International Journal of Mathematics And its Applications
Volume 3, Issue 2 (2015), 177–183.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Super Pair Sum Labeling of H-graphs

Research Article

J.Venkateswari[1], R.Vasuki[2]* and P.Sugirtha[3]

1 Department of Mathematics, Dr. Sivanthi Aditanar College of Engineering, Tiruchendur-628 215, Tamil Nadu, India.

2 Department of Mathematics, Dr. Sivanthi Aditanar College of Engineering, Tiruchendur-628 215, Tamil Nadu, India.

3 Department of Mathematics, Dr. Sivanthi Aditanar College of Engineering, Tiruchendur-628 215, Tamil Nadu, India.

Abstract: Let G be a graph with p vertices and q edges.The graph G is said to be a super pair sum labeling if there exists a bijection f from $V(G)\cup E(G)$ to $\left\{0, \pm 1, \pm 2, \dots, \pm\left(\frac{p+q-1}{2}\right)\right\}$ when $p+q$ is odd and from $V(G)\cup E(G)$ to $\left\{\pm 1, \pm 2, \dots, \pm\left(\frac{p+q}{2}\right)\right\}$ when $p+q$ is even such that $f(uv)=f(u)+f(v)$. A graph that admits a super pair sum labeling is called a super pair sum graph. In this paper, we investigate super pair sum labeling of some H-graphs.

MSC: 05C78.

Keywords: labeling, super pair sum labeling, super pair sum graph.

1. Introduction

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V,E)$ be a graph with p vertices and q edges. For notations and terminology we follow [2]. Path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n. $K_{1,m}$ is called a star and it is denoted by S_m. The bistar $B_{m,n}$ is the graph obtained from K_2 by identifying the center vertices of $K_{1,m}$ and $K_{1,n}$ at the end vertices of K_2 respectively. $B_{m,n}$ is often denoted by $B(m)$. The union of two graphs G_1 and G_2 is a graph $G_1\cup G_2$ with $V(G_1\cup G_2)=V(G_1)\cup V(G_2)$ and $E(G_1\cup G_2)=E(G_1)\cup E(G_2)$. The H-graph of a path P_n, denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \dots, v_n$ and $u_1, u_2, \dots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even. The corona of a graph G on p vertices $v_1, v_2, \dots, v_p$ is the graph obtained from G by adding p new vertices $u_1, u_2, \dots, u_p$ and the new edges u_iv_i for $1\le i\le p$. The corona of G is denoted by $G\odot K_1$. The graph $P_n\odot K_1$ is called a comb. The 2-corona of a graph G, denoted by $G\odot S_2$ is a graph obtained from G by identifying the center vertex of the star S_2 at each vertex of G. The graceful labelings of graphs was first introduced by Rosa in 1961[1] and super vertex graceful labeling of some standard graphs was discussed in [5]. The concept of pair sum labeling was introduced and studied by R. Ponraj et al. [3, 4]. Let G be a (p,q) graph. A one-one map $f: V(G)\to\{\pm 1, \pm 2, \dots, \pm p\}$ is said to be a pair sum labeling if the induced edge function

* E-mail: vasukisehar@gmail.com

$f_e : E(G) \to Z - \{0\}$ defined by $f_e(uv) = f(u) + f(v)$ is one-one and $f_e(E(G))$ is either of the form $\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q}{2}}\}$ or $\{\pm k_1, \pm k_2, \ldots, \pm k_{\frac{q-1}{2}}\}$ according as q is even or odd. A graph with a pair sum labeling defined on it is called a pair sum graph.

Motivated by Ponraj and Parthiban, R. Vasuki et al. introduced the concept of super pair sum labeling [6] and discussed the super pair sum behaviour of some standard graphs like path, $K_{1,m}$, bistar $B_{m,n}$ for $m \geq 1, n \geq 1, P[2n; S_m]$ for $n \geq 1, m \geq 1$, comb, $C_{2n}, K_{1,m} \cup K_{1,n}$ and the caterpillar $S(x_1, x_2, \ldots, x_n)$. A graph G with p vertices and q edges is said to have a super pair sum labeling if there exists a bijection f from $V(G) \cup E(G)$ to $\left\{0, \pm 1, \pm 2, \ldots, \pm \left(\frac{p+q-1}{2}\right)\right\}$ when $p+q$ is odd and from $V(G) \cup E(G)$ to $\left\{\pm 1, \pm 2, \ldots, \pm \left(\frac{p+q}{2}\right)\right\}$ when $p+q$ is even such that $f(uv) = f(u) + f(v)$. A graph that admits a super pair sum labeling is called a super pair sum graph.

A super pair sum labeling of $P_7 \odot K_1$ is shown in Figure 1.

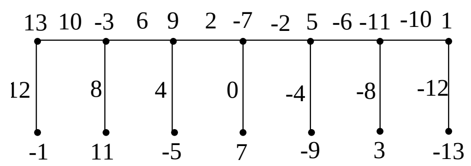

Figure 1.

In this paper, we prove that the graphs H-graph H_n, corona of a H-graph, 2-corona of a H-graph and the disconnected H-graph $2H_n$ for $n \geq 3$ are super pair sum graphs.

2. Super Pair Sum Graphs

Theorem 2.1. *The H-graph G is a super pair sum graph.*

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of the H-graph G. The graph G has $2n$ vertices and $2n-1$ edges.

Define $f : V(G) \cup E(G) \to \{0, \pm 1, \pm 2, \ldots, \pm(2n-1)\}$ as follows:

$$f(u_i) = \begin{cases} -i, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 2n - i + 1, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} n - i + 1, & \text{if } n \text{ is odd, } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ -(n+i), & \text{if } n \text{ is odd, } 1 \leq i \leq n \text{ and } i \text{ is even} \\ -(n+i), & \text{if } n \text{ is even, } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ n - i + 1, & \text{if } n \text{ is even, } 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(u_i u_{i+1}) = 2n - 2i, \quad 1 \leq i \leq n-1$$

$$f(v_i v_{i+1}) = -2i, \quad 1 \leq i \leq n-1$$

$$f\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 0, \quad if\ n\ is\ odd \text{ and}$$

$$f\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 0, \quad if\ n\ is\ even.$$

It can be verified that f is a super pair sum labeling and hence G is a super pair sum graph. For example, a super pair sum labeling of H_7 and H_8 are shown in Figure 2.

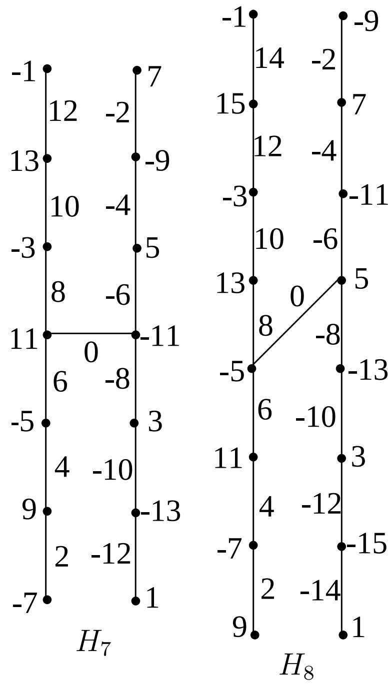

Figure 2.

□

Theorem 2.2. *The graph $H_n \odot K_1$ is a super pair sum graph.*

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices on the path of length $n-1$. Let u_i' and v_i' be the pendant vertices at u_i and v_i respectively, for $1 \leq i \leq n$. The graph $H_n \odot K_1$ has $4n$ vertices and $4n-1$ edges.

Define $f : V(G) \cup E(G) \rightarrow \{0, \pm 1, \pm 2, \ldots, \pm(4n-1)\}$ as follows:

$$f(u_{2i-1}) = 4n - 4i + 3, \quad 1 \leq i \leq \left\lfloor \frac{n+1}{2} \right\rfloor$$

$$f(u_{2i}) = 1 - 4i, \quad 1 \leq i \leq \left\lceil \frac{n-1}{2} \right\rceil$$

$$f(u_{2i-1}') = 3 - 4i, \quad 1 \leq i \leq \left\lfloor \frac{n+1}{2} \right\rfloor$$

$$f(u_{2i}') = 4n - 4i + 1, \quad 1 \leq i \leq \left\lceil \frac{n-1}{2} \right\rceil$$

$$f(v_{2i-1}) = \begin{cases} -2n - 4i + 3, & 1 \leq i \leq \frac{n+1}{2} \text{ and } n \text{ is odd} \\ 2n - 4i + 3, & 1 \leq i \leq \frac{n}{2} \text{ and } n \text{ is even} \end{cases}$$

$$f(v_{2i}) = \begin{cases} 2n - 4i + 1, & 1 \leq i \leq \frac{n-1}{2} \text{ and } n \text{ is odd} \\ -2n - 4i + 1, & 1 \leq i \leq \frac{n}{2} \text{ and } n \text{ is even} \end{cases}$$

$$f(v_{2i-1}') = \begin{cases} 2n - 4i + 3, & 1 \leq i \leq \frac{n+1}{2} \text{ and } n \text{ is odd} \\ -2n - 4i + 3, & 1 \leq i \leq \frac{n}{2} \text{ and } n \text{ is even} \end{cases}$$

$$f(v'_{2i}) = \begin{cases} -2n-4i+1, & 1 \le i \le \frac{n-1}{2} \text{ and } n \text{ is odd} \\ 2n-4i+1, & 1 \le i \le \frac{n}{2} \text{ and } n \text{ is even} \end{cases}$$

$$f(u_i u_{i+1}) = 4n-4i, \quad 1 \le i \le n-1$$

$$f(u_i u'_i) = 4n-4i+2, \quad 1 \le i \le n$$

$$f(v_i v_{i+1}) = -4i, \quad 1 \le i \le n-1$$

$$f(v_i v'_i) = 2-4i, \quad 1 \le i \le n$$

$$f\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 0, \quad if\ n\ is\ odd \text{ and}$$

$$f\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 0, \quad if\ n\ is\ even.$$

Thus, f is a super pair sum labeling and hence $H_n \odot K_1$ is a super pair sum graph. For example, a super pair sum labeling of $H_7 \odot K_1$ and $H_8 \odot K_1$ are shown in Figure 3.

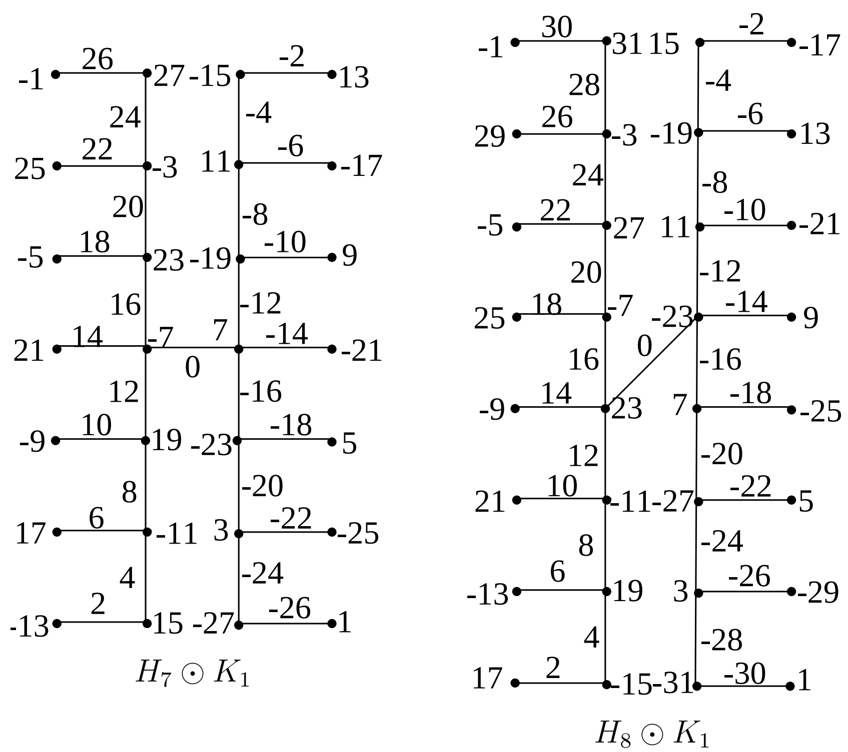

Figure 3.

□

Theorem 2.3. *The graph $H_n \odot S_2$ is a super pair sum graph.*

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices on the path of length $n-1$. Let $u_i u'_i, u_i u''_i$ be the path attached at u_i and $v_i v'_i, v_i v''_i$ be the path attached at $v_i, 1 \le i \le n$. The graph $H_n \odot S_2$ has $6n$ vertices and $6n-1$ edges. Define

$f : V(G) \cup E(G) \to \{0, \pm 1, \pm 2, \ldots, \pm(6n-1)\}$ as follows:

$$f(u_{2i-1}) = 6n - 6i + 5, \quad 1 \le i \le \left\lfloor \frac{n+1}{2} \right\rfloor$$
$$f(u_{2i}) = 1 - 6i, \quad 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil$$
$$f(u'_{2i-1}) = 5 - 6i, \quad 1 \le i \le \left\lfloor \frac{n+1}{2} \right\rfloor$$
$$f(u'_{2i}) = 6n - 6i + 3, \quad 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil$$
$$f(u''_{2i-1}) = 3 - 6i, \quad 1 \le i \le \left\lfloor \frac{n+1}{2} \right\rfloor$$

$$f(u''_{2i}) = 6n - 6i + 1, \quad 1 \le i \le \left\lceil \frac{n-1}{2} \right\rceil$$
$$f(v_{2i-1}) = \begin{cases} -3n - 6i + 4, & 1 \le i \le \frac{n+1}{2} \text{ and } n \text{ is odd} \\ 3n - 6i + 5, & 1 \le i \le \frac{n}{2} \text{ and } n \text{ is even} \end{cases}$$
$$f(v_{2i}) = \begin{cases} 3n - 6i + 2, & 1 \le i \le \frac{n-1}{2} \text{ and } n \text{ is odd} \\ -3n - 6i + 1, & 1 \le i \le \frac{n}{2} \text{ and } n \text{ is even} \end{cases}$$
$$f(v'_{2i-1}) = \begin{cases} 3n - 6i + 6, & 1 \le i \le \frac{n+1}{2} \text{ and } n \text{ is odd} \\ -3n - 6i + 5, & 1 \le i \le \frac{n}{2} \text{ and } n \text{ is even} \end{cases}$$
$$f(v'_{2i}) = \begin{cases} -3n - 6i + 2, & 1 \le i \le \frac{n-1}{2} \text{ and } n \text{ is odd} \\ 3n - 6i + 3, & 1 \le i \le \frac{n}{2} \text{ and } n \text{ is even} \end{cases}$$
$$f(v''_{2i-1}) = \begin{cases} 3n - 6i + 4, & 1 \le i \le \frac{n+1}{2} \text{ and } n \text{ is odd} \\ -3n - 6i + 3, & 1 \le i \le \frac{n}{2} \text{ and } n \text{ is even} \end{cases}$$
$$f(v''_{2i}) = \begin{cases} -3n - 6i, & 1 \le i \le \frac{n-1}{2} \text{ and } n \text{ is odd} \\ 3n - 6i + 1, & 1 \le i \le \frac{n}{2} \text{ and } n \text{ is even} \end{cases}$$
$$f(u_i u_{i+1}) = 6n - 6i, \quad 1 \le i \le n - 1$$
$$f(u_i u'_i) = 6n - 6i + 4, \quad 1 \le i \le n$$
$$f(u_i u''_i) = 6n - 6i + 2, \quad 1 \le i \le n$$
$$f(v_i v_{i+1}) = -6i, \quad 1 \le i \le n - 1$$
$$f(v_i v'_i) = -6i + 4, \quad 1 \le i \le n$$
$$f(v_i v''_i) = -6i + 2, \quad 1 \le i \le n$$
$$f\left(u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}\right) = 0, \quad if\ n\ is\ odd \text{ and}$$
$$f\left(u_{\frac{n}{2}+1} v_{\frac{n}{2}}\right) = 0, \quad if\ n\ is\ even.$$

It can be verified that f is a super pair sum labeling and hence $H_n \odot S_2$ is a super pair sum graph. For example, a super pair sum labeling of $H_5 \odot S_2$ and $H_6 \odot S_2$ are shown in Figure 4.

□

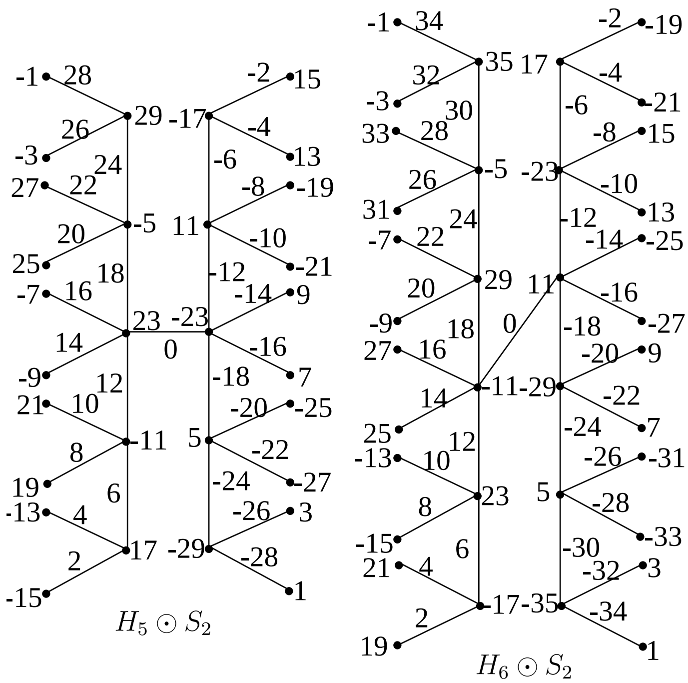

Figure 4.

Theorem 2.4. *The disconnected graph $2H_n$ for $n \geq 3$ is a super pair sum graph.*

Proof. Let $u'_1, u'_2, \ldots, u'_n$ and $v'_1, v'_2, \ldots, v'_n$ be the vertices of the first copy of H_n and $u''_1, u''_2, \ldots, u''_n$ and $v''_1, v''_2, \ldots, v''_n$ be the vertices of the second copy of H_n. The graph $2H_n$ has $4n$ vertices and $4n-2$ edges. Define $f : V(2H_n) \cup E(2H_n) \rightarrow \{\pm 1, \pm 2, \ldots, \pm 4n-1\}$ as follows:

$$f(u'_i) = \begin{cases} i, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ -(4n+1)+i, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(v'_i) = \begin{cases} -(3n+1)+i, & \text{if } n \text{ is odd, } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ n+i, & \text{if } n \text{ is odd, } 1 \leq i \leq n \text{ and } i \text{ is even} \\ n+i, & \text{if } n \text{ is even, } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ -(3n+1)+i, & \text{if } n \text{ is even, } 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(u''_i) = \begin{cases} 2n+i, & 1 \leq i \leq n \text{ and } i \text{ is odd} \\ -(2n+1)+i, & 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(v''_i) = \begin{cases} -(n+1)+i, & \text{if } n \text{ is odd, } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ 3n+i, & \text{if } n \text{ is odd, } 1 \leq i \leq n \text{ and } i \text{ is even} \\ 3n+i, & \text{if } n \text{ is even, } 1 \leq i \leq n \text{ and } i \text{ is odd} \\ -(n+1)+i, & \text{if } n \text{ is even, } 1 \leq i \leq n \text{ and } i \text{ is even} \end{cases}$$

$$f(u_i'u_{i+1}') = -4n + 2i, \quad 1 \le i \le n-1$$

$$f(v_i'v_{i+1}') = -2n + 2i, \quad 1 \le i \le n-1$$

$$f(u_i''u_{i+1}'') = 2i, \quad 1 \le i \le n-1$$

$$f(v_i''v_{i+1}'') = 2n + 2i, \quad 1 \le i \le n-1$$

$$f\left(u'_{\frac{n+1}{2}}v'_{\frac{n+1}{2}}\right) = -2n \quad if\ n\ is\ odd$$

$$f\left(u''_{\frac{n+1}{2}}v''_{\frac{n+1}{2}}\right) = 2n \quad if\ n\ is\ odd$$

$$f\left(u'_{\frac{n}{2}+1}v'_{\frac{n}{2}}\right) = -2n \quad if\ n\ is\ even$$

$$f\left(u''_{\frac{n}{2}+1}v''_{\frac{n}{2}}\right) = 2n \quad if\ n\ is\ even.$$

Thus, f is a super pair sum labeling and hence $2H_n$ is a super pair sum graph for $n \ge 3$. For example, a super pair sum labeling of $2H_7$ and $2H_8$ are shown in Figure 5.

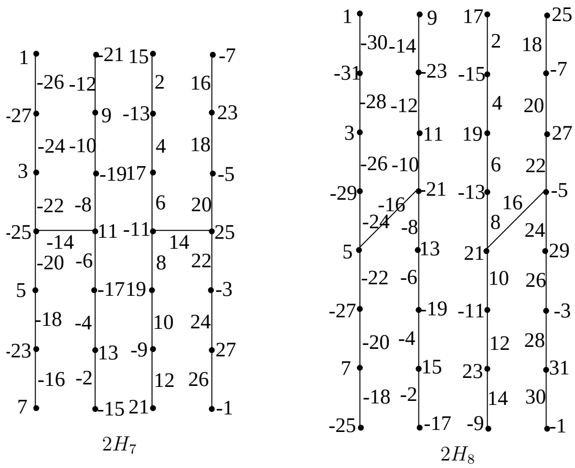

Figure 5.

□

References

[1] J.A.Gallian, *A dynamic survey of graph labeling*, The Electronic J. Combin., (17)(2010).

[2] F.Harary, *Graph Theory*, Narosa publishing House, New Delhi, (1998).

[3] R.Ponraj, J.Vijaya Xavier Parthipan and R.Kala, *Some results on pair sum labelings of graphs*, International Journal of Mathematical Combinatorics, (4)(2010), 53-61.

[4] R.Ponraj, J.Vijaya Xavier Parthipan and R.Kala, *A note on pair sum graphs*, Journal of Scientific Research, (3)(2)(2011), 321-329.

[5] Sin-Min Lee, *Elo leung and Ho Kuen Ng, On super vertex-graceful unicyclic graphs*, Czechoslovak Math. J., (59)(134)(2009), 1-22.

[6] R.Vasuki and S.Arockiaraj, *Super pair sum labeling of graphs*, (communicated).