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Some Properties of r-T_2 Spaces

Research Article

A.A.Ramadan[1]*, M. El Dardery[2] and Y.C.Kim[3]

1 Department of Mathematics, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt.

2 Faculty of Science, Department of Mathematics, Fayoum University, Fayoum, Egypt.

3 Department of Mathematics, Kangnung National University,Korea.

Abstract: We introduce r-T_2 spaces in fuzzy topological spaces in view of Šostak and investigate some properties of r-T_2 spaces. Moreover, we study properties of subspaces and products of r-T_2 spaces.

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1. Introduction and preliminaries

Šostak introduced the fundamental concept of a fuzzy topological structure as an extension of both crisp topology and Chang fuzzy topology [3]. It has been developed in many directions [6, 12, 13, 18]. Many separation axioms were introduced in fuzzy topological spaces in a sense of Chang or Lowen [1, 2, 5, 10, 16, 17, 21]. Srivastava [24] introduced separation axioms in a view of the definition of Hazra.et.al [8]. In this paper, we define r-T_2 space in fuzzy topological spaces in a view of the definition of Sostak. We investigate some properties of r-T_2 spaces. In particular, we study properties of subspaces and products of r-T_2 spaces. Throughout this paper, let X be a nonempty set, $I = [0, 1]$ and $I_0 = (0, 1]$. For $\alpha \in I$, $\bar{\alpha}(x) = \alpha$ for all $x \in X$. A *fuzzy point* x_t for $t \in I_0$ is an element of I^X such that

$$x_t(y) = \begin{cases} t, & \text{if } y = x, \\ 0, & \text{if } y \neq x. \end{cases}$$

The set of all fuzzy points in X is denoted by $Pt(X)$. A fuzzy point $x_t \in \lambda$ iff $t \leq \lambda(x)$. A fuzzy set λ is quasi-coincident with μ, denoted by $\lambda \; q \; \mu$, if there exists $x \in X$ such that $\lambda(x) + \mu(x) > 1$. If λ is not quasi-coincident with μ, we denote $\lambda \; \bar{q} \; \mu$.

* *E-mail: aramadan58@gmail.com*

Definition 1.1 ([6, 8, 12, 20, 22]). *A function $\tau : I^X \to I$ is called a fuzzy topology on X if it satisfies the following conditions:*

(O1) $\tau(\bar{0}) = \tau(\bar{1}) = 1$.

(O2) $\tau(\mu_1 \wedge \mu_2) \geq \tau(\mu_1) \wedge \tau(\mu_2)$ *for each* $\mu_1, \mu_2 \in I^X$.

(O3) $\tau(\bigvee_{i\in\Gamma} \mu_i) \geq \bigwedge_{i\in\Gamma} \tau(\mu_i)$ *for any* $\{\mu_i\}_{i\in\Gamma} \subset I^X$. *The pair (X, τ) is called a fuzzy topological space.*

Let τ_1 and τ_2 be fuzzy topologies on X. We say τ_1 is *finer* than τ_2 (τ_2 is *coarser* than τ_1) if $\tau_2(\mu) \leq \tau_1(\mu)$ for all $\mu \in I^X$.

Theorem 1.2 ([8]). *Let (X, τ) be a fuzzy topological space. For each $r \in I_0, \lambda \in I^X$, we define an operator $C_\tau I^X \times I_0 \to I^X$ as follows:*

$$C_\tau(\lambda, r) = \bigwedge\{\mu \mid \mu \geq \lambda, \tau(\bar{1} - \mu) \geq r\}.$$

Then it satisfies the following properties:

(1) $C_\tau(\bar{0}, r) = \bar{0}, C_\tau(\bar{1}, r) = \bar{1}$, *for all* $r \in I_0$.

(2) $C_\tau(\lambda, r) \geq \lambda$.

(3) $C_\tau(\lambda_1, r) \leq C_\tau(\lambda_2, r)$, *if* $\lambda_1 \leq \lambda_2$.

(4) $C_\tau(\lambda \vee \mu, r) = C_\tau(\lambda, r) \vee C_\tau(\mu, r)$, *for all* $r \in I_0$.

(5) $C_\tau(\lambda, r) \leq C_\tau(\lambda, r')$, *if* $r \leq r'$, *where* $r, r' \in I_0$.

(6) $C_\tau(C_\tau(\lambda, r), r) = C_\tau(\lambda, r)$.

Definition 1.3 ([11]). *Let $\bar{0} \notin \Theta_X$ be a subset of I^X. A function $\beta : \Theta_X \to I$ is called a fuzzy topological base on X if it satisfies the following conditions:*

(B1) $\beta(\bar{1}) = 1$.

(B2) $\beta(\mu_1 \wedge \mu_2) \geq \beta(\mu_1) \wedge \beta(\mu_2)$, *for all* $\mu_1, \mu_2 \in \Theta_X$.

A fuzzy topological base β always *generates* a fuzzy topology τ_β on X in the following sense:

Theorem 1.4 ([11]). *Let β be a fuzzy topological base on X. Define the function $\tau_\beta : I^X \to I$ as follows: for each $\mu \in I^X$,*

$$\tau_\beta(\mu) = \begin{cases} \bigvee\{\bigwedge_{i\in J} \beta(\mu_i)\}, & \text{if } \mu = \bigvee_{i\in J} \mu_i, \ \ \mu_j \in \Theta_X, \\ 1, & \text{if } \mu = \bar{0}, \\ 0, & \text{otherwise} \end{cases}$$

where the first $\bigvee$ is taken over all families $\{\mu_i \in \Theta_X \mid \mu = \bigvee_{i\in J} \mu_i\}$. Then (X, τ_β) is a fuzzy topological space.

Let (X, τ_1) and (Y, τ_2) be fuzzy topological spaces. A function $f : (X, \tau_1) \to (Y, \tau_2)$ is called fuzzy continuous if $\tau_2(\mu) \leq \tau_1(f^{-1}(\mu))$ for all $\mu \in I^Y$.

Theorem 1.5 ([11]). *Let $(X_i, \tau_i)_{i\in\Gamma}$ be fuzzy topological spaces and X a set and $f_i : X \to X_i$ a function , for each $i \in \Gamma$. Let $\Theta_X = \{\bar{0} \neq \mu = \bigwedge_{i\in F} f_i^{-1}(\nu_i) | \tau_i(\nu_i) > 0,\ i \in F\}$ be given, for every finite index set $F \subset \Gamma$. Define a function $\beta : \Theta_X \to I$ on X by*

$$\beta(\mu) = \bigvee\{\bigwedge_{i\in F} \tau_i(\nu_i) \mid \mu = \bigwedge_{i\in F} f_i^{-1}(\nu_i)\}$$

where the first $\bigvee$ is taken over all finite index subset F of Γ. Then:

(1) β is a fuzzy topological base on X.

(2) The fuzzy topology τ_β generated by β is the coarsest fuzzy topology on X for which each $i \in \Gamma$, f_i is fuzzy continuous.

(3) A map $f : (Z, \tau_Z) \to (X, \tau_\beta)$ is fuzzy continuous iff for each $i \in \Gamma$, $f_i \circ f$ is fuzzy continuous.

From Theorem 1.5, we can define the following definitions.

Definition 1.6 ([11]). *Let (X, τ) be a fuzzy topological space and A be a subset of X. The pair $(A, \tau|_A)$ is said to be a subspace of (X, τ) if $\tau|_A$ is the coarsest fuzzy topology on A for which the inclusion map i is fuzzy continuous.*

Definition 1.7 ([11]). *Let X be the product $\prod_{i\in\Gamma} X_i$ of the family $\{(X_i, \tau_i) \mid i \in \Gamma\}$ of fuzzy topological spaces. The the coarsest fuzzy topology $\tau = \bigotimes \tau_i$ on X for which each the projections $\pi_i : X \to X_i$ is fuzzy continuous is called the product fuzzy topology of $\{\tau_i \mid i \in \Gamma\}$, and (X, τ) is called the product fuzzy topology space.*

2. The Properties of r-T_2 Spaces

Definition 2.1. *Let (X, τ) be a fuzzy topological space. A fuzzy set $\mu \in I^X$ is called a r-$\mathcal{Q}_\tau$ open neighborhood of x_t if $x_t\ q\ \mu$ and $\tau(\mu) \geq r$. We denote*

$$\mathcal{Q}_\tau(x_t, r) = \{\mu \in I^X \mid x_t\ q\ \mu,\ \tau(\mu) \geq r\}.$$

Definition 2.2. *A fuzzy topological space (X, τ) is said to be a r-T_2-space if for each $x_t, y_s \in Pt(X)$ such that $x \neq y$, there exist $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_s, r)$ such that $\lambda \wedge \mu = \bar{0}$.*

Theorem 2.3. *A fuzzy topological space (X, τ) is r-T_2 iff for each $x_t, y_s \in Pt(X)$ such that $x \neq y$, and $t, s < 1$, there exist $\lambda, \mu \in I^X$ such that $x_t \in \lambda,\ \tau(\lambda) \geq r$, $y_s \in \mu,\ \tau(\mu) \geq r$ and $\lambda \wedge \mu = \bar{0}$.*

Proof. ($\Rightarrow$) For each $x_t, y_s \in Pt(X)$ such that $x \neq y$, and $t, s < 1$, $x_{1-t}, y_{1-s} \in Pt(X)$. Since (X, τ) is r-T_2, there exist $\lambda \in Q_\tau(x_{1-t}, r)$ and $\mu \in Q_\tau(y_{1-s}, r)$ such that $\lambda \wedge \mu = \bar{0}$. Thus, $\lambda \in Q_\tau(x_{1-t}, r)$ implies $x_t \in \lambda$ and $\tau(\lambda) \geq r$. Thus, $\mu \in Q_\tau(y_{1-s}, r)$ implies $y_s \in \mu$ and $\tau(\mu) \geq r$.

($\Leftarrow$) Let $x_t, y_s \in Pt(X)$ such that $x \neq y$. Let $t, s < 1$. For $x_{1-t}, y_{1-s} \in Pt(X)$, there exist $\lambda, \mu \in I^X$ such that $x_{1-t} \in \lambda, \tau(\lambda) \geq r$, $y_{1-s} \in \mu, \tau(\mu) \geq r$ and $\lambda \wedge \mu = \bar{0}$. It implies $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_s, r)$.

If $t = 1$ or $s = 1$, let $t = 1$ and $s < 1$. There exists $0 < p < 1$ such that $x_p, y_{1-s} \in Pt(X)$. Then there exist $\lambda, \mu \in I^X$ such that $x_p \in \lambda\ \ \tau(\lambda) \geq r,\ \ y_{1-s} \in \mu\ \ \tau(\mu) \geq r$ and $\lambda \wedge \mu = \bar{0}$. Thus, $x_t\ q\ \lambda$ and $y_s\ q\ \mu$. Hence $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_s, r)$ such that $\lambda \wedge \mu = \bar{0}$. Hence (X, τ) is r-T_2. □

Theorem 2.4. *A fuzzy topological space (X,τ) is r-T_2 iff for each $x_t, y_t \in Pt(X)$ such that $x \neq y$, there exist $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_t, r)$ such that $\lambda \wedge \mu = \bar{0}$.*

Proof. ($\Rightarrow$) It is trivial.
($\Leftarrow$) Let $x_t, y_s \in Pt(X)$ such that $x \neq y$ and $t < s$. Since for each $x_t, y_t \in Pt(X)$, there exist $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_t, r)$ such that $\lambda \wedge \mu = \bar{0}$, then $y_t \; q \; \mu$ implies $y_s \; q \; \mu$. Hence (X,τ) is r-T_2. □

Definition 2.5 ([16]). *Let D be a directed set. A function $S : D \to Pt(X)$ is called a fuzzy net.*

Definition 2.6. *Let (X,τ) be a fuzzy topological space, $\mu \in I^X$, $x_t \in Pt(X)$ and $r \in I_0$. A fuzzy point x_t is called a fuzzy r-limit point of S, denoted by $S \xrightarrow{r} x_t$, if for every $\mu \in Q_\tau(x_t, r)$, there exists $n_0 \in D$ such that for each $n \in D$ with $n \geq n_0$, we have $S(n) \; q \; \mu$. We denote*

$$\lim_\tau(S, r) = \bigvee \{x_t \in Pt(X) \mid x_t \text{ is a fuzzy } r\text{-limit point of } S\}.$$

For $\lambda \in I^X$, we denote $supp(\lambda) = \{x \in X \mid \lambda(x) > 0\}$ and $|supp(\lambda)|$ is the cardinal number of $supp(\lambda)$.

Theorem 2.7. *Let (X,τ) be a fuzzy topological space. Then the following statements are equivalent.*

(1) (X,τ) is r-T_2.

(2) For each fuzzy net S, $|supp(lim_\tau(S, r))| \leq 1$.

Proof. (1)$\Rightarrow$ (2) Suppose there exists a fuzzy net $S : D \to Pt(X)$ such that $|supp(lim_\tau(S, r))| \geq 2$. There exist $x \neq y \in supp(lim_\tau(S, r))$ such that $S \xrightarrow{r} x_t, y_s$. Since (X,τ) is r-T_2, there exist $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_s, r)$ such that $\lambda \wedge \mu = \bar{0}$. Since $S \xrightarrow{r} x_t, y_s$, there exist n_1, n_2 such that

$$\forall n \geq n_1, \; S(n) \; q \; \lambda,$$
$$\forall n \geq n_2, \; S(n) \; q \; \mu.$$

Since D is a directed set, there exists $n_3 \geq n_1, n_2$ such that

$$\forall n \geq n_3, \; S(n) \; q \; \lambda, \; S(n) \; q \; \mu.$$

It implies $S(n) \; q \; \lambda \wedge \mu$, for all $n \geq n_3$. Since $\lambda \wedge \mu = \bar{0}$, it is a contradiction.
(2)$\Rightarrow$ (1) Let (X,τ) be not r-T_2. Then there exist $x_t, y_s \in Pt(X)$ with $x \neq y$ such that for all $\lambda \in Q_\tau(x_t, r)$ and for all $\mu \in Q_\tau(y_s, r)$, we have $\lambda \wedge \mu \neq \bar{0}$. Define a relation on $D = \{\lambda \wedge \mu \mid \lambda \in Q_\tau(x_t, r), \mu \in Q_\tau(y_s, r)\}$ by

$$\lambda_1 \wedge \mu_1 \prec \lambda_2 \wedge \mu_2 \text{ iff } \lambda_1 \geq \lambda_2, \mu_1 \geq \mu_2.$$

Then $(D, \prec)$ is a directed set. For each $\lambda \wedge \mu \in D$, since $\lambda \wedge \mu \neq \bar{0}$, there exist $z \in X$ and $p \in I_0$ such that $(\lambda \wedge \mu)(z) > 1 - p > 0$. Then $z_p \; q \; \lambda \wedge \mu$. Thus, we can define a fuzzy net $S : D \to Pt(X)$ by

$$S(\lambda \wedge \mu) = z_p, \text{ that is, } S(\lambda \wedge \mu) = \lambda \wedge \mu.$$

For every $\lambda \in Q_\tau(x_t, r)$, there exists $\lambda = \lambda \wedge \bar{1} \in D$ such that for all $\rho \in D$ with $\lambda \prec \rho$, we have $S(\rho) \; q \; \rho$. Since $\rho \leq \lambda$, we have $S(\rho) \; q \; \lambda$. Hence $x_t \in \lim_\tau(S, r)$. Similarly, $y_s \in lim_\tau(S, r)$. Thus, $|supp(lim_\tau(S, r))| \geq 2$. □

Example 2.8. *Let $X = \{x, y\}$ be a set. We define a fuzzy topology $\tau : I^X \to I$ as follows:*

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}; \\ \frac{1}{2}, & \text{if } \lambda = x_1; \\ \frac{1}{2}, & \text{if } \lambda = y_1; \\ 0, & \text{otherwise.} \end{cases}$$

For each $x_t, y_s \in Pt(X)$ such that $x \neq y$, for $0 < r \leq \frac{1}{2}$, there exist $x_1 \in Q_\tau(x_t, r)$ and $y_1 \in Q_\tau(y_s, r)$ such that $x_1 \wedge y_1 = \bar{0}$. Hence (X, τ) is r-T_2, for $0 < r \leq \frac{1}{2}$. Moreover, we easily show that (X, τ) is not r-T_2, for $r > \frac{1}{2}$. Let N be a natural number set. Define a fuzzy net $S : N \to Pt(X)$ by

$$S(n) = \begin{cases} x_{0.4}, & \text{if } n = 2m, \\ y_{0.3}, & \text{if } n = 2m + 1. \end{cases}$$

We can show $lim_\tau(S, \frac{1}{2}) = Opt$. Lett. 0 from (1) to (2).

(1) x_t for $t \in I_0$ is not a fuzzy $\frac{1}{2}$-limit point of S, for $x_1 \in Q_\tau(x_t, \frac{1}{2})$ and for each $n \in N$, there exists $2n + 1 \in N$ such that $2n + 1 \geq n$ and $S(2n + 1)\ \bar{q}\ x_1$.

(2) y_s for $s \in I_0$ is not a fuzzy $\frac{1}{2}$-limit point of S, for $y_1 \in Q_\tau(y_s, \frac{1}{2})$ and for each $n \in N$, there exists $2n + 1 \in N$ such that $2n + 1 \geq n$ and $S(2n)\ \bar{q}\ y_1$. Thus, $|supp(\lim_\tau(S, \frac{1}{2}))| = 0$.

Example 2.9. *Let $X = \{x, y\}$ be a set. We define a fuzzy topology $\tau : I^X \to I$ as follows:*

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{1}{2}, & \text{if } \lambda = x_{0.4}, \\ 0, & \text{otherwise.} \end{cases}$$

For $x_{0.3}, y_{0.5} \in Pt(X)$, since $Q_\tau(x_{0.3}, r) = Q_\tau(y_{0.5}, r) = \{\bar{1}\}$, for each $r \in I_0$, (X, τ) is not r-T_2. Let N be a natural number set. Define a fuzzy net $S : N \to Pt(X)$ by

$$S(n) = x_{a_n},\ a_n = 0.5 + (-1)^n 0.2.$$

(1) x_t for $t \leq 0.6$ is a fuzzy r-limit point of S, for $\bar{1} \in Q_\tau(x_t, r)$ and for all $n \in N$,we have $S(n)\ q\ \bar{1}$.

(2) x_t for $0.6 < t$ and $0 < r \leq \frac{1}{2}$ is not a fuzzy r-limit point of S, for $x_{0.4} \in Q_\tau(x_t, r)$ and for each $n \in N$, there exists $2n + 1 \in N$ such that $2n + 1 \geq n$ and $S(2n + 1) = x_{0.3}\ \overline{q}\ x_{0.4}$.

(3) y_s for $s \in I_0$ is a fuzzy r-limit point of S, for $\bar{1} \in Q_\tau(y_s, r)$ and for all $n \in N$,we have $S(n)\ q\ \bar{1}$.

From (1) to (3), put $\mu(x) = 0.6$ and $\mu(y) = 1$, we obtain

$$\lim_\tau(S, r) = \begin{cases} \mu, & \text{if } 0 < r \leq \frac{1}{2}, \\ \bar{1}, & \text{if } r > \frac{1}{2}. \end{cases}$$

Thus, $|supp(\lim_\tau(S, r))| = 2$.

Define $\triangle_X \in I^{X\times X}$ as follows:

$$\triangle_X(x,y) = \begin{cases} 1, & \text{if } x = y, \\ 0, & \text{if } x \neq y. \end{cases}$$

Theorem 2.10. *Let (X,τ) be r-T_2. Then $C_{\tau\otimes\tau}(\triangle_X, r) = \triangle_X$ where $\tau\otimes\tau$ is a product fuzzy topology on $X\times X$.*

Proof. We only show that $C_{\tau\otimes\tau}(\triangle_X, r) \leq \triangle_X$ from Theorem 1.2 (2). Suppose $C_{\tau\otimes\tau}(\triangle_X, r) \not\leq \triangle_X$. Then there exist $(x,y) \in X\times X$ and $t \in I_0$ such that

$$C_{\tau\otimes\tau}(\triangle_X, r)(x,y) > t > \triangle_X(x,y). \tag{1}$$

Since $\triangle_X(x,y) < t$. Then $x \neq y$. Since (X,τ) is r-T_2, for $x_t, y_t \in Pt(X)$, there exist $\lambda \in Q_\tau(x_t, r)$ and $\mu \in Q_\tau(y_t, r)$ such that $\lambda \wedge \mu = \bar{0}$. Put $\rho = \pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu)$. Then $\tau\otimes\tau(\rho) \geq \tau(\lambda) \wedge \tau(\mu) \geq r$. Moreover, since $x_t \; q \; \lambda$ and $y_t \; q \; \mu$, we have

$$(\pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu))(x,y) + t = \lambda(x) \wedge \mu(y) + t > 1.$$

Thus, $\rho \in Q_{\tau\otimes\tau}((x,y)_t, r)$. Since, for all $x \in X$,

$$\rho(x,x) = (\pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu))(x,x) = \lambda(x) \wedge \mu(x) = 0,$$

we have $\rho \leq \bar{1} - \triangle_X$. So, $\triangle_X \leq \bar{1} - \rho$ and $\tau\otimes\tau(\rho) \geq r$ implies

$$\triangle_X \leq C_{\tau\otimes\tau}(\triangle_X, r) \leq \bar{1} - \rho.$$

Since $(x,y)_t \; q \; \rho$,

$$C_{\tau\otimes\tau}(\triangle_X, r)(x,y) \leq (\bar{1} - \rho)(x,y) < t.$$

It is a contradiction for the equation (1). □

Theorem 2.11. *Let $\tau\otimes\tau$ be a product fuzzy topology on $X\times X$ of a fuzzy topological space (X,τ). If $C_{\tau\otimes\tau}(\triangle_X, r) = \triangle_X$, then (X,τ) is $(r-\epsilon)$-T_2, for arbitrary $\epsilon > 0$.*

Proof. Let $x_t, y_t \in Pt(X)$ with $x \neq y$. Since $C_{\tau\otimes\tau}(\triangle_X, r) = \triangle_X$, by the definition $C_{\tau\otimes\tau}$ of Theorem 1.2 and Definition 1.1 (O3), we have $\tau\otimes\tau(\bar{1} - \triangle_X) \geq r$. Put $\rho = \bar{1} - \triangle_X$. Then $\rho(x,y) = 1$ implies $(x,y)_t \; q \; \rho$. Let β be a base for $\tau\otimes\tau$. Since $\tau\otimes\tau(\rho) \geq r$, by Theorem 1.4, for $\epsilon > 0$, there exists a family $\{\rho_i \mid \rho = \bigvee_{i\in\Gamma} \rho_i\}$ such that

$$\tau\otimes\tau(\rho) \geq \bigwedge_{i\in\Gamma} \beta(\rho_i) > r - \epsilon.$$

Since $(x,y)_t \; q \; (\rho = \bigvee_{i\in\Gamma} \rho_i)$, there exists $i \in \Gamma$ such that $(x,y)_t \; q \; \rho_i$ and $\beta(\rho_i) > r - \epsilon$. From Theorem 1.5, there exist $\lambda, \mu \in I^X$ such that

$$\rho_i = \pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu), \; \beta(\rho_i) \geq \tau(\lambda) \wedge \tau(\mu) \geq r - \epsilon.$$

Therefore $\tau(\lambda) \geq r - \epsilon$, $\tau(\mu) \geq r - \epsilon$. Furthermore, since $(x,y)_t \; q \; \rho_i$, we have

$$(x,y)_t \; q \; \pi_1^{-1}(\lambda) \Rightarrow (\pi_1^{-1}(\lambda)(x,y) = \lambda(x)) + t > 1,$$
$$(x,y)_t \; q \; \pi_2^{-1}(\mu) \Rightarrow (\pi_2^{-1}(\mu)(x,y) = \mu(y)) + t > 1.$$

Hence $\lambda \in Q_\tau(x_t, r-\epsilon)$ and $\mu \in Q_\tau(y_t, r-\epsilon)$. Moreover, for each $x \in X$,

$$\begin{aligned}(\lambda \wedge \mu)(x) &= \pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu)(x,x)\\ &= \rho_i(x,x) \leq \rho(x,x) = 0,\end{aligned}$$

because $\rho(x,x) = (Opt.\ Lett.\ 1 - \triangle_X)(x,x) = 0$. Thus, by Theorem 2.4, (X,τ) is $(r-\epsilon)$-T_2, for arbitrary $\epsilon > 0$. □

Example 2.12. *Let $X = \{a,b\}$ and $X \times X$ be sets. We define a fuzzy topology $\tau : I^X \to I$ as follows:*

$$\tau(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1},\\ \frac{3}{4}, & \text{if } \lambda = b_1,\\ \frac{1}{2}, & \text{if } \lambda = a_1,\\ 0, & \text{otherwise.}\end{cases}$$

For each $a_t, b_s \in Pt(X)$ and $0 < r \leq \frac{1}{2}$, there exist $a_1 \in Q_\tau(a_t, r)$ and $b_1 \in Q_\tau(b_s, r)$ such that $a_1 \wedge b_1 = \bar{0}$. Hence (X,τ) is r-T_2, for $0 < r \leq \frac{1}{2}$. Let β be a base of product fuzzy topological space $(X \times X, \tau \otimes \tau)$. From Theorem 1.5, since

$$\begin{aligned}(a,b)_1 &= \pi_1^{-1}(a_1) \wedge \pi_2^{-1}(b_1),\\ \beta((a,b)_1) &= \tau(a_1) \wedge \tau(b_1) = \frac{1}{2}.\end{aligned}$$

Since

$$\begin{aligned}(b,a)_1 &= \pi_2^{-1}(a_1) \wedge \pi_1^{-1}(b_1),\\ \beta((b,a)_1) &= \tau(a_1) \wedge \tau(b_1) = \frac{1}{2}.\end{aligned}$$

Thus, $\tau \otimes \tau((a,b)_1 \vee (b,a)_1) = \frac{1}{2}$. Since $\triangle_X = \bar{1} - (a,b)_1 \vee (b,a)_1$, $C_{\tau\otimes\tau}(\triangle_X, \frac{1}{2}) = \triangle_X$.

Theorem 2.13. *Let $f : (X,\tau) \to (Y,\tau_1)$ and $g : (X,\tau) \to (Z,\tau_2)$ be fuzzy continuous. Define a function $h : X \to Y \times Z$ by $h(x) = (f(x), g(x))$. Then $h : (X,\tau) \to (Y \times Z, \tau_1 \otimes \tau_2)$ is fuzzy continuous where $\tau_1 \otimes \tau_2$ is a product fuzzy topology of (Y,τ_1) and (Z,τ_2).*

Proof. Suppose there exists $\rho \in I^{Y\times Z}$ such that $\tau(h^{-1}(\rho)) < \tau_1 \otimes \tau_2(\rho)$. Let β be a base for $\tau_1 \otimes \tau_2$. By the definition of $\tau_1 \otimes \tau_2$, there exists a family $\{\rho_i \mid \rho = \bigvee_{i\in\Gamma} \rho_i\}$ such that

$$\tau(h^{-1}(\rho)) < \bigwedge_{i\in\Gamma} \beta(\rho_i) \leq \tau_1 \otimes \tau_2(\rho).$$

By the definition of β of Theorem 1.5, for each $i \in \Gamma$, there exist $\lambda_i \in I^Y$ and $\mu_i \in I^Z$ with $\rho_i = \pi_1^{-1}(\lambda_i) \wedge \pi_2^{-1}(\mu_i)$ such that

$$\tau(h^{-1}(\rho)) < \bigwedge_{i\in\Gamma} (\tau_1(\lambda_i) \wedge \tau_2(\mu_i)) \leq \bigwedge_{i\in\Gamma} \beta(\rho_i). \tag{2}$$

On the other hand, $(\pi_1 \circ h)^{-1}(\lambda_i)(x) = \lambda_i(\pi_1(h(x))) = \lambda_i(f(x)) = f^{-1}(\lambda_i)(x)$ for all $x \in X$, similarly, $(\pi_2 \circ h)^{-1}(\mu_i) = g^{-1}(\mu_i)$. Thus, we have

$$\begin{aligned} h^{-1}(\rho_i) &= h^{-1}(\pi_1^{-1}(\lambda_i) \wedge \pi_2^{-1}(\mu_i)) \\ &= h^{-1}(\pi_1^{-1}(\lambda_i)) \wedge h^{-1}(\pi_2^{-1}(\mu_i)) \\ &= (\pi_1 \circ h)^{-1}(\lambda_i) \wedge (\pi_2 \circ h)^{-1}(\mu_i) \\ &= f^{-1}(\lambda_i) \wedge g^{-1}(\mu_i). \end{aligned}$$

It follows

$$\begin{aligned} \tau(h^{-1}(\rho)) &= \tau(h^{-1}(\bigvee_{i\in\Gamma} \rho_i)) \\ &\geq \bigwedge_{i\in\Gamma} \tau(h^{-1}(\rho_i)) \\ &= \bigwedge_{i\in\Gamma} (\tau(f^{-1}(\lambda_i) \wedge g^{-1}(\mu_i))) \\ &\geq \bigwedge_{i\in\Gamma} (\tau(f^{-1}(\lambda_i)) \wedge \tau(g^{-1}(\mu_i))) \end{aligned}$$

(Since f and g are fuzzy continuous,)

$$\geq \bigwedge_{i\in\Gamma} (\tau_1(\lambda_i) \wedge \tau_2(\mu_i)).$$

It is a contradiction for the equation (2). □

Theorem 2.14. *Let $f, g : (X, \tau_1) \to (Y, \tau_2)$ be fuzzy continuous. Let (Y, τ) be r-T_2 and $C_{\tau_1}(\chi_A, r) = \chi_X$ where A is a subset of X, χ_A and χ_X are characteristic functions. If $f(a) = g(a)$ for all $a \in A$, then $f(x) = g(x)$ for all $x \in X$.*

Proof. Suppose that there exists $x \in X - A$ with $f(x) \neq g(x)$. Since (Y, τ_2) is r-T_2, for $f(x)_1, g(x)_1 \in Pt(Y)$, there exist $\lambda \in Q_{\tau_2}(f(x)_1, r)$ and $\mu \in Q_{\tau_2}(g(x)_1, r)$ such that $\lambda \wedge \mu = \bar{0}$. Since $f(x)_1 \; q \; \lambda$ and $g(x)_1 \; q \; \mu$, we have

$$(f^{-1}(\lambda) \wedge g^{-1}(\mu))(x) > 0 \tag{3}$$

On the other hand, let $\pi_i : Y \times Y \to Y$ be projection maps for each $i \in \{1, 2\}$ and $h : X \to Y \times Y$ defined by $h(x) = (f(x), g(x))$. We have

$$\begin{aligned} &\lambda \wedge \mu = \bar{0} \\ &\Rightarrow \pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu) \wedge \triangle_Y = \bar{0} \\ &\Rightarrow h^{-1}(\pi_1^{-1}(\lambda) \wedge \pi_2^{-1}(\mu)) \wedge h^{-1}(\triangle_Y) = \bar{0} \\ &\Rightarrow f^{-1}(\lambda) \wedge g^{-1}(\mu) \wedge h^{-1}(\triangle_Y) = \bar{0} \\ &\Rightarrow h^{-1}(\triangle_Y) \leq \bar{1} - (f^{-1}(\lambda) \wedge g^{-1}(\mu)). \end{aligned}$$

Since (Y,τ_2) is r-T_2, by Theorem 2.10, $C_{\tau_2\otimes\tau_2}(\triangle_Y,r)=\triangle_Y$. Since $h:(X,\tau_1)\to(Y\times Y,\tau_2\otimes\tau_2)$ is fuzzy continuous, by Theorem 1.5 (3),

$$C_{\tau_1}(h^{-1}(\triangle_Y),r)\leq h^{-1}(C_{\tau_2\otimes\tau_2}(\triangle_Y,r))=h^{-1}(\triangle_Y).$$

Hence, by Theorem 1.2 (2), $C_{\tau_1}(h^{-1}(\triangle_Y),r)=h^{-1}(\triangle_Y)$. Furthermore, since $C_{\tau_1}(\chi_A,r)=\chi_X$ and $\chi_A\leq h^{-1}(\triangle_Y)$, we have

$$\begin{aligned}\chi_X=C_{\tau_1}(\chi_A,r)&\leq C_{\tau_1}(h^{-1}(\triangle_Y),r)=h^{-1}(\triangle_Y)\\&\leq\bar{1}-(f^{-1}(\lambda)\wedge g^{-1}(\mu)).\end{aligned}$$

Thus, $\chi_X(x)=1$ but $\bar{1}(x)-(f^{-1}(\lambda)\wedge g^{-1}(\mu))(x)<1$. It is a contradiction for the equation (3). □

Theorem 2.15. *If $f:(X,\tau_1)\to(Y,\tau_2)$ is fuzzy continuous and (Y,τ_2) is r-T_2, then $C_{\tau_1\otimes\tau_2}(\chi_{G(f)},r)=\chi_{G(f)}$ where $G(f)=\{(x,y)\in X\times Y|\ y=f(x)\}$.*

Proof. Let $id_Y:(Y,\tau_2)\to(Y,\tau_2)$ be an identity map. Since $f\circ\pi_1:(X\times Y,\tau_1\otimes\tau_2)\to(Y,\tau_2)$ and $id_Y\circ\pi_2:(X\times Y,\tau_1\otimes\tau_2)\to(Y,\tau_2)$ are fuzzy continuous, by Theorem 2.13, $f\times id_Y:(X\times Y,\tau_1\otimes\tau_2)\to(Y\times Y,\tau_2\otimes\tau_2)$ is fuzzy continuous. From Theorem 1.5 (3), it implies

$$C_{\tau_1\otimes\tau_2}((f\times id_Y)^{-1}(\triangle_Y),r)\leq(f\times id_Y)^{-1}(C_{\tau_2\otimes\tau_2}(\triangle_Y,r)).$$

Since (Y,τ_2) is r-T_2, we have $C_{\tau_2\otimes\tau_2}(\triangle_Y,r)=\triangle_Y$. By Theorem 1.3 (2),

$$C_{\tau_1\otimes\tau_2}((f\times id_Y)^{-1}(\triangle_Y),r)=(f\times id_Y)^{-1}(\triangle_Y).$$

Since $(f\times id_Y)^{-1}(\triangle_Y)=\chi_{G(f)}$, we have $C_{\tau_1\otimes\tau_2}(\chi_{G(f)},r)=\chi_{G(f)}$ □

Theorem 2.16. *Every subspace of r-T_2 spaces is a r-T_2 space.*

Proof. Let $(A,\tau|_A)$ be a subspace of a r-T_2 space (X,τ). Let $a_t,b_s\in Pt(A)$ such that $a\neq b$. Then $a_t,b_s\in Pt(X)$ such that $a\neq b$. Since (X,τ) is r-T_2, there exists $\lambda\in Q_\tau(a_t,r)\mu\in Q_\tau(b_s,r)$ such that $\lambda\wedge\mu=\bar{0}$. Since $\tau_A(i^{-1}(\lambda))\geq\tau(\lambda)\geq r$ and $a_t\ q\ i^{-1}(\lambda)$, we have $i^{-1}(\lambda)\in Q_{\tau|_A}(a_t,r)$. Similarly, $i^{-1}(\mu)\in Q_{\tau|_A}(b_s,r)$. Moreover, $i^{-1}(\lambda)\wedge i^{-1}(\mu)=\bar{0}$. Hence $(A,\tau|_A)$ is r-T_2. □

Theorem 2.17. *Let $\{(X_i,\tau_i)\mid i\in\Gamma\}$ be a family of r-T_2 spaces. Let (X,τ) be the product fuzzy topological space of $\{(X_i,\tau_i)\mid i\in\Gamma\}$. Then (X,τ) is r-T_2.*

Proof. Let $x_t,y_s\in Pt(X)$ such that $x\neq y$. Then there exists $i\in\Gamma$ such that $(\pi_i(x))_t,(\pi_i(y))_s\in Pt(X_i)$ with $\pi_i(x)\neq\pi_i(y)$. Since (X_i,τ_i) is r-T_2, there exist $\lambda\in Q_{\tau_i}((\pi_i(x))_t,r)$ and $\mu\in Q_{\tau_i}((\pi_i(y))_s,r)$ with $\lambda\wedge\mu=\bar{0}$. Since $\pi_i(x_t)=(\pi_i(x))_t\ q\ \lambda$ iff $x_t\ q\ \pi_i^{-1}(\lambda)$, we have

$$\pi_i^{-1}(\lambda)\in Q_\tau(x_t,r).$$

Similarly, $\pi_i^{-1}(\mu)\in Q_\tau(y_s,r)$. Moreover, $\pi_i^{-1}(\lambda)\wedge\pi_i^{-1}(\mu)=\bar{0}$. Therefore, (X,τ) is a $r-T_2$ space. □

Theorem 2.18. *Let $\{(X_i, \tau_i) \mid i \in \Gamma\}$ be a family of fuzzy topological spaces. Let (X, τ) be their product fuzzy topological space. If (X, τ) is a r-T_2 space, then (X_j, τ_j) is a $(r-\epsilon)$-T_2 space for each $\epsilon > 0$ and for each $j \in \Gamma$.*

Proof. Let $(x^j)_t, (y^j)_s \in Pt(X_j)$ such that $x^j \neq y^j$. Then there exist $x^i \in X_i$ for all $i \in \Gamma - \{j\}$ such that $x \neq y \in X$ with

$$\pi_i(x) = \begin{cases} x^i, & \text{if } i \in \Gamma - \{j\}, \\ x^j, & \text{if } i = j, \end{cases}$$

$$\pi_i(y) = \begin{cases} x^i, & \text{if } i \in \Gamma - \{j\}, \\ y^j, & \text{if } i = j. \end{cases}$$

Since (X, τ) is r-T_2 space, there exist

$$\rho \in Q_\tau(x_t, r),\ \omega \in Q_\tau(y_s, r),\ \rho \wedge \omega = \bar{0}$$

Let β be a base for τ. Since $\tau(\rho) \geq r$ and $\tau(\omega) \geq r$, for $\epsilon > 0$, there exists two families $\{\rho_k \mid \rho = \bigvee_{k\in K} \rho_k\}$ and $\{\omega_m \mid \omega = \bigvee_{m\in M} \omega_m\}$ such that

$$\tau(\rho) \geq \bigwedge_{k\in K} \beta(\rho_k) > r - \epsilon,$$
$$\tau(\omega) \geq \bigwedge_{m\in M} \beta(\omega_m) > r - \epsilon.$$

Since $x_t\ q\ (\rho = \bigvee_{k\in K} \rho_k)$ and $y_s\ q\ (\omega = \bigvee_{m\in M} \omega_m)$, there exist $k \in K$ and $m \in M$ such that

$$x_t\ q\ \rho_k,\ \beta(\rho_k) > r - \epsilon,$$
$$y_s\ q\ \omega_m,\ \beta(\omega_m) > r - \epsilon,$$

$$\rho_k \wedge \omega_m = \bar{0}. \tag{4}$$

Then there exist two family: $\{\lambda_i \mid \rho_k = \bigwedge_{i\in F_1} \pi_i^{-1}(\lambda_i)\}$ and $\{\mu_l \mid \omega_m = \bigwedge_{l\in F_2} \pi_l^{-1}(\mu_l)\}$, where F_1 and F_2 are finite subsets of Γ such that

$$\beta(\rho_k) \geq \bigwedge_{i\in F_1} \tau_i(\lambda_i) > r - \epsilon,$$
$$\beta(\omega_m) \geq \bigwedge_{l\in F_2} \tau_l(\mu_l) > r - \epsilon.$$

Without loss of generality, we may assume $j \in F = F_1 = F_2$ because, if necessary, we can take $F = F_1 \cup F_2 \cup \{j\}$ such that

$$\lambda_i = \bar{1},\ \forall\, i \in F_2 \cup \{j\} \text{ and } \mu_l = \bar{1},\ \forall\, l \in F_1 \cup \{j\}.$$

Hence we have

$$\beta(\rho_k) \geq \bigwedge_{i\in F} \tau_i(\lambda_i) > r - \epsilon, \beta(\omega_m) \qquad \geq \bigwedge_{i\in F} \tau_i(\mu_i) > r - \epsilon.$$

Since $x_t \; q \; \rho_k$ and $y_s \; q \; \omega_m$, for each $j \in F$,

$$x_t \; q \; \pi_j^{-1}(\lambda_j), \; y_s \; q \; \pi_j^{-1}(\mu_j).$$

Hence

$$\lambda_j \in Q_{\tau_j}(x_t^j, r-\epsilon), \; \mu_j \in Q_{\tau_j}(y_s^j, r-\epsilon).$$

We only show that $\lambda_j \wedge \mu_j = \bar{0}$. Suppose there exists $z^j \in X^j$ such that

$$(\lambda_j \wedge \mu_j)(z^j) > 0. \tag{5}$$

Then there exist $x^i \in X_i$ for all $i \in \Gamma - \{j\}$ and $z \in X$ with

$$\pi_i(z) = \begin{cases} x^i, & \text{if } i \in \Gamma - \{j\}, \\ z^j, & \text{if } i = j. \end{cases}$$

Since $x_t \; q \; \rho_k$ and $y_s \; q \; \omega_m$, we have

$$t > (\bigvee_{i \in F-\{j\}} (\bar{1} - \lambda_i)(\pi_i(x))) \vee (\bar{1} - \lambda_j)(x^j)$$
$$s > (\bigvee_{i \in F-\{j\}} (\bar{1} - \mu_i)(\pi_i(x)) \vee (\bar{1} - \mu_j)(y^j).$$

It implies

$$t > \bigvee_{i \in F-\{j\}} (\bar{1} - \lambda_i)(\pi_i(x)), \tag{6}$$
$$s > \bigvee_{i \in F-\{j\}} (\bar{1} - \mu_i)(\pi_i(x)). \tag{7}$$

Moreover, from (5),

$$((\bar{1} - \lambda_j) \vee (\bar{1} - \mu_j))(z^j) < 1. \tag{8}$$

Hence, by (6), (7) and (8), we have

$$(\bar{1} - \rho_k)(z) = (\bigvee_{i \in F-\{j\}} (\bar{1} - \lambda_i)(x^i)) \vee (\bar{1} - \lambda_j)(z^j) < 1$$
$$(\bar{1} - \omega_m)(z) = (\bigvee_{i \in F-\{j\}} (\bar{1} - \mu_i)(x^i)) \vee (\bar{1} - \mu_j)(z^j) < 1.$$

Therefore $(\rho_k \wedge \omega_m)(z) > 0$. It is a contradiction for the equation (4). Hence (X_j, τ_j) is $(r-\epsilon)$-T_2. □

Example 2.19. *Let $X = \{a\}$, $Y = \{b, c\}$ and $X \times Y = \{(a, b), (a, c)\}$ be sets. We define fuzzy topologies τ_1, τ_2 as follows:*

$$\tau_1(\lambda) = \begin{cases} 1, & \textit{if } \lambda = \bar{0} \textit{ or } \bar{1}, \\ 0, & \textit{otherwise,} \end{cases}$$

$$\tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{3}{4}, & \text{if } \lambda = c_1, \\ \frac{1}{2}, & \text{if } \lambda = b_1, \\ 0, & \text{otherwise.} \end{cases}$$

Then (X, τ_1) *is* $r - T_2$*-space for all* $r \in I_0$ *and* (Y, τ_2) *is* r*-*T_2*-space for all* $0 < r \leq \frac{1}{2}$*. We obtain the product fuzzy topology* $\tau_1 \times \tau_2 : I^{X \times Y} \to I$ *as follows:*

$$\tau_1 \otimes \tau_2(\lambda) = \begin{cases} 1, & \text{if } \lambda = \bar{0} \text{ or } \bar{1}, \\ \frac{3}{4}, & \text{if } \lambda = (a, c)_1, \\ \frac{1}{2}, & \text{if } \lambda = (a, b)_1, \\ 0, & \text{otherwise.} \end{cases}$$

Hence $(X \times Y, \tau_1 \otimes \tau_2)$ *is* r*-*T_2 *for* $0 < r \leq \frac{1}{2}$.

References

[1] D.M. Ali, *A note on some* FT_2 *concepts*, Fuzzy sets and Systems, 42(1991), 381-386.

[2] S.S.Benchalli and G.P.Siddapur, *On level spaces of fuzzy topological spaces*, Bull. of Math. Anal. Appl., 1(2)(2009), 57-65.

[3] C.L.Chang, *Fuzzy topological spaces*, J. Math. Anal. Appl.,24(1968), 182-190.

[4] S.L.Chen and J.S.Cheng, *On convergence of nets of L-fuzzy sets*, J. Fuzzy Math., 2(1994), 517-524.

[5] J.S. Cheng, *On separation axioms in L- fuzzy toplogical spaces*, Proceedings of Int.Conf. on Machine Learning and Cyberneties,(2005), 2534-2539.

[6] M.Demirci, *Neighborhood structures in smooth topological spaces*, Fuzzy sets and Systems, 92(1997), 123-128.

[7] S.Ganguly and S.Saha, *On separation axioms and* T_i*-fuzzy continuity*, Fuzzy sets and Systems, 16(1985), 265-275.

[8] R.N.Hazra, S.K.Samanta and K.C.Chattopadhyay, *Gradation of openness: Fuzzy topology*, Fuzzy sets and Systems, 49(2)(1992), 237-242.

[9] U.Höhle, S.E.Rodabaugh, *Mathematics of Fuzzy Sets, Logic, Topology and Measure Theory*, The Handbooks of Fuzzy Sets Series, Volume 3, Kluwer Academic Publishers, Dordrecht, (1999).

[10] B.Hutton and I.Reilly, *Separation axioms in fuzzy topological spaces*, Fuzzy sets and Systems, 3(1980), 93-104.

[11] Y.C.Kim, *Initial smooth fuzzy topological spaces*, J. of Fuzzy Logic and Intelligent Systems (in Korea), 8(3)(1998), 88-94.

[12] Y.C.Kim, A.A.Ramadan and M.Usama, *(L,M)- topologies and (2,M)-fuzzifying topologies*, International of Fuzzy Reveiw, 2(2)(2007), 129-144.

[13] Y.C.Kim, A.A.Ramadan and M.Usama, *(L,M)- Neighborhood spaces*, Kangweon- Kyungli Math. Jour., 15(2)(2007), 121-133.

[14] S.E.Rodabaugh, E.P.Klement, *Toplogical And Algebraic Structures In Fuzzy Sets*, The Handbook of Recent Developments in the Mathematics of Fuzzy Sets, Kluwer Academic Publishers, (Boston/Dordrecht/London), (2003)

[15] R.Lowen, *Fuzzy topological spaces and fuzzy compactness*, J. Math. Anal. Appl., 56(1976), 621-633.

[16] Liu Ying-Ming and Luo Mao-Kang, *Fuzzy topology*, World Scientific Publishing Co., Singapore, (1997).

[17] V.L.G. Nayagam and G.Sivaraman, *Semi- Hausdorff fuzzy filters*, Int. J. of Math. and Math. Sci., (2007), 1-10.

[18] W.Peeters, *Subspaces of smooth fuzzy topologies and initial smooth fuzzy structures*, Fuzzy sets and Systems, 104(1999), 423-433.

[19] Pu Pao-Ming and Liu Ying-Ming, *Fuzzy topology I; Neighborhood structure of a fuzzy point and Moore-Smith convergence*, J.Math.Anal. Appl., 76(1980), 571-599.

[20] A.A.Ramadan, *Smooth topological spaces*,Fuzzy sets and Systems, 48(3)(1992), 371-375.

[21] P.V.Ramakrishnan and V.L.G. Nayagam, *Nearly fuzzy Hausdorff spaces*, Indian J.pure and Applied Math., 31(6)(2000), 695-711.

[22] A.P. Šostak, *On a fuzzy topological structure*, Rend. Circ. Matem. Palermo Ser.II, 11(1985), 89-103.

[23] A.K.Singh, S.N.Lai and R.Srivastava, *Separation axioms in intuitionistic fuzzy toplogical spaces*, Advances in Fuzzy Systems, (2012), 1-7.

[24] R.Srivastava, *On separation axioms in a newly defined fuzzy topology*, Fuzzy sets and Systems, 62(1994), 341-346.