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On Construction of Even Vertex Odd Mean Graphs

Research Article

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Abstract: A graph G with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function $f : V(G) \to \{0, 2, 4, \ldots, 2q-2, 2q\}$ such that the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. In this paper we discuss the construction of two kinds of even vertex odd mean graphs. Here we prove that $(P_n; S_1)$ for $n \geq 1$, $(P_{2n}; S_m)$ for $m \geq 2, n \geq 1$, $(P_m; C_n)$ for $n \equiv 0 (mod\ 4)$, $m \geq 1$, $[P_m; C_n]$ for $n \equiv 0 (mod\ 4)$, $m \geq 1$ and $[P_m; C_n^{(2)}]$ for $n \equiv 0 (mod\ 4)$, $m \geq 1$ are even vertex odd mean graphs.

MSC: 05C78.

Keywords: Labeling, even vertex odd mean labeling, even vertex odd mean graph.

1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology we follow [4]. Path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. The graph $P_2 \times P_2 \times P_2$ is called a cube and is denoted by Q_3. Let C_n be a cycle with fixed vertex v and $(P_m; C_n)$ the graph obtained from m copies of C_n and the path $P_m : u_1, u_2, \ldots, u_m$ by joining u_i with the vertex v of the i^{th} copy of C_n by means of an edge for $1 \leq i \leq m$.

Let S_m be a star with vertices $v_0, v_1, v_2, \ldots, v_m$ and let $(P_{2n}; S_m)$ be the graph obtained from $2n$ copies of S_m and the path $P_{2n} : u_1, u_2, \ldots, u_{2n}$ by joining u_j with the vertex v_0 of the j^{th} copy of S_m by means of an edge, for $1 \leq j \leq 2n, (P_n; S_1)$ the graph obtained from n copies of S_1 and the path $P_n : u_1, u_2, \ldots, u_n$ by joining u_j with the vertex v_0 of the j^{th} copy of S_1 by means of an edge, for $1 \leq j \leq n$. Suppose $C_n : v_1 v_2 \ldots v_n v_1$ be a cycle of length n. Let $[P_m; C_n]$ be the graph obtained from m copies of C_n with vertices $v_{1_1}, v_{1_2}, \ldots, v_{1_n}, v_{2_1}, \ldots, v_{2_n}, \ldots, v_{m_1}, \ldots, v_{m_n}$ and joining v_{i_j} and $v_{(i+1)_j}$ by means of an edge, for some j and $1 \leq i \leq m-1$.

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Let $C_n^{(2)}$ be a friendship graph. Define $[P_m; C_n^{(2)}]$ to be the graph obtained from m copies of $C_n^{(2)}$ and the path $P_m : u_1u_2 \dots u_m$ by joining u_i with the center vertex of the i^{th} copy of $C_n^{(2)}$ for $1 \leq i \leq m$. The graceful labelings of graphs was first introduced by Rosa in 1961 [1] and R.B. Gnanajothi introduced odd graceful graphs [3]. The concept of mean labeling was first introduced by S. Somasundaram and R. Ponraj [7]. Further some more results on mean graphs are discussed in [6, 8, 9]. A graph G is said to be a mean graph if there exists an injective function f from $V(G)$ to $\{0, 1, 2, \dots, q\}$ such that the induced map f^* from $E(G)$ to $\{1, 2, 3, \dots, q\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection.

In [5], K. Manickam and M. Marudai introduced odd mean labeling of a graph. A graph G is said to be odd mean if there exists an injective function f from $V(G)$ to $\{0, 1, 2, 3, \dots, 2q-1\}$ such that the induced map f^* from $E(G)$ to $\{1, 3, 5, \dots, 2q-1\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection. The concept of even mean labeling was introduced and studied by B. Gayathri and R. Gopi [2]. A function f is called an even mean labeling of a graph G with p vertices and q edges if f is an injection from the vertices of G to the set $\{2, 4, \dots, 2q\}$ such that when each edge uv is assigned the label $\frac{f(u)+f(v)}{2}$, then the resulting edge labels are distinct. A graph which admits an even mean labeling is said to be even mean graph.

Motivated by these, R. Vasuki et al. introduced the concept of even vertex odd mean labeling [10] and discussed the even vertex odd mean behaviour of some standard graphs. A graph G with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function $f : V(G) \to \{0, 2, 4, \dots, 2q-2, 2q\}$ such that the induced map $f^* E(G) \to \{1, 3, 5, \dots, 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. An even vertex odd mean labeling of $K_{2,5}$ is shown in Figure 1.

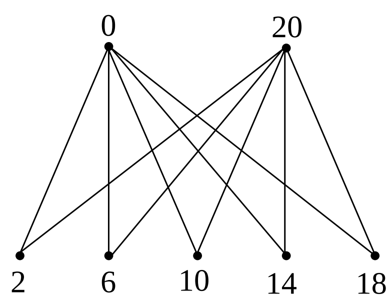

Figure 1.

In this paper, we prove that, the graphs $(P_n; S_1)$ for $n \geq 1$, $(P_{2n}; S_m)$ for $m \geq 2, n \geq 1$, $(P_m; C_n)$ for $n \equiv 0(mod\ 4)$, $m \geq 1$, $[P_m; C_n]$ for $n \equiv 0(mod\ 4)$, $m \geq 1$, and $[P_m; C_n^{(2)}]$ for $n \equiv 0(mod\ 4)$, $m \geq 1$ are even vertex odd mean graphs.

2. Even Vertex Odd Mean Graphs $(P_m; G)$

Let G be a graph with fixed vertex v and let $(P_m; G)$ be the graph obtained from m copies of G and the path P_m : $u_1, u_2, \dots, u_m$ by joining u_i with the vertex v of the i^{th} copy of G by means of an edge for $1 \leq i \leq m$.

Theorem 2.1. *$(P_{2n}; S_m)$ is an even vertex odd mean graph, $m \geq 2, n \geq 1$.*

Proof. Let $u_1, u_2, \dots, u_{2n}$ be the vertices of P_{2n}. Let $v_{0_j}, v_{1_j}, v_{2_j}, \dots, v_{m_j}$ be the vertices of the j^{th} copy of S_m, where v_{0_j} is the center vertex, $1 \leq j \leq 2n$.

Define $f : V(P_{2n}; S_m) \to \{0, 2, 4, \ldots, 2q-2, 2q = 4n(m+2) - 2\}$ as follows:

$$f(u_j) = \begin{cases} (2m+4)(j-1)+2, & 1 \le j \le 2n \text{ and } j \text{ is odd} \\ (2m+4)j - 4, & 1 \le j \le 2n \text{ and } j \text{ is even} \end{cases}$$

$$f(v_{0_j}) = \begin{cases} (2m+4)(j-1), & 1 \le j \le 2n \text{ and } j \text{ is odd} \\ (2m+4)j - 2, & 1 \le j \le 2n \text{ and } j \text{ is even} \end{cases}$$

$$f(v_{i_j}) = \begin{cases} (2m+4)(j-1)+4i+2, & 1 \le i \le m, 1 \le j \le 2n \text{ and } j \text{ is odd} \\ (2m+4)(j-2)+4i, & 1 \le i \le m, 1 \le j \le 2n \text{ and } j \text{ is even.} \end{cases}$$

For the vertex labeling f, the induced edge labeling f^* is obtained as follows:

$$f^*(u_j u_{j+1}) = (2m+4)j - 1, \quad 1 \le j \le 2n-1$$

$$f^*(u_j v_{0_j}) = \begin{cases} (2m+4)(j-1)+1, & 1 \le j \le 2n \text{ and } j \text{ is odd} \\ (2m+4)j - 3, & 1 \le j \le 2n \text{ and } j \text{ is even} \end{cases}$$

$$f^*(v_{0_j} v_{i_j}) = \begin{cases} (2m+4)(j-1)+2i+1, & 1 \le i \le m, 1 \le j \le 2n \\ & \text{and } j \text{ is odd} \\ (2m+4)(j-1)+2i-1, & 1 \le i \le m, 1 \le j \le 2n \\ & \text{and } j \text{ is even.} \end{cases}$$

It can be verified that f is an even vertex odd mean labeling and hence $(P_{2n}; S_m)$ is an even vertex odd mean graph for $n \ge 1$ and $m \ge 2$. For example, an even vertex odd mean labeling of $(P_6; S_4)$ is shown in Figure 2.

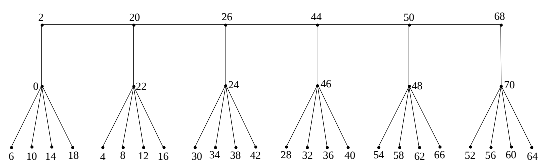

Figure 2. $(P_6; S_4)$

□

Theorem 2.2. *The graph $(P_n; S_1), n \ge 1$ is an even vertex odd mean graph.*

Proof. Let $u_1, u_2, \ldots, u_n$ be the vertices of P_n. Let v_{0_j}, and v_{1_j} be the vertices.
Define $f : V(P_n; S_1) \to \{0, 2, 4, \ldots, 2q-2, 2q = 6n-2\}$ as follows:

$$f(u_j) = \begin{cases} 6j-6, & 1 \le j \le n \text{ and } j \text{ is odd} \\ 6j-2, & 1 \le j \le n \text{ and } j \text{ is even} \end{cases}$$

$$f(v_{0_j}) = 6j - 4, \quad 1 \le j \le n$$

$$f(v_{1_j}) = \begin{cases} 6j-2, & 1 \le j \le n \text{ and } j \text{ is odd} \\ 6j-6, & 1 \le j \le n \text{ and } j \text{ is even.} \end{cases}$$

The induced edge labels are obtained as follows:

$$f^*(u_j u_{j+1}) = 6j-1, \quad 1 \le j \le n-1$$

$$f^*(u_j v_{0_j}) = \begin{cases} 6j-5, & 1 \le j \le n \text{ and } j \text{ is odd} \\ 6j-3, & 1 \le j \le n \text{ and } j \text{ is even} \end{cases}$$

$$f^*(v_{0_j} v_{1_j}) = \begin{cases} 6j-3, & 1 \le j \le n \text{ and } j \text{ is odd} \\ 6j-5, & 1 \le j \le n \text{ and } j \text{ is even.} \end{cases}$$

Thus, f is an even vertex odd mean labeling. Hence $(P_n; S_1)$ is an even vertex odd mean graph for any $n \ge 1$. For example, an even vertex odd mean labeling of $(P_7; S_1)$ is shown in Figure 3.

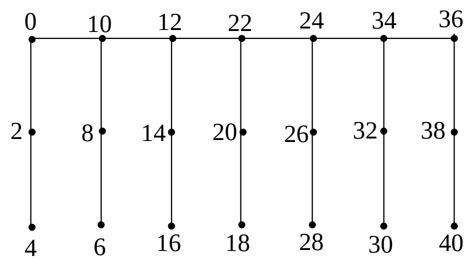

Figure 3. $(P_7; S_1)$

$\square$

Theorem 2.3. *$(P_m; C_n)$ is an even vertex odd mean graph, for $n \equiv 0 (mod\ 4)$ and $m \ge 1$.*

Proof. Let $v_{i_1}, v_{i_2}, \ldots, v_{i_n}$ be the vertices in the i^{th} copy of C_n, $1 \le i \le m$ and $u_1, u_2, \ldots, u_m$ be the vertices of P_m. In $(P_m; C_n)$, u_i is joined with v_{i_1} by an edge, for each i, $1 \le i \le m$.

Define $f : V(P_m; C_n) \to \{0, 2, 4, \ldots, 2q-2, 2q = (2n+4)m - 2\}$ as follows.

$$f(u_i) = \begin{cases} 2(n+2)(i-1), & 1 \le i \le m \text{ and } i \text{ is odd} \\ 2(n+2)i - 2, & 1 \le i \le m \text{ and } i \text{ is even.} \end{cases}$$

For $1 \le i \le m$ and i is odd,

$$f(v_{i_j}) = \begin{cases} 2(n+2)(i-1) + 2j, & 1 \le j \le \frac{n}{2} \\ 2(n+2)(i-1) + 2j + 4, & \frac{n}{2} + 1 \le j \le n \text{ and } j \text{ is odd} \\ 2(n+2)(i-1) + 2j, & \frac{n}{2} + 2 \le j \le n \text{ and } j \text{ is even.} \end{cases}$$

For $1 \le i \le m$ and i is even,

$$f(v_{i_j}) = \begin{cases} 2(n+2)i - 2(j+1), & 1 \le j \le \frac{n}{2} \\ 2(n+2)i - 2(j+3), & \frac{n}{2} + 1 \le j \le n \text{ and } j \text{ is odd} \\ 2(n+2)i - 2(j+1), & \frac{n}{2} + 2 \le j \le n \text{ and } j \text{ is even.} \end{cases}$$

The induced edge labels are obtained as follows:

$$f^*(u_iu_{i+1}) = 2i(n+2) - 1, \quad 1 \le i \le m-1.$$

For $1 \le i \le m$ and i is odd,

$$f^*(v_{i_j}v_{i_{(j+1)}}) = \begin{cases} 2(n+2)(i-1) + 2j + 1, & 1 \le j \le \frac{n}{2} - 1 \\ 2(n+2)(i-1) + 2j + 3, & \frac{n}{2} \le j \le n-1 \end{cases}$$

$$f^*(v_{i_n}v_{i_1}) = 2(n+2)(i-1) + n + 1$$

$$f^*(u_iv_{i_1}) = 2(n+2)(i-1) + 1.$$

For $1 \le i \le m$ and i is even,

$$f^*(v_{i_j}v_{i_{(j+1)}}) = \begin{cases} 2(n+2)i - (2j+3), & 1 \le j \le \frac{n}{2} - 1 \\ 2(n+2)i - (2j+5), & \frac{n}{2} \le j \le n-1 \end{cases}$$

$$f^*(v_{i_n}v_{i_1}) = 2(n+2)i - n - 3$$

$$f^*(u_iv_{i_1}) = 2(n+2)i - 3.$$

Thus, f is an even vertex odd mean labeling and hence $(P_m; C_n)$ is an even vertex odd mean graph for $n \equiv 0(mod\ 4)$ and $m \ge 1$. For example, an even vertex odd mean labeling of $(P_4; C_8)$ is shown in Figure 4.

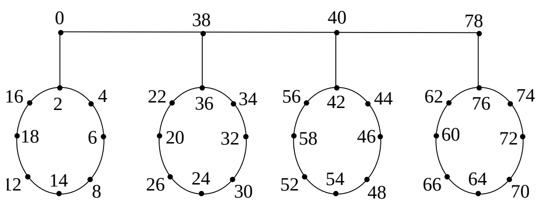

Figure 4. $(P_4; C_8)$

□

3. Even Vertex Odd Mean Graphs $[P_m; G]$

Let G be a graph with fixed vertex v and let $[P_m; G]$ be the graph obtained from m copies of G by joining v_{i_j} and $v_{(i+1)_j}$ by means of an edge for some j and $1 \le i \le m-1$.

Theorem 3.1. *$[P_m; C_n]$ is an even vertex odd mean graph, for $n \equiv 0(mod\ 4)$ and $m \ge 1$.*

Proof. Let $u_1, u_2, \ldots, u_m$ be the vertices of P_m. Let $v_{i_1}, v_{i_2}, \ldots, v_{i_n}$ be the vertices of the i^{th} copy of C_n, $1 \le i \le m$ and joining $v_{i_j}(= u_i)$ and $v_{(i+1)_j}(= u_{i+1})$ by means of an edge for some j.
Define $f : V([P_m; C_n]) \to \{0, 2, 4, \ldots, 2q-2, 2q = (2n+2)m - 2\}$ as follows:

For $1 \leq i \leq m$ and i is odd,

$$f(v_{i_j}) = \begin{cases} 2(n+1)(i-1)+2j-2, & 1 \leq j \leq \frac{n}{2} \\ 2(n+1)(i-1)+2j+2, & \frac{n}{2}+1 \leq j \leq n \text{ and } j \text{ is odd} \\ 2(n+1)(i-1)+2j-2, & \frac{n}{2}+2 \leq j \leq n \text{ and } j \text{ is even.} \end{cases}$$

For $1 \leq i \leq m$ and i is even,

$$f(v_{i_j}) = \begin{cases} 2(n+1)i-2j, & 1 \leq j \leq \frac{n}{2} \\ 2(n+1)i-2(j+2), & \frac{n}{2}+1 \leq j \leq n \text{ and } j \text{ is odd} \\ 2(n+1)i-2j, & \frac{n}{2}+2 \leq j \leq n \text{ and } j \text{ is even.} \end{cases}$$

The induced edge labels are obtained as follows:

$$f^*(v_{i_1}v_{(i+1)_1}) = 2(n+1)i-1, \quad 1 \leq i \leq m-1$$

For $1 \leq i \leq m$ and i is odd,

$$f^*(v_{i_j}v_{(i+1)_j}) = \begin{cases} 2(n+1)(i-1)+2j-1, & 1 \leq j \leq \frac{n}{2} \\ 2(n+1)(i-1)+2j+1, & \frac{n}{2} \leq j \leq n-1 \end{cases}$$

$$f^*(v_{i_n}v_{i_1}) = 2(n+1)i-(n+3).$$

For $1 \leq i \leq m$ and i is even,

$$f^*(v_{i_j}v_{(i+1)_j}) = \begin{cases} 2(n+1)i-2j-1, & 1 \leq j \leq \frac{n}{2}-1 \\ 2(n+1)i-2j-3, & \frac{n}{2} \leq j \leq n-1 \end{cases}$$

$$f^*(v_{i_n}v_{i_1}) = 2(n+1)i-(n+1).$$

Thus, f is an even vertex odd mean labeling and hence, $[P_m; C_n]$ is an even vertex odd mean graph for $n \equiv 0(mod\ 4)$ and $m \geq 1$. For example, an even vertex odd mean labeling of $[P_5; C_8]$ is shown in Figure 5.

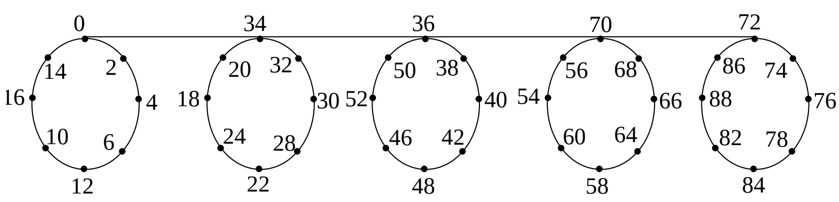

Figure 5. $[P_5; C_8]$

□

Theorem 3.2. *$[P_m; C_n^{(2)}]$ is an even vertex odd mean graph, for $n \equiv 0(mod\ 4)$ and $m \geq 1$.*

Proof. Let $u_1, u_2, \ldots, u_m$ be the vertices of P_m and each vertex $u_i, 1 \leq i \leq m$ is attached with the common vertex in the i^{th} copy of $C_n^{(2)}$. Let v'_{i_j} and v''_{i_j} for $1 \leq j \leq n$ be the vertices in the i^{th} copy of $C_n^{(2)}$, in which $v'_{i_1} = v''_{i_1}$, $1 \leq i \leq m$.

Define $f : V([P_m; C_n^{(2)}]) \rightarrow \{0, 2, 4, \ldots, 2q-2, 2q = (4n+2)m - 2\}$ as follows:

For $1 \leq i \leq m$,

$$f(v'_{i_j}) = \begin{cases} (4n+2)i - 2(n+j), & 1 \leq j \leq 2 \\ (4n+2)(i-1) + 2j - 6, & 3 \leq j \leq \frac{n}{2} + 2 \\ (4n+2)(i-1) + 2j - 2, & \frac{n}{2} + 3 \leq j \leq n \text{ and } j \text{ is odd} \\ (4n+2)(i-1) + 2j - 6, & \frac{n}{2} + 3 \leq j \leq n \text{ and } j \text{ is even} \end{cases}$$

$$f(v''_{i_j}) = \begin{cases} (4n+2)i - 2(n-j+2), & 2 \leq j \leq \frac{n}{2} \\ (4n+2)i - 2n + 2j, & \frac{n}{2} + 1 \leq j \leq n \text{ and } j \text{ is odd} \\ (4n+2)i - 2(n-j+2), & \frac{n}{2} + 2 \leq j \leq n \text{ and } j \text{ is even} \end{cases}$$

The induced edge labels are obtained as follows:

For $1 \leq i \leq m$,

$$f^*(v'_{i_j} v'_{(i+1)_j}) = \begin{cases} (4n+2)(i-1) + 2j - 5, & 3 \leq j \leq \frac{n}{2} + 1 \\ (4n+2)(i-1) + 2j - 3, & \frac{n}{2} + 2 \leq j \leq n \end{cases}$$

$$f^*(v'_{i_1} v'_{i_2}) = (4n+2)i - (2n+3)$$

$$f^*(v'_{i_2} v'_{i_3}) = (4n+2)i - 3(n+1)$$

$$f^*(v''_{i_j} v''_{(i+1)_j}) = \begin{cases} (4n+2)i - 2n + 2j - 5, & 1 \leq j \leq \frac{n}{2} - 1 \\ (4n+2)i - 2n + 2j - 1, & \frac{n}{2} \leq j \leq n \end{cases}$$

$$f^*(v''_{i_n} v''_{i_1}) = (4n+2)i - (n+3).$$

Thus, f is an even vertex odd mean labeling. Hence, $[P_m; C_n^{(2)}]$ is an even vertex odd mean graph for $n \equiv 0 (mod\ 4)$ and $m \geq 1$. For example, an even vertex odd mean labeling of $[P_4; C_8^{(2)}]$ is shown in Figure 6.

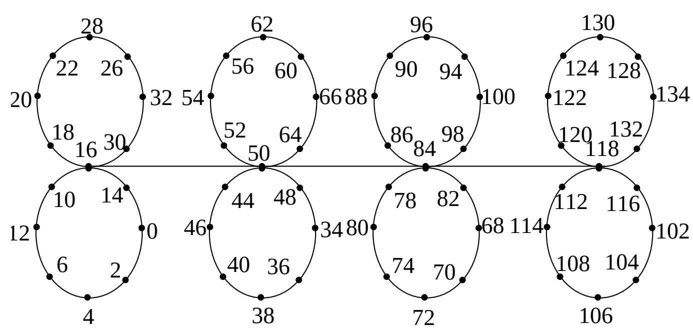

Figure 6. $[P_4; C_8^{(2)}]$

□

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