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Separation Axioms via q-$\mathcal{I}$-open Sets

Research Article

P.Kalaiselvi[1] and N.Rajesh[2]*

1 Department of Mathematics, Bharathidasan University College, Orathanadu, Thanjavur, TamilNadu, India.

2 Department of Mathematics, Rajah Serfoji Government College, Thanjavur, Tamilnadu, India.

Abstract: In this paper, q-$\mathcal{I}$-open sets are used to define and study some weak separation axioms in ideal topological spaces.

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1. Introduction

The subject of ideals in topological spaces has been introduced and studied by Kuratowski [4] and Vaidyanathasamy [5]. An ideal $\mathcal{I}$ on a topological space (X, τ) is a nonempty collection of subsets of X which satisfies (i) $A \in \mathcal{I}$ and $B \subset A$ implies $B \in \mathcal{I}$ and (ii) $A \in \mathcal{I}$ and $B \in \mathcal{I}$ implies $A \cup B \in \mathcal{I}$. Given a topological space (X, τ) with an ideal $\mathcal{I}$ on X and if $\mathcal{P}(X)$ is the set of all subsets of X, a set operator $(.)^\star$: $\mathcal{P}(X) \to \mathcal{P}(X)$, called the local function [5] of A with respect to τ and $\mathcal{I}$, is defined as follows: For $A \subset X$, $A^\star(\tau, \mathcal{I}) = \{x \in X | U \cap A \notin \mathcal{I}$ for every open neighbourhood U of $x\}$. A Kuratowski closure operator $\mathrm{Cl}^\star(.)$ for a topology $\tau^\star(\tau, \mathcal{I})$ called the $\star$-topology, finer than τ is defined by $\mathrm{Cl}^\star(A) = A \cup A^\star(\tau, \mathcal{I})$ where there is no chance of confusion, $A^\star(\mathcal{I})$ is denoted by $A^\star$. If $\mathcal{I}$ is an ideal on X, then $(X, \tau, \mathcal{I})$ is called an ideal topological space. In this paper, q-$\mathcal{I}$-open sets are used to define some weak separation axioms and to study some of their basic properties.

2. Preliminaries

For a subset A of a topological space (X, τ), we denote the closure of A and the interior of A by $\mathrm{Cl}(A)$ and $\mathrm{Int}(A)$, respectively. A subset S of an ideal topological space $(X, \tau, \mathcal{I})$ is quasi $\mathcal{I}$-open [1] if $S \subset \mathrm{Cl}(\mathrm{Int}(S^*))$. The complement of a q-$\mathcal{I}$-open set is called a q-$\mathcal{I}$-closed set [1]. The intersection of all q-$\mathcal{I}$-closed sets containing S is called the q-$\mathcal{I}$-closure of S and is denoted by $q\mathcal{I}\,\mathrm{Cl}(S)$. The q-$\mathcal{I}$-Interior of S is defined by the union of all q-$\mathcal{I}$-open sets contained in S and is denoted by $q\mathcal{I}\,\mathrm{Int}(S)$. The set of all q-$\mathcal{I}$-open sets of $(X, \tau, \mathcal{I})$ is denoted by $Q\mathcal{I}O(X)$. The set of all q-$\mathcal{I}$-open sets of $(X, \tau, \mathcal{I})$ containing a point $x \in X$ is denoted by $Q\mathcal{I}O(X, x)$.

* *E-mail: nrajesh_topology@yahoo.co.in*

Definition 2.1. *A function $f\colon (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{J})$ is said to be q-$\mathcal{I}$-continuous [1] (resp. q-$\mathcal{I}$-irresolute [1]) if the inverse image of every open (resp. q–$\mathcal{J}$-open) set in Y is q-$\mathcal{I}$-open in X.*

Definition 2.2. *An ideal topological space $(X,\tau,\mathcal{I})$ is said to be q-$\mathcal{I}$-regular if for each closed set F of X and each point $x \in X\backslash F$, there exist disjoint q-$\mathcal{I}$-open sets U and V such that $F \subset U$ and $x \in V$.*

3. q-$\mathcal{I}$-T_0 Spaces

Definition 3.1. *An ideal topological space $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_0 if for any distinct pair of points in X, there is a q-$\mathcal{I}$-open set containing one of the points but not the other.*

Theorem 3.1. *An ideal topological space $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_0 if and only if for each pair of distinct points x, y of X, $q\mathcal{I}\operatorname{Cl}(\{x\}) \neq q\mathcal{I}\operatorname{Cl}(\{y\})$.*

Proof. Let $(X,\tau,\mathcal{I})$ be a q-$\mathcal{I}$-T_0 space and x, y be any two distinct points of X. There exists a q-$\mathcal{I}$-open set G containing x or y, say, x but not y. Then $X\backslash G$ is a q-$\mathcal{I}$-closed set which does not contain x but contains y. Since $\in q\mathcal{I}\operatorname{Cl}(\{y\})$ is the smallest q-$\mathcal{I}$-closed set containing y, $q\mathcal{I}\operatorname{Cl}(\{y\}) \subset X\backslash G$, and so $x \notin q\mathcal{I}\operatorname{Cl}(\{y\})$. Consequently, $q\mathcal{I}\operatorname{Cl}(\{x\}) \neq q\mathcal{I}\operatorname{Cl}(\{y\})$. Conversely, let x, $y \in X$, $x \neq y$ and $q\mathcal{I}\operatorname{Cl}(\{x\}) \neq q\mathcal{I}\operatorname{Cl}(\{y\})$. Then there exists a point $z \in X$ such that z belongs to one of the two sets, say, $q\mathcal{I}\operatorname{Cl}(\{x\})$ but not to $q\mathcal{I}\operatorname{Cl}(\{y\})$. If we suppose that $x \in q\mathcal{I}\operatorname{Cl}(\{y\})$, then $z \in q\mathcal{I}\operatorname{Cl}(\{x\}) \subset q\mathcal{I}\operatorname{Cl}(\{y\})$, which is a contradiction. So $x \in X\backslash q\mathcal{I}\operatorname{Cl}(\{y\})$, where $X\backslash q\mathcal{I}\operatorname{Cl}(\{y\})$ is a q-$\mathcal{I}$-open set and does not contain y. This shows that $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_0. □

Definition 3.2 ([2])**.** *Let A and X_0 be subsets of an ideal topological space $(X,\tau,\mathcal{I})$ such that $A \subset X_0 \subset X$. Then $(X_0,\tau_{|X_0},\mathcal{I}_{|X_0})$ is an ideal topological space with an ideal $\mathcal{I}_{|X_0} = \{I \in \mathcal{I} | I \subset X_0\} = \{I \cap X_0 | I \in \mathcal{I}\}$.*

Lemma 3.1. *[[1]] Let A and X_0 be subsets of an ideal topological space $(X,\tau,\mathcal{I})$. If $A \in Q\mathcal{I}O(X)$ and X_0 is open in $(X,\tau,\mathcal{I})$, then $A \cap X_0 \in Q\mathcal{I}O(X_0)$.*

Theorem 3.2. *Every open subspace of a q-$\mathcal{I}$-T_0 space is q-$\mathcal{I}$-T_0.*

Proof. Let Y be an open subspace of a q-$\mathcal{I}$-T_0 space $(X,\tau,\mathcal{I})$ and x, y be two distinct points of Y. Then there exists a q-$\mathcal{I}$-open set A in X containing x or y, say, x but not y. Now by Lemma 3.1, $A \cap Y$ is a q-$\mathcal{I}$-open set in Y containing x but not y. Hence $(Y,\tau_{|Y},\mathcal{I}_{|Y})$ is q–$\mathcal{I}_{|Y}$-T_0. □

Definition 3.3. *A function $f\colon (X,\tau,\mathcal{I}) \to (Y,\sigma)$ is said to be point q-$\mathcal{I}$-closure one-to-one if and only if x, $y \in X$ such that $q\mathcal{I}\operatorname{Cl}(\{x\}) \neq q\mathcal{I}\operatorname{Cl}(\{y\})$, then $q\mathcal{I}\operatorname{Cl}(\{f(x)\}) \neq q\mathcal{I}\operatorname{Cl}(\{f(y)\})$.*

Theorem 3.3. *If $f\colon (X,\tau,\mathcal{I}) \to (Y,\sigma)$ is point-q-$\mathcal{I}$-closure one-to-one and $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_0, then f is one-to-one.*

Proof. Let x and y be any two distinct points of X. Since $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_0, $q\mathcal{I}\operatorname{Cl}(\{x\}) \neq q\mathcal{I}\operatorname{Cl}(\{y\})$ by Theorem 3.1. But f is point-q-$\mathcal{I}$-closure one-to-one implies that $q\mathcal{I}\operatorname{Cl}(\{f(x)\}) \neq q\mathcal{I}\operatorname{Cl}(\{f(y)\})$. Hence $f(x) \neq f(y)$. Thus, f is one-to-one. □

Theorem 3.4. *Let $f\colon (X,\tau,\mathcal{I}) \to (Y,\sigma)$ be a function from q-$\mathcal{I}$-T_0 space $(X,\tau,\mathcal{I})$ into a topological space (Y,σ). Then f is point-q-$\mathcal{I}$-closure one-to-one if and only if f is one-to-one.*

Proof. The proof follows from Theorem 3.3. □

Theorem 3.5. *Let $f\colon (X,\tau,\mathcal{I}) \to (Y,\sigma,\mathcal{I})$ be an injective q-$\mathcal{I}$-irresolute function. If Y is q-$\mathcal{I}$-T_0, then $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_0.*

Proof. Let x, $y \in X$ with $x \neq y$. Since f is injective and Y is q-$\mathcal{I}$-T_0, there exists a q-$\mathcal{I}$-open set V_x in Y such that $f(x) \in V_x$ and $f(y) \notin V_x$ or there exists a q-$\mathcal{I}$-open set V_y in Y such that $f(y) \in V_y$ and $f(x) \notin V_y$ with $f(x) \neq f(y)$. By q-$\mathcal{I}$-irresoluteness of f, $f^{-1}(V_x)$ is q-$\mathcal{I}$-open set in $(X,\tau,\mathcal{I})$ such that $x \in f^{-1}(V_x)$ and $y \notin f^{-1}(V_x)$ or $f^{-1}(V_y)$ is q-$\mathcal{I}$-open set in $(X,\tau,\mathcal{I})$ such that $y \in f^{-1}(V_y)$ and $x \notin f^{-1}(V_y)$. This shows that $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_0. □

4. q-$\mathcal{I}$-T_1 Spaces

Definition 4.1. *An ideal topological space $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_1 if to each pair of distinct points x, y of X, there exists a pair of q-$\mathcal{I}$-open sets, one containing x but not y and the other containing y but not x.*

Theorem 4.1. *For an ideal topological space $(X,\tau,\mathcal{I})$, each of the following statements are equivalent:*

(1) $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_1;

(2) Each one point set is q-$\mathcal{I}$-closed in X;

(3) Each subset of X is the intersection of all q-$\mathcal{I}$-open sets containing it;

(4) The intersection of all q-$\mathcal{I}$-open sets containing the point $x \in X$ is the set $\{x\}$.

Proof. (1)⇒(2): Let $x \in X$. Then by (1), for any $y \in X$, $y \neq x$, there exists a q-$\mathcal{I}$-open set V_y containing y but not x. Hence $y \in V_y \subset X\backslash\{x\}$. Now varying y over $X\backslash\{x\}$ we get $X\backslash\{x\} = \cup\,\{V_y\colon\ y \in X\backslash\{x\}\}$. So $X\backslash\{x\}$ being a union of q-$\mathcal{I}$-open set. Accordingly $\{x\}$ is q-$\mathcal{I}$-closed.
(2)⇒(1): Let x, $y \in X$ and $x \neq y$. Then by (2), $\{x\}$ and $\{y\}$ are q-$\mathcal{I}$-closed sets. Hence $X\backslash\{x\}$ is a q-$\mathcal{I}$-open set containing y but not x and $X\backslash\{y\}$ is a q-$\mathcal{I}$-open set containing x but not y. Therefore, $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_1.
(2)⇒(3): If $A \subset X$, then for each point $y \notin A$, there exists a set $X\backslash\{y\}$ such that $A \subset X\backslash\{y\}$ and each of these sets $X\backslash\{y\}$ is q-$\mathcal{I}$-open. Hence $A = \cap\,\{\ X\backslash\{y\}\colon\ \mathrm{y} \in X\backslash A\}$ so that the intersection of all q-$\mathcal{I}$-open sets containing A is the set A itself.
(3)⇒(4): Obvious.
(4)⇒(1): Let x, $y \in X$ and $x \neq y$. Hence there exists a q-$\mathcal{I}$-open set U_x such that $x \in U_x$ and $y \notin U_x$. Similarly, there exists a q-$\mathcal{I}$-open set U_y such that $y \in U_y$ and $x \notin U_y$. Hence $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_1. □

Theorem 4.2. *Every open subspace of a q-$\mathcal{I}$-T_1 space is q-$\mathcal{I}$-T_1.*

Proof. Let A be an open subspace of a q-$\mathcal{I}$-T_1 space $(X,\tau,\mathcal{I})$. Let $x \in A$. Since $(X,\tau,\mathcal{I})$ is q-$\mathcal{I}$-T_1, $X\backslash\{x\}$ is q-$\mathcal{I}$-open in $(X,\tau,\mathcal{I})$. Now, A being open, $A \cap (X\backslash\{\mathrm{x}\}) = A\backslash\{x\}$ is q-$\mathcal{I}$-open in A by Lemma 3.1. Consequently, $\{x\}$ is q-$\mathcal{I}$-closed in A. Hence by Theorem 4.1, A is q-$\mathcal{I}$-T_1. □

Theorem 4.3. *Let X be a T_1 space and $f\colon (X,\tau) \to (Y,\sigma,\mathcal{I})$ a q-$\mathcal{I}$-closed surjective function. Then $(Y,\sigma,\mathcal{I})$ is q-$\mathcal{I}$-T_1.*

Proof. Suppose $y \in Y$. Since f is surjective, there exists a point $x \in X$ such that $y = f(x)$. Since X is T_1, $\{x\}$ is closed in X. Again by hypothesis, $f(\{x\}) = \{y\}$ is q-$\mathcal{I}$-closed in Y. Hence by Theorem 4.1, Y is q-$\mathcal{I}$-T_1. □

Definition 4.2. *A point $x \in X$ is said to be a q-$\mathcal{I}$-limit point of A if and only if for each $V \in Q\mathcal{I}O(X)$, $U \cap (A \setminus \{x\}) \neq \varnothing$ and the set of all q-$\mathcal{I}$-limit points of A is called the q-$\mathcal{I}$-derived set of A and is denoted by q-$\mathcal{I}d(A)$.*

Theorem 4.4. *If $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_1 and $x \in$ q-$\mathcal{I}d(A)$ for some $A \subset X$, then every q-$\mathcal{I}$-neighbourhood of x contains infinitely many points of A.*

Proof. Suppose U is a q-$\mathcal{I}$-neighbourhood of x such that $U \cap A$ is finite. Let $U \cap A = \{x_1, x_2, \ldots x_n\} = B$. Clearly B is a q-$\mathcal{I}$-closed set. Hence $V = (U \cap A) \backslash (B \backslash \{x\})$ is a q-$\mathcal{I}$-neighbourhood of point x and $V \cap (A \backslash \{x\}) = \varnothing$, which implies that $x \in q$-$\mathcal{I}$d(A), which contradicts our assumption. Therefore, the given statement in the theorem is true. □

Theorem 4.5. *In a q-$\mathcal{I}$-T_1 space $(X, \tau, \mathcal{I})$, q-$\mathcal{I}d(A)$ is q-$\mathcal{I}$-closed for any subset A of X.*

Proof. As the proof of the theorem is easy, it is omitted. □

Theorem 4.6. *Let $f\colon (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{I})$ be an injective and q-$\mathcal{I}$-irresolute function. If $(Y, \sigma, \mathcal{I})$ is q-$\mathcal{I}$-T_1, then $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_1.*

Proof. Proof is similar to Theorem 3.5 □

Definition 4.3. *An ideal topological space $(X, \tau, \mathcal{I})$ is said to be q-$\mathcal{I}$-R_0 [3] if and only if for every q-$\mathcal{I}$-open sets contains the q-$\mathcal{I}$-closure of each of its singletons.*

Theorem 4.7. *An ideal topological space $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_1 if and only if it is q-$\mathcal{I}$-T_0 and q-$\mathcal{I}$-R_0.*

Proof. Let $(X, \tau, \mathcal{I})$ be a q-$\mathcal{I}$-T_1 space. Then by definition and as every q-$\mathcal{I}$-T_1 space is q-$\mathcal{I}$-R_0, it is clear that $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_0 and q-$\mathcal{I}$-R_0 space. Conversely, suppose that $(X, \tau, \mathcal{I})$ is both q-$\mathcal{I}$-T_0 and q-$\mathcal{I}$-R_0. Now, we show that $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_1 space. Let $x, y \in X$ be any pair of distinct points. Since $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_0, there exists a q-$\mathcal{I}$-open set G such that $x \in G$ and $y \notin G$ or there exists a q-$\mathcal{I}$-open set H such that $y \in H$ and $x \notin H$. Suppose $x \in G$ and $y \notin G$. As $x \in G$ implies the $q\mathcal{I}\,\mathrm{Cl}(\{x\}) \subset G$. As $y \notin G$, $y \notin q\mathcal{I}\,\mathrm{Cl}(\{x\})$. Hence $y \in H = X \backslash q\mathcal{I}\,\mathrm{Cl}(\{x\})$ and it is clear that $x \notin H$. Hence, it follows that there exist q-$\mathcal{I}$-open sets G and H containing x and y respectively such that $y \notin G$ and $x \notin H$. This implies that $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_1. □

5. q-$\mathcal{I}$-T_2 Spaces

Definition 5.1. *An ideal topological space $(X, \tau, \mathcal{I})$ is said to be q-$\mathcal{I}$-T_2 space if for each pair of distinct points x, y of X, there exists a pair of disjoint q-$\mathcal{I}$-open sets, one containing x and the other containing y.*

Theorem 5.1. *For an ideal topological space $(X, \tau, \mathcal{I})$, the following statements are equivalent:*

(1) $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_2;

(2) Let $x \in X$. For each $y \neq x$, there exists $U \in Q\mathcal{I}O(X, x)$ and $y \in q\mathcal{I}\,\mathrm{Cl}(U)$.

(3) For each $x \in X$, $\cap\{q\mathcal{I}\,\mathrm{Cl}(U_x) : U_x$ is a q-$\mathcal{I}$-neighbourhood of $x\} = \{x\}$.

(4) The diagonal $\triangle = \{(x, x) : x \in X\}$ is q-$\mathcal{I}$-closed in $X \times X$.

Proof. (1)⇒(2): Let $x \in X$ and $y \neq x$. Then there exist disjoint q-$\mathcal{I}$-open sets U and V such that $x \in U$ and $y \in V$. Clearly, $X\backslash V$ is q-$\mathcal{I}$-closed, $q\mathcal{I}\,\mathrm{Cl}(U) \subset X\backslash V$ and therefore $y \notin q\mathcal{I}\,\mathrm{Cl}(U)$.

(2)⇒(3): If $y \neq x$, then there exists $U \in Q\mathcal{I}O(X, x)$ and $y \notin q\mathcal{I}\,\mathrm{Cl}(U)$. So $y \notin \cap\{q\mathcal{I}\,\mathrm{Cl}(U) : U \in Q\mathcal{I}O(X, x)\}$.

(3)⇒(4): We prove that $X\backslash\triangle$ is q-$\mathcal{I}$-open. Let $(x, y) \notin \triangle$. Then $y \neq x$ and since $\cap\{q\mathcal{I}\,\mathrm{Cl}(U) : U \in Q\mathcal{I}O(X, x)\} = \{x\}$, there is some $U \in Q\mathcal{I}O(X, x)$ and $y \notin q\mathcal{I}\,\mathrm{Cl}(U)$. Since $U \cap X\backslash q\mathcal{I}\,\mathrm{Cl}(U) = \emptyset$, $U \times (X\backslash q\mathcal{I}\,\mathrm{Cl}(U))$ is q-$\mathcal{I}$-open set such that $(x, y) \in U \times (X\backslash q\mathcal{I}\,\mathrm{Cl}(U)) \subset X\backslash\triangle$.

(4)⇒(5): If $y \neq x$, then $(x, y) \notin \triangle$ and thus there exist $U, V \in Q\mathcal{I}O(X)$ such that $(x, y) \in U \times V$ and $(U \times V) \cap \triangle = \emptyset$. Clearly, for the q-$\mathcal{I}$-open sets U and V we have $x \in U$, $y \in V$ and $U \cap V = \emptyset$. □

Corollary 5.1. *An ideal toological space is $(X, \tau, \mathcal{I})$ q-$\mathcal{I}$-T_2 if and only if each singleton subsets of X is q-$\mathcal{I}$-closed.*

Corollary 5.2. *An ideal toological space $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_2 if and only if two distinct points of X have disjoint q-$\mathcal{I}$-closure.*

Theorem 5.2. *Every q-$\mathcal{I}$-regular T_0-space is q-$\mathcal{I}$-T_2.*

Proof. Let $(X, \tau, \mathcal{I})$ be a q-$\mathcal{I}$-regular T_0 space and x, $y \in X$ such that $x \neq y$. Since X is T_0, there exists an open set V containing one of the points, say, x but not y. Then $y \in X\backslash V$, $X\backslash V$ is closed and $x \notin X\backslash V$. By q-$\mathcal{I}$-regularity of X, there exist q-$\mathcal{I}$-open sets G and H such that $x \in G$, $y \in X\backslash V \subset H$ and $G \cap H = \varnothing$. Hence $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_2. □

Theorem 5.3. *Every open subspace of a q-$\mathcal{I}$-T_2 space is q-$\mathcal{I}$-T_2.*

Proof. Proof is similar to Theorem 4.2 □

Theorem 5.4. *If $f\colon (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is injective, open and q-$\mathcal{I}$-continuous and Y is T_2, then $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_2.*

Proof. Since f is injective, $f(x) \neq f(y)$ for each x, $y \in X$ and $x \neq y$. Now Y being T_2, there exist open sets G, H in Y such that $f(x) \in G$, $f(y) \in H$ and $G \cap H = \varnothing$. Let $U = f^{-1}(G)$ and $V = f^{-1}(H)$. Then by hypothesis, U and V are q-$\mathcal{I}$-open in X. Also $x \in f^{-1}(G) = U$, $y \in f^{-1}(H) = V$ and $U \cap V = f^{-1}(G) \cap f^{-1}(H) = \varnothing$. Hence $(X, \tau, \mathcal{I})$ is q-$\mathcal{I}$-T_2. □

Definition 5.2. *A function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma, \mathcal{J})$ is called strongly q-$\mathcal{I}$-open if the image of every q-$\mathcal{I}$-open subset of $(X, \tau, \mathcal{I})$ is q–$\mathcal{J}$-oen in $(Y, \sigma, \mathcal{J})$.*

Theorem 5.5. *Let $(X, \tau, \mathcal{I})$ be an ideal topological space, R an equivalence relation in X and $p : (X, \tau, \mathcal{I}) \to X|R$ the identification function. If $R \subset (X \times X)$ and p is a strongly q-$\mathcal{I}$-open function, then $X|R$ is q-$\mathcal{I}$-T_2*

Proof. Let $p(x)$ and $p(y)$ be the distinct members of $X|R$. Since x and y are not related, $R \subset (X \times X)$ is q-$\mathcal{I}$-closed in $X \times X$. There are q-$\mathcal{I}$-open sets U and V such that $x \in U$ and $y \in V$ and $U \times V \subset X\backslash R$. Thus $p(U)$ and $p(V)$ are disjoint q-$\mathcal{I}$-open sets in $X|R$ since p is strongly q-$\mathcal{I}$-open. □

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