tamil Nadu, India.

2 Department of Mathematics, Avinashilingam University, Coimbatore, Tamil Nadu, India.

Abstract: In this paper we introduce the notion of intuitionistic fuzzy contra regular generalized semipre continuous mappings. Furthermore we provide some properties of intuitionistic fuzzy contra regular generalized semipre continuous mappings and discuss some interesting theorems.

Keywords: Intuitionistic fuzzy topology, Intuitionistic fuzzy closed sets, Intuitionistic fuzzy regular generalized semipre closed sets, Intuitionistic fuzzy regular generalized semipre open sets, Intuitionistic fuzzy contra regular generalized semipre continuous mappings.

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1. Introduction

The notion of intuitionistic fuzzy sets by Atanassov [1] was a breakthrough towards the evolution of intuitionistic fuzzy topology. Using this notion, Coker [3] constructed the basic concepts of intuitionistic fuzzy topological spaces. Later this was followed by the introduction of intuitionistic fuzzy regular generalized semipreclosed sets by Vaishnavy, V and Jayanthi, D [10] in 2015 which was simultaneously followed by the introduction of intuitionistic fuzzy regular generalized semipre continuous mappings [12] by the same authors. We now extend our idea towards intuitionistic fuzzy contra regular generalized semipre continuous mappings and discuss some of their properties.

2. Preliminaries

Definition 2.1 ([1]). An intuitionistic fuzzy set (IFS in short) A is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$, where the function $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A , respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $IFS(X)$, the set of all intuitionistic fuzzy sets in X . An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A \rangle$ instead of denoting $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$.

* E-mail: siingam@yahoo.com

Definition 2.2 ([1]). Let A and B be two IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$

(c) $A^c = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$

(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$

(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$

The intuitionistic fuzzy sets $0 \sim \langle x, 0, 1 \rangle$ and $1 \sim \langle x, 1, 0 \rangle$ are respectively the empty set and the whole set of X .

Definition 2.3 ([2]). An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0 \sim, 1 \sim \in \tau$

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called the intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X . The compliment A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition 2.4 ([2]). Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{G / G \text{ is an IFOS in } X \text{ and } G \subseteq A\}$$

$$\text{cl}(A) = \cap \{K / K \text{ is an IFCS in } X \text{ and } A \subseteq K\}$$

Note that for any IFS A in (X, τ) , we have $\text{cl}(A^c) = (\text{int}(A))^c$ and $\text{int}(A^c) = (\text{cl}(A))^c$.

Definition 2.5 ([4]). An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy pre closed set (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$

(ii) intuitionistic fuzzy closed set (IF_α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs. The family of all IFPCSs, IF_α CSs and (respectively IFPOSs, IF_α OSs) of an IFTS (X, τ) are respectively denoted by $IFPC(X)$, $IF_\alpha C(X)$ (respectively $IFPO(X)$, $IF_\alpha O(X)$).

Definition 2.6 ([13]). An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

(i) intuitionistic fuzzy semi-pre closed set (IFSPCS in short) if there exists an IFPCS B such that $\text{int}(B) \subseteq A \subseteq B$

(ii) intuitionistic fuzzy semi-pre open set (IFSPOS in short) if there exists an IFPOS B such that $B \subseteq A \subseteq cl(B)$.

The family of all IFSPCSs (respectively IFSPOSs) of an IFTS (X, τ) is denoted by $IFSPC(X)$ (respectively $IFSPC(X)$). Every IFSCS (respectively IFSCS) and every IFPCS (respectively IFPOS) is an IFSPCS (respectively IFSPOS). But the separate converses need not hold in general.

Definition 2.7 ([7]). Let A be an IFS in an IFTS (X, τ) . Then the semi-pre interior and the semi-pre closure of A are defined as

$$spint(A) = \cup\{G|G \text{ is an IFSPOS in } X \text{ and } G \subseteq A\}$$

$$spcl(A) = \cap\{K|K \text{ is an IFSPCS in } X \text{ and } A \subseteq K\}$$

Note that for any IFS A in (X, τ) , we have $spcl(A^c) = (spint(A))^c$ and $spint(A^c) = (spcl(A))^c$.

Definition 2.8 ([8]). An IFS A is an

(i) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = cl(int(A))$

(ii) intuitionistic fuzzy regular open set (IFROS in short) if $A = int(cl(A))$.

Definition 2.9 ([3]). An intuitionistic fuzzy point (IFP in short), written as $p_{(\alpha, \beta)}$, is defined to be an intuitionistic fuzzy set of X given by

$$p_{(\alpha, \beta)}(x) = \begin{cases} (\alpha, \beta), & \text{if } x = p, \\ (0, 1), & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha, \beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.10 ([8]). Two IFSs are said to be q -coincident ($A_q B$ in short) if and only if there exists an element $x \in X$ such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.11 ([10]). An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy regular generalized semipreclosed set (IFRGSPCS in short) if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in (X, τ) .

Definition 2.12 ([11]). The complement A^c of an IFRGSPCS A in an IFTS (X, τ) is called an intuitionistic fuzzy regular generalized semipreopen set (IFRGSPOS in short) in X .

Definition 2.13 ([11]). If every IFRGSPCS in (X, τ) is an IFSPCS in (X, τ) , then the space can be called as an intuitionistic fuzzy regular semipre $T_{1/2}$ space (IFRSPT $_{1/2}$ in short).

Definition 2.14 ([4]). Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy continuous (IF continuous in short) mapping if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.

Definition 2.15 ([12]). A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an intuitionistic fuzzy regular generalized semipre continuous (IFRGSP continuous in short) mapping if $f^{-1}(V)$ is an IFRGSPCS in (X, τ) for every IFCS V in (Y, σ) .

Definition 2.16 ([6]). Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an

(i) intuitionistic fuzzy contra continuous mapping (IFC continuous mapping in short) if $f^{-1}(B) \in IFO(X)$ for each IFCS B in Y

(ii) intuitionistic fuzzy contra α -continuous mapping (IF $_{\alpha}$ C continuous mapping in short) if $f^{-1}(B) \in IF_{\alpha}O(X)$ for each IFCS B in Y

(iii) intuitionistic fuzzy contra pre continuous mapping (IFPC continuous mapping in short) if $f^{-1}(B) \in IFPO(X)$ for each IFCS B in Y

Corollary 2.17. Let $A, A_i (i \in J)$ be intuitionistic fuzzy sets in X and $B, B_j (j \in K)$ be intuitionistic fuzzy sets in Y and $f : X \rightarrow Y$ be a function. Then

(a) $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$

(b) $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$

(c) $A \subseteq f^{-1}(f(A))$ [If f is injective, then $A = f^{-1}(f(A))$]

(d) $f(f^{-1}(B)) \subseteq B$ [If f is surjective, then $B = f(f^{-1}(B))$]

(e) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$

(f) $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$

(g) $f^{-1}(0 \sim) = 0 \sim$

(h) $f^{-1}(1 \sim) = 1 \sim$

(i) $f^{-1}(B^c) = (f^{-1}(B))^c$

3. Contra Regular Generalized Semipre Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

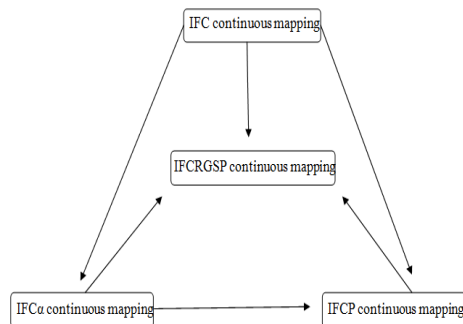
In this section we introduce the notion of intuitionistic fuzzy contra regular generalized semipre continuous mappings and discuss few of its properties.

Definition 3.1. A mapping $f : X \rightarrow Y$ is said to be an intuitionistic fuzzy contra regular generalized semipre continuous (IFCRGSP continuous in short) mapping if $f^{-1}(A)$ is an IFRGSPCS in X for every IFOS A in Y .

Example 3.2. Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$ and $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7) \rangle$. Then $\tau = \{0 \sim, G_1, 1 \sim\}$ and $\sigma = \{0 \sim, G_2, 1 \sim\}$ are IFTs on X and Y respectively.

Then, $IFPC(X) = \{0 \sim, 1 \sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1] / \text{either } \mu_b \geq 0.6 \text{ or } \mu_b < 0.4 \text{ whenever } \mu_a \geq 0.5, \mu_a + \nu_a \leq 1 \text{ and } \mu_b + \nu_b \leq 1\}$. Therefore, $IFSPC(X) = \{0 \sim, 1 \sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1] / \mu_a + \nu_a \leq 1 \text{ and } \mu_b + \nu_b \leq 1\}$. Now $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7) \rangle$ is an IFOS in Y . Therefore $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$. We have $\text{spcl}(f^{-1}(G_2)) = f^{-1}(G_2)$. We have $f^{-1}(G_2) \subseteq G_1$. Hence $\text{spcl}(f^{-1}(G_2)) \subseteq G_1$, where G_1 is an IFROS in X . This implies $f^{-1}(G_2)$ is an IFRGSPCS in X . Therefore f is an IFCRGSP continuous mapping.

Remark 3.3. Every IFC continuous mapping, IF $_{\alpha}$ C continuous mapping, IFCP continuous mapping is an IFCRGSP continuous mapping but their converses need not hold in general. This can be observed from the following diagram and examples.



Example 3.4. Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$ and $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7) \rangle$. Then $\tau = \{0 \sim, G_1, 1 \sim\}$ and $\sigma = \{0 \sim, G_2, 1 \sim\}$ are IFTs on X and Y respectively.

Then, $IFPC(X) = \{0 \sim, 1 \sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1] / \text{either } \mu_b \geq 0.6 \text{ or } \mu_b < 0.4 \text{ whenever } \mu_a \geq 0.5, \mu_a + \nu_a \leq 1 \text{ and } \mu_b + \nu_b \leq 1\}$. Therefore, $IFSPC(X) = \{0 \sim, 1 \sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1] / \mu_a + \nu_a \leq 1 \text{ and } \mu_b + \nu_b \leq 1\}$. Now $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7) \rangle$ is an IFOS in Y . Therefore $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$. We have $spcl(f^{-1}(G_2)) = f^{-1}(G_2)$. We have $f^{-1}(G_2) \subseteq G_1$. Hence $spcl(f^{-1}(G_2)) \subseteq G_1$, where G_1 is an IFROS in X . This implies $f^{-1}(G_2)$ is an IFRGSPCS in X . Therefore f is an IFCRGSP continuous mapping. We have $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7) \rangle$ is an IFOS in Y . But $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$ is not an IFCS in X , since $cl(f^{-1}(G_2)) = G_1^c \neq f^{-1}(G_2)$. This implies f is not an IFC continuous mapping. Further we have $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$ is not an IFCS in X , since $cl(int(cl(f^{-1}(G_2)))) = cl(int(G_1^c)) = cl(G_1) = G_1^c \not\subseteq f^{-1}(G_2)$. Therefore f is not an IFC_α continuous mapping. Hence f is an IFCRGSP continuous mapping but it is neither an IFC continuous mapping nor an IFC_α continuous mapping.

Example 3.5. Let $X = \{a, b\}$ and $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.4), (0.5, 0.6) \rangle$ and $G_2 = \langle y, (0.5, 0.7), (0.5, 0.3) \rangle$. Then $\tau = \{0 \sim, G_1, 1 \sim\}$ and $\sigma = \{0 \sim, G_2, 1 \sim\}$ are IFTs on X and Y respectively.

Then, $IFPC(X) = \{0 \sim, 1 \sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1] / \mu_b < 0.6 \text{ whenever } \mu_a \geq 0.5, \mu_a < 0.5 \text{ whenever } \mu_b \geq 0.6, \mu_a + \nu_a \leq 1 \text{ and } \mu_b + \nu_b \leq 1\}$. Therefore, $IFSPC(X) = \{0 \sim, 1 \sim, \mu_a \in [0, 1], \mu_b \in [0, 1], \nu_a \in [0, 1], \nu_b \in [0, 1] / \mu_b < 0.6 \text{ whenever } \mu_a \geq 0.5, \mu_a < 0.5 \text{ whenever } \mu_b \geq 0.6, \mu_a + \nu_a \leq 1 \text{ and } \mu_b + \nu_b \leq 1\}$. Now $G_2 = \langle y, (0.5, 0.7), (0.5, 0.3) \rangle$ is an IFOS in Y . Therefore $f^{-1}(G_2) = \langle x, (0.5, 0.7), (0.5, 0.3) \rangle$. We have $spcl(f^{-1}(G_2)) = f^{-1}(G_2)$. We have $f^{-1}(G_2^c) \subseteq 1 \sim$. Hence $spcl(f^{-1}(G_2^c)) \subseteq 1 \sim$, where $1 \sim$ is an IFROS in X . This implies $f^{-1}(G_2^c)$ is an IFRGSPCS in X . Therefore f is an IFCRGSP continuous mapping. We have $G_2 = \langle y, (0.5, 0.7), (0.5, 0.3) \rangle$ is an IFOS in Y . But since $cl(int(f^{-1}(G_2))) = cl(G_1) = 1 \not\subseteq f^{-1}(G_2)$, it is not an IFPCS in X . Hence f is not an IFCP continuous mapping.

Theorem 3.6. Let $f : X \rightarrow Y$ be a mapping. Then f is an IFCRGSP continuous mapping if and only if $f^{-1}(A)$ is an IFRGSPCS in X for every IFCS A in Y .

Proof. **Necessity:** Let A be an IFCS in Y . Then A^c is an IFOS in Y . By hypothesis, $f^{-1}(A^c)$ is an IFRGSPCS in X . Since, $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRGSPCS in X .

Sufficiency: Let A be an IFOS in Y . Then A^c is an IFCS in Y . By hypothesis, $f^{-1}(A^c)$ is an IFRGSPCS in X . Since,

$f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFRGSPCS in X . Thus f is an IFCRGSP continuous mapping. \square

Theorem 3.7. *Let $f : X \rightarrow Y$ be a bijective mapping. Suppose that one of the following properties hold:*

(i) $f^{-1}(cl(B)) \subseteq int(spcl(f^{-1}(B)))$ for each IFS B in Y

(ii) $cl(spint(f^{-1}(B))) \subseteq f^{-1}(int(B))$ for each IFS B in Y

(iii) $f(cl(spint(A))) \subseteq int(f(A))$ for each IFS A in X

(iv) $f(cl(A)) \subseteq int(f(A))$ for each IFSPoS A in X

Then f is an IFCRGSP continuous mapping.

Proof. (i) \Rightarrow (ii) is obvious by taking complement of (i).

(ii) \Rightarrow (iii) Let $A \subseteq X$. Put $B = f(A)$ in Y . This implies $A = f^{-1}(f(A)) = f^{-1}(B)$ in X . Now $cl(spint(A)) = cl(spint(f^{-1}(B))) \subseteq f^{-1}(int(B))$ by (ii). Therefore $f(cl(spint(A))) \subseteq f(f^{-1}(int(B))) = int(B) = int(f(A))$.

(iii) \Rightarrow (iv) Let $A \subseteq X$ be an IFSPoS. Then $spint(A) = A$. By hypothesis, $f(cl(spint(A))) \subseteq int(f(A))$. Therefore $f(cl(A)) = f(cl(spint(A))) \subseteq int(f(A))$.

Suppose (iv) holds. Let A be an IFOS in Y . Then $f^{-1}(A)$ is an IFS in X and $spint(f^{-1}(A))$ is an IFSPoS in X . Hence by hypothesis, $f(cl(spint(f^{-1}(A)))) \subseteq int(f(spint(f^{-1}(A)))) \subseteq int(f(f^{-1}(A))) = int(A) \subseteq A$. Therefore $cl(spint(f^{-1}(A))) = f^{-1}(f(cl(spint(f^{-1}(A)))) \subseteq f^{-1}(A)$. Now $cl(int(f^{-1}(A))) \subseteq cl(spint(f^{-1}(A))) \subseteq f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFPCS in X and hence an IFRGSPCS in X [10]. Thus f is an IFCRGSP continuous mapping. \square

Theorem 3.8. *Let $f : X \rightarrow Y$ be a mapping. Suppose that one of the following properties hold:*

(i) $f(spcl(A)) \subseteq int(f(A))$ for each IFS A in X

(ii) $spcl(f^{-1}(B)) \subseteq f^{-1}(int(B))$ for each IFS B in Y

(iii) $f^{-1}(cl(B)) \subseteq spint(f^{-1}(B))$ for each IFS B in Y

Then f is an IFCRGSP continuous mapping.

Proof. (i) \Rightarrow (ii) Let $B \subseteq Y$. Then $f^{-1}(B)$ is an IFS in X . By hypothesis, $f(spcl(f^{-1}(B))) \subseteq int(f(f^{-1}(B))) \subseteq int(B)$. Now $spcl(f^{-1}(B)) \subseteq f^{-1}(f(spcl(f^{-1}(B)))) \subseteq f^{-1}(int(B))$.

(ii) \Rightarrow (iii) is obvious by taking complement in (ii).

Suppose (iii) holds. Let A be an IFCS in Y . Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X . Now $f^{-1}(A) = f^{-1}(cl(A)) \subseteq spint(f^{-1}(A)) \subseteq f^{-1}(A)$, by hypothesis. This implies $f^{-1}(A)$ is an IFSPoS in X and hence an IFRGSPoS in X [11].

Therefore f is an IFCRGSP continuous mapping. \square

Theorem 3.9. *Let $f : X \rightarrow Y$ be a bijective mapping. Then f is an IFCRGSP continuous mapping if $cl(f(A)) \subseteq f(spint(A))$ for every IFS A in X .*

Proof. Let A be an IFCS in Y . Then $cl(A) = A$ and $f^{-1}(A)$ is an IFS in X . By hypothesis $cl(f(f^{-1}(A))) \subseteq f(spint(f^{-1}(A)))$. Since f is onto, $f(f^{-1}(A)) = A$. Therefore $A = cl(A) = cl(f(f^{-1}(A))) \subseteq f(spint(f^{-1}(A)))$. Now $f^{-1}(A) \subseteq f^{-1}(f(spint(f^{-1}(A)))) = spint(f^{-1}(A)) \subseteq f^{-1}(A)$. Hence $f^{-1}(A)$ is an IFSPoS in X and hence an IFRGSPoS in X [11]. Thus f is an IFCRGSP continuous mapping. \square

Theorem 3.10. *If $f : X \rightarrow Y$ is an IFCRGSP continuous mapping, where X is an IFRSPT_{1/2} space, then the following conditions hold:*

- (i) $spcl(f^{-1}(B)) \subseteq f^{-1}(int(spcl(B)))$ for every IFOS in Y
- (ii) $f^{-1}(cl(spint(B))) \subseteq spint(f^{-1}(B))$ for every IFCS B in Y .

Proof.

- (i) Let $B \subseteq Y$ be an IFOS. By hypothesis $f^{-1}(B)$ is an IFRGSPCS in X . Since X is an IFRSPT_{1/2} space, $f^{-1}(B)$ is an IFSPCS in X . This implies $spcl(f^{-1}(B)) = f^{-1}(B) \subseteq f^{-1}(int(B)) \subseteq f^{-1}(int(spcl(B)))$.
- (ii) can be proved easily by taking the complement of (i).

□

Theorem 3.11. *If $f : X \rightarrow Y$ is an IFCRGSP continuous mapping and $g : Y \rightarrow Z$ is an IF continuous mapping then $g \circ f : X \rightarrow Z$ is an IFCRGSP continuous mapping.*

Proof. Let A be an IFOS in Z . Then $g^{-1}(A)$ is an IFOS in Y , since g is an IF continuous mapping. Since f is an IFCRGSP continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFRGSPCS in X . Therefore $g \circ f$ is an IFCRGSP continuous mapping. □

Theorem 3.12. *If $f : X \rightarrow Y$ is an IFCRGSP continuous mapping and $g : Y \rightarrow Z$ is an IFC continuous mapping then $g \circ f : X \rightarrow Z$ is an IFRGSP continuous mapping.*

Proof. Let A be an IFOS in Z . Then $g^{-1}(A)$ is an IFCS in Y , since g is an IFC continuous mapping. Since f is an IFCRGSP continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFRGSPOS in X . Therefore $g \circ f$ is an IFRGSP continuous mapping. □

Theorem 3.13. *For a mapping $f : X \rightarrow Y$, where X is an IFRSPT_{1/2} space, the following are equivalent:*

- (i) f is an IFCRGSP continuous mapping
- (ii) For every IFCS A in Y and for any IFP $p(\alpha, \beta) \in X$, if $f(p(\alpha, \beta))_q A$ then $p(\alpha, \beta)_q spint(f^{-1}(A))$
- (iii) For every IFCS A in Y and for any $p(\alpha, \beta) \in X$, if $f(p(\alpha, \beta))_q A$ then there exists an IFRGSPOS B such that $p(\alpha, \beta)_q B$ and $f(B) \subset A$.

Proof. (i) \Rightarrow (ii) Let f be an IFCRGSP continuous mapping. Let $A \subset Y$ be an IFCS and let $p(\alpha, \beta) \in X$. Also let $f(p(\alpha, \beta))_q A$ then $p(\alpha, \beta)_q f^{-1}(A)$. By hypothesis $f^{-1}(A)$ is an IFRGSPOS in X . Since X is an IFRSPT_{1/2} space, $f^{-1}(A)$ is an IFSPCS in X . Hence $spint(f^{-1}(A)) = f^{-1}(A)$. This implies $p(\alpha, \beta)_q spint(f^{-1}(A))$.

(ii) \Rightarrow (i) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFCS in X . Let $p(\alpha, \beta) \in X$ and let $f(p(\alpha, \beta))_q A$ then $p(\alpha, \beta)_q f^{-1}(A)$. By hypothesis this implies $p(\alpha, \beta)_q spint(f^{-1}(A))$. That is $f^{-1}(A) \subseteq spint(f^{-1}(A))$. But $spint(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $spint(f^{-1}(A)) = f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFSPCS in X and hence an IFRGSPOS in X [11]. This implies f is an IFCRGSP continuous mapping.

(ii) \Rightarrow (iii) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFCS in X . Let $p(\alpha, \beta) \in X$. Also let $f(p(\alpha, \beta))_q A$ then $p(\alpha, \beta)_q f^{-1}(A)$. By hypothesis this implies $p(\alpha, \beta)_q \text{spint}(f^{-1}(A))$. That is $f^{-1}(A) \subseteq \text{spint}(f^{-1}(A))$. But $\text{spint}(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $\text{spint}(f^{-1}(A)) = f^{-1}(A)$. Thus $f^{-1}(A)$ is an IFSPPOS in X and hence an IFRGSPPOS in X [11]. Let $f^{-1}(A) = B$. Therefore $p(\alpha, \beta)_q B$ and $f(B) \subseteq f(f^{-1}(A)) \subseteq A$.

(iii) \Rightarrow (ii) Let $A \subseteq Y$ be an IFCS then $f^{-1}(A)$ is an IFCS in X . Let $p(\alpha, \beta) \in X$. Also let $f(p(\alpha, \beta))_q A$ then $p(\alpha, \beta)_q f^{-1}(A)$. By hypothesis there exists an IFRGSPPOS B in X such that $p(\alpha, \beta)_q B$ and $f(B) \subseteq A$. Let $B = f^{-1}(A)$. Since X is an IFRSPT_{1/2} space, $f^{-1}(A)$ is an IFSPPOS in X . Therefore $p(\alpha, \beta)_q \text{spint}(f^{-1}(A))$. \square

Theorem 3.14. *A mapping $f : X \rightarrow Y$ is an IFCRGSP continuous mapping if $f^{-1}(\text{spcl}(B)) \subseteq \text{int}(f^{-1}(B))$ for every IFS B in Y .*

Proof. Let $B \subseteq Y$ be an IFCS. Then $\text{cl}(B) = B$. Since every IFCS is an IFSPCS [13], $\text{spcl}(B) = B$. Now by hypothesis, $f^{-1}(B) = f^{-1}(\text{spcl}(B)) \subseteq \text{int}(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFOS in X and hence an IFRGSPPOS in X [11]. Therefore f is an IFCRGSP continuous mapping. \square

Theorem 3.15. *A mapping $f : X \rightarrow Y$ is an IFCRGSP continuous mapping, where X is an IFRSPT_{1/2} space if and only if $f^{-1}(\text{spcl}(B)) \subseteq \text{spint}(f^{-1}(\text{cl}(B)))$ for every IFS B in Y .*

Proof. **Necessity:** Let $B \subseteq Y$ be an IFS. Then $\text{cl}(B)$ is an IFCS in Y . By hypothesis, $f^{-1}(\text{cl}(B))$ is an IFRGSPPOS in X . Since X is an IFRSPT_{1/2} space, $f^{-1}(\text{cl}(B))$ is an IFSPPOS in X . Therefore $f^{-1}(\text{spcl}(B)) \subseteq f^{-1}(\text{cl}(B)) = \text{spint}(f^{-1}(\text{cl}(B)))$.

Sufficiency: Let $B \subseteq Y$ be an IFCS. Then $\text{cl}(B) = B$. By hypothesis, $f^{-1}(\text{spcl}(B)) \subseteq \text{spint}(f^{-1}(\text{cl}(B))) = \text{spint}(f^{-1}(B))$. But $\text{spcl}(B) = B$. Therefore $f^{-1}(B) = f^{-1}(\text{spcl}(B)) \subseteq \text{spint}(f^{-1}(B)) \subseteq f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFSPPOS in X and hence an IFRGSPPOS in X [11]. Hence f is an IFCRGSP continuous mapping. \square

Theorem 3.16. *An IF continuous mapping $f : X \rightarrow Y$ is an IFCRGSP continuous mapping if $\text{IFRGSP}(X) = \text{IFRGSPC}(X)$.*

Proof. Let $A \subseteq Y$ be an IFOS. By hypothesis, $f^{-1}(A)$ is an IFOS in X and hence is an IFRGSPPOS in X [11]. Thus $f^{-1}(A)$ is an IFRGSPCS in X , since $\text{IFRGSP}(X) = \text{IFRGSPC}(X)$. Therefore f is an IFCRGSP continuous mapping. \square

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