International Journal of Mathematics And its Applications
Volume 3, Issue 3–A (2015), 75–83.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Weakly (1,2)*-g^*-closed Sets

Research Article

K.M.Dharmalingam[1], G.Thanavalli[2] and O.Ravi[3]*

1 Department of Mathematics, The Madura College, Madurai, Tamil Nadu, India.
2 Department of Mathematics, SBM College of Engineering & Technology, Thamaraipadi, Dindigul, Tamil Nadu, India.
3 Department of Mathematics, P.M.Thevar College, Usilampatti, Tamil Nadu, India.

Abstract: The aim of this paper is to introduce a new class of (1,2)*-generalized closed sets called weakly (1,2)*-g^*-closed sets.

MSC: 54E55.

Keywords: (1,2)*-g^*-closed set, (1,2)*-wg^*-closed set, (1,2)*-g^*-continuous map, (1,2)*-g^*-irresolute map, weakly (1,2)*-g^*-open map, weakly (1,2)*-g^*-continuous map.

1. Introduction

Thamilisai [21] studied and investigated the properties of the notion of (1,2)*-g^*-closed sets. In this paper, we introduce a new class of (1,2)*-generalized closed sets called weakly (1,2)*-g^*-closed sets which contains the above mentioned class. Also, we investigate the relationships among the related (1,2)*-generalized closed sets.

2. Preliminaries

Throughout this paper, X, Y and Z denote bitopological spaces (X, τ_1, τ_2), (Y, σ_1, σ_2) and (Z, η_1, η_2) respectively.

Definition 2.1. *Let A be a subset of a bitopological space X. Then A is called $\tau_{1,2}$-open [16] if $A = P \cup Q$, for some $P \in \tau_1$ and $Q \in \tau_2$. The complement of $\tau_{1,2}$-open set is called $\tau_{1,2}$-closed. The family of all $\tau_{1,2}$-open (resp. $\tau_{1,2}$-closed) sets of X is denoted by (1,2)*-O(X) (resp. (1,2)*-C(X)).*

Definition 2.2 ([16])**.** *Let A be a subset of a bitopological space X. Then*

1. *the $\tau_{1,2}$-interior of A, denoted by $\tau_{1,2}$-int(A), is defined by $\cup$ { $U : U \subseteq A$ and U is $\tau_{1,2}$-open};*
2. *the $\tau_{1,2}$-closure of A, denoted by $\tau_{1,2}$-cl(A), is defined by $\cap$ { $U : A \subseteq U$ and U is $\tau_{1,2}$-closed}.*

Remark 2.3 ([16])**.** *Notice that $\tau_{1,2}$-open subsets of X need not necessarily form a topology.*

Definition 2.4. *Let A be a subset of a bitopological space X. Then A is called*

* *E-mail: siingam@yahoo.com*

1. *(1,2)*-α-open set [16] if $A \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))). The complement of (1,2)*-α-open set is (1,2)*-α-closed. The (1,2)*-α-closure [18] of a subset A of X, denoted by (1,2)*-αcl(A), is defined to be the intersection of all (1,2)*-α-closed sets of X containing A. It is known that (1,2)*-αcl(A) is (1,2)*-α-closed set.*
2. *regular (1,2)*-open set [19] if $A = \tau_{1,2}$-int($\tau_{1,2}$-cl(A)). The complement of regular (1,2)*-open set is regular (1,2)*-closed.*
3. *(1,2)*-π-open [12] if the finite union of regular (1,2)*-open sets.*
4. *(1,2)*-semi-closed [16] if $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)) $\subseteq$ A.*
5. *(1,2)*-semi-open [16] if $A \subseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A)).*

Definition 2.5. *Let A be a subset of a bitopological space X. Then A is called*

1. *a (1,2)*-generalized closed (briefly, (1,2)*-g-closed) set [17] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X. The complement of (1,2)*-g-closed set is called (1,2)*-g-open set.*
2. *(1,2)*-g^*-closed set [21] if $\tau_{1,2}$-cl(A) $\subseteq$ U whenever $A \subseteq U$ and U is (1,2)*-g-open in X. The complement of (1,2)*-g^*-closed set is called (1,2)*-g^*-open set. The family of all (1,2)*-g^*-open sets of X is denoted by (1,2)*-$G^*O(X)$.*

Definition 2.6. *A function $f : X \to Y$ is called:*

1. *(1,2)*-continuous [16] if $f^{-1}(V)$ is a $\tau_{1,2}$-closed set in X for every $\sigma_{1,2}$-closed set V of Y.*
2. *perfectly (1,2)*-continuous [20] if $f^{-1}(V)$ is $\tau_{1,2}$-clopen in X for every regular (1,2)*-open set V of Y.*
3. *(1,2)*-R-map [17] if $f^{-1}(V)$ is regular (1,2)*-open in X for every regular (1,2)*-open set V of Y.*
4. *(1,2)*-open [16] if f(V) is $\sigma_{1,2}$-open in Y for every $\tau_{1,2}$-open set V of X.*
5. *(1,2)*-closed [16] if f(V) is $\sigma_{1,2}$-closed in Y for every $\tau_{1,2}$-closed set V of X.*

Definition 2.7 ([15]). *A subset A of a bitopological space X is called:*

1. *a weakly (1,2)*-g-closed (briefly, (1,2)*-wg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U whenever $A \subseteq U$ and U is $\tau_{1,2}$-open in X.*
2. *a weakly (1,2)*-πg-closed (briefly, (1,2)*-wπg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U whenever $A \subseteq U$ and U is (1,2)*-π-open in X.*
3. *a regular (1,2)*-weakly generalized closed (briefly, (1,2)*-rwg-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U whenever $A \subseteq U$ and U is regular (1,2)*-open in X.*

3. Weakly (1,2)*-g^*-closed Sets

We introduce the definition of weakly (1,2)*-g^*-closed sets in bitopological spaces and study the relationships of such sets.

Definition 3.1. *A subset A of a bitopological space X is called a weakly (1,2)*-g^*-closed (briefly, (1,2)*-wg^*-closed) set if $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U whenever $A \subseteq U$ and U is (1,2)*-g-open in X.*

Theorem 3.2. *Every (1,2)*-g^*-closed set is (1,2)*-wg^*-closed but not conversely.*

Example 3.3. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{a, b\}, X\}$. Then the sets in $\{\phi, \{a, b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{c\}, X\}$ are called $\tau_{1,2}$-closed. Then the set $\{a\}$ is (1,2)*-wg*-closed set but it is not a (1,2)*-g*-closed in X.*

Theorem 3.4. *Every (1,2)*-wg*-closed set is (1,2)*-wg-closed but not conversely.*

Proof. Let A be any (1,2)*-wg*-closed set and U be any $\tau_{1,2}$-open set containing A. Then U is a (1,2)*-g-open set containing A. We have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U. Thus, A is (1,2)*-wg-closed. □

Example 3.5. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X\}$ and $\tau_2 = \{\phi, \{a\}, X\}$. Then the sets in $\{\phi, \{a\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Then the set $\{a, b\}$ is (1,2)*-wg-closed but it is not a (1,2)*-wg*-closed.*

Theorem 3.6. *Every (1,2)*-wg*-closed set is (1,2)*-wπg-closed but not conversely.*

Proof. Let A be any (1,2)*-wg*-closed set and U be any (1,2)*-π-open set containing A. Then U is a (1,2)*-g-open set containing A. We have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U. Thus, A is (1,2)*-wπg-closed. □

Example 3.7. *In Example 3.5, the set $\{a, c\}$ is (1,2)*-wπg-closed but it is not a (1,2)*-wg*-closed.*

Theorem 3.8. *Every (1,2)*-wg*-closed set is (1,2)*-rwg-closed but not conversely.*

Proof. Let A be any (1,2)*-wg*-closed set and U be any regular (1,2)*-open set containing A. Then U is a (1,2)*-g-open set containing A. We have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ U. Thus, A is (1,2)*-rw$g$-closed. □

Example 3.9. *In Example 3.5, the set $\{a\}$ is (1,2)*-rwg-closed but it is not a (1,2)*-wg*-closed.*

Theorem 3.10. *If a subset A of a bitopological space X is both $\tau_{1,2}$-closed and (1,2)*-g-closed, then it is (1,2)*-wg*-closed in X.*

Proof. Let A be a (1,2)*-g-closed set in X and U be any $\tau_{1,2}$-open set containing A. Then U $\supseteq$ $\tau_{1,2}$-cl(A) $\supseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))). Since A is $\tau_{1,2}$-closed, U $\supseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) and hence (1,2)*-wg*-closed in X. □

Theorem 3.11. *If a subset A of a bitopological space X is both $\tau_{1,2}$-open and (1,2)*-wg*-closed, then it is $\tau_{1,2}$-closed.*

Proof. Since A is both $\tau_{1,2}$-open and (1,2)*-wg*-closed, A $\supseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) = $\tau_{1,2}$-cl(A) and hence A is $\tau_{1,2}$-closed in X. □

Corollary 3.12. *If a subset A of a bitopological space X is both $\tau_{1,2}$-open and (1,2)*-wg*-closed, then it is both regular (1,2)*-open and regular (1,2)*-closed in X.*

Theorem 3.13. *Let X be a bitopological space and A $\subseteq$ X be $\tau_{1,2}$-open. Then, A is (1,2)*-wg*-closed if and only if A is (1,2)*-g*-closed.*

Proof. Let A be (1,2)*-g^*-closed. By Theorem 3.2, it is (1,2)*-wg*-closed. Conversely, let A be (1,2)*-wg*-closed. Since A is $\tau_{1,2}$-open, by Theorem 3.11, A is $\tau_{1,2}$-closed. Hence A is (1,2)*-g^*-closed. □

Theorem 3.14. *If a set A of X is (1,2)*-wg*-closed, then $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) − A contains no non-empty (1,2)*-g-closed set.*

Proof. Let F be a (1,2)*-g-closed set such that F $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) − A. Since F^c is (1,2)*-g-open and A $\subseteq$ F^c, from the definition of (1,2)*-wg*-closedness it follows that $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) $\subseteq$ F^c. i.e., F $\subseteq$ ($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)))^c. This implies that F $\subseteq$ ($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))) $\cap$ ($\tau_{1,2}$-cl($\tau_{1,2}$-int(A)))^c = ϕ. □

Theorem 3.15. *If a subset A of a bitopological space X is (1,2)*-nowhere dense, then it is (1,2)*-wg*-closed.*

Proof. We know that a subset A of X is (1,2)*-nowhere dense if $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)) = $\emptyset$. Since $\tau_{1,2}$-int(A) $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)) and A is (1,2)*-nowhere dense, $\tau_{1,2}$-int(A) = ϕ. Therefore $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)) = ϕ and hence A is (1,2)*-wg*-closed in X. □

The converse of Theorem 3.15 need not be true as seen in the following example.

Example 3.16. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and $\tau_{1,2}$-closed. Then the set $\{a\}$ is (1,2)*-wg*-closed set but not (1,2)*-nowhere dense in X.*

Remark 3.17. *The following examples show that (1,2)*-wg*-closedness and (1,2)*-semi-closedness are independent.*

Example 3.18. *In Example 3.3, we have the set $\{a, c\}$ is (1,2)*-wg*-closed set but not (1,2)*-semi-closed in X.*

Example 3.19. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{c\}, \{a, c\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Then the set $\{a\}$ is (1,2)*-semi-closed set but not (1,2)*-wg*-closed in X.*

Remark 3.20. *From the above discussions and known results, we obtain the following diagram for a subset of a bitopological space, where $A \to B$ represents A implies B but not conversely.*

Diagram

$$\tau_{1,2}\text{-closed} \Rightarrow (1,2)^*\text{-}wg^*\text{-closed} \Rightarrow (1,2)^*\text{-}wg\text{-closed} \Rightarrow (1,2)^*\text{-}w\pi g\text{-closed} \Rightarrow (1,2)^*\text{-}rwg\text{-closed}$$

Definition 3.21. *A subset A of a bitopological space X is called (1,2)*-wg*-open set if A^c is (1,2)*-wg*-closed in X.*

Proposition 3.22. *Every (1,2)*-g^*-open set is (1,2)*-wg*-open but not conversely.*

Theorem 3.23. *A subset A of a bitopological space X is (1,2)*-wg*-open if $G \subseteq \tau_{1,2}$-int($\tau_{1,2}$-cl(A)) whenever $G \subseteq A$ and G is (1,2)*-g-closed.*

Proof. Let A be any (1,2)*-wg*-open. Then A^c is (1,2)*-wg*-closed. Let G be a (1,2)*-g-closed set contained in A. Then G^c is a (1,2)*-g-open set containing A^c. Since A^c is (1,2)*-wg*-closed, we have $\tau_{1,2}$-cl($\tau_{1,2}$-int(A^c)) $\subseteq G^c$. Therefore G $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)).

Conversely, we suppose that G $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl(A)) whenever G $\subseteq$ A and G is (1,2)*-g-closed. Then G^c is a (1,2)*-g-open set containing A^c and $G^c \supseteq (\tau_{1,2}$-int($\tau_{1,2}$-cl(A)))^c. It follows that $G^c \supseteq \tau_{1,2}$-cl($\tau_{1,2}$-int(A^c)). Hence A^c is (1,2)*-wg*-closed and so A is (1,2)*-wg*-open. □

Definition 3.24. *Let $f : X \to Y$ be a function. Then f is said to be*

1. *contra (1,2)*-g^*-continuous if the inverse image of every $\sigma_{1,2}$-open set in Y is (1,2)*-g^*-closed set in X.*
2. *(1,2)*-g^*-irresolute if the inverse image of every (1,2)*-g^*-closed set in Y is (1,2)*-g^*-closed set in X.*

Theorem 3.25. *The following are equivalent for a function $f : X \to Y$:*

1. *f is contra (1,2)*-g^*-continuous.*
2. *the inverse image of every $\sigma_{1,2}$-closed set of Y is (1,2)*-g^*-open in X.*

Proof. Let U be any $\sigma_{1,2}$-closed set of Y. Since Y \U is $\sigma_{1,2}$-open, then by (1), it follows that f^{-1}(Y \U) = X \f^{-1}(U)is (1,2)*-g^*-closed. This shows that $f^{-1}(U)$ is (1,2)*-g^*-open in X. Converse is similar. □

4. Weakly (1,2)*-g^*-continuous Functions

Definition 4.1. *Let X and Y be two bitopological spaces. A function $f : X \to Y$ is called weakly (1,2)*-g^*-continuous (briefly, (1,2)*-wg^*-continuous) if $f^{-1}(U)$ is a (1,2)*-wg^*-open set in X for each $\sigma_{1,2}$-open set U of Y.*

Example 4.2. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{a\}, Y\}$. Then the sets in $\{\phi, \{a\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-closed. The function $f : X \to Y$ defined by $f(a) = b$, $f(b) = c$ and $f(c) = a$ is (1,2)*-wg^*-continuous, because every $\sigma_{1,2}$-open subset of Y is (1,2)*-wg^*-closed in X.*

Theorem 4.3. *Every (1,2)*-g^*-continuous function is (1,2)*-wg^*-continuous.*

Proof. It follows from Theorem 3.2. □

The converse of Theorem 4.3 need not be true as seen in the following example.

Example 4.4. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{b\}, Y\}$. Then the sets in $\{\phi, \{b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a, c\}, Y\}$ are called $\sigma_{1,2}$-closed. Let $f : X \to Y$ be the identity function. Then f is (1,2)*-wg^*-continuous but not (1,2)*-g^*-continuous.*

Theorem 4.5. *A function $f : X \to Y$ is (1,2)*-wg^*-continuous if and only if $f^{-1}(U)$ is a (1,2)*-wg^*-closed set in X for each $\sigma_{1,2}$-closed set U of Y.*

Proof. Let U be any $\sigma_{1,2}$-closed set of Y. According to the assumption $f^{-1}(U^c) = X \setminus f^{-1}(U)$ is (1,2)*-wg^*-open in X, so $f^{-1}(U)$ is (1,2)*-wg^*-closed in X. The converse can be proved in a similar manner. □

Definition 4.6. *A bitopological space X is said to be locally (1,2)*-g^*-indiscrete if every (1,2)*-g^*-open set of X is $\tau_{1,2}$-closed in X.*

Theorem 4.7. *Let $f : X \to Y$ be a function. If f is contra (1,2)*-g^*-continuous and X is locally (1,2)*-g^*-indiscrete, then f is (1,2)*- continuous.*

Proof. Let V be a $\sigma_{1,2}$-closed in Y. Since f is contra (1,2)*-g^*-continuous, $f^{-1}(V)$ is (1,2)*-g^*-open in X. Since X is locally (1,2)*-g^*-indiscrete, $f^{-1}(V)$ is $\tau_{1,2}$-closed in X. Hence f is (1,2)*-continuous. □

Theorem 4.8. *Let $f : X \to Y$ be a function. If f is contra (1,2)*-g^*-continuous and X is locally (1,2)*-g^*-indiscrete, then f is (1,2)*-wg^*-continuous.*

Proof. Let $f : X \to Y$ be contra (1,2)*-g^*-continuous and X is locally (1,2)*-g^*-indiscrete. By Theorem 4.7, f is (1,2)*-continuous, then f is (1,2)*-wg^*-continuous. □

Proposition 4.9. *If $f : X \to Y$ is perfectly (1,2)*-continuous and (1,2)*-wg^*-continuous, then it is (1,2)*-R-map.*

Proof. Let V be any regular (1,2)*-open subset of Y. According to the assumption, $f^{-1}(V)$ is both $\tau_{1,2}$-open and $\tau_{1,2}$-closed in X. Since $f^{-1}(V)$ is $\tau_{1,2}$-closed, it is (1,2)*-wg^*-closed. We have $f^{-1}(V)$ is both $\tau_{1,2}$-open and (1,2)*-wg^*-closed. Hence, by Corollary 3.12, it is regular (1,2)*-open in X, so f is (1,2)*-R-map. □

Definition 4.10. *A bitopological space X is called (1,2)*-g^*-compact if every cover of X by (1,2)*-g^*-open sets has finite subcover.*

Definition 4.11. *A bitopological space X is called weakly (1,2)*-g^*-compact (briefly, (1,2)*-wg*-compact) if every (1,2)*-wg*-open cover of X has a finite subcover.*

Remark 4.12. *Every (1,2)*-wg*-compact space is (1,2)*-g^*-compact.*

Theorem 4.13. *Let $f : X \to Y$ be surjective (1,2)*-wg*-continuous function. If X is (1,2)*-wg*-compact, then Y is (1,2)*-compact.*

Proof. Let $\{A_i : i \in I\}$ be an $\sigma_{1,2}$-open cover of Y. Then $\{ f^{-1}(A_i) : i \in I\}$ is a (1,2)*-wg^*-open cover in X. Since X is (1,2)*-wg^*-compact, it has a finite subcover, say $\{ f^{-1}(A_1), f^{-1}(A_2),...., f^{-1}(A_n)\}$. Since f is surjective $\{A_1, A_2,...., A_n\}$ is a finite subcover of Y and hence Y is (1,2)*-compact. □

Definition 4.14. *A bitopological space X is called weakly (1,2)*-g^*-connected (briefly, (1,2)*-wg*-connected) if X cannot be written as the disjoint union of two non-empty (1,2)*-wg*-open sets.*

Definition 4.15. *A bitopological space X is called (1,2)*-g^*-connected if X cannot be written as the disjoint union of two non-empty (1,2)*-g^*-open sets.*

Definition 4.16. *A bitopological space X is called almost (1,2)*-connected if X cannot be written as the disjoint union of two non-empty regular (1,2)*-open sets.*

Theorem 4.17. *If a bitopological space X is (1,2)*-wg*-connected, then X is almost (1,2)*-connected (resp. (1,2)*-g^*-connected).*

Proof. It follows from the fact that each regular (1,2)*-open set (resp. (1,2)*-g^*-open set) is (1,2)*-wg^*-open. □

Theorem 4.18. *For a bitopological space X, the following statements are equivalent:*

1. *X is (1,2)*-wg*-connected.*
2. *The empty set ϕ and X are only subsets which are both (1,2)*-wg*-open and (1,2)*-wg*-closed.*
3. *Each (1,2)*-wg*-continuous function from X into a discrete space Y which has at least two points is a constant function.*

Proof. (1) ⇒ (2). Let $S \subseteq X$ be any proper subset, which is both (1,2)*-wg^*-open and (1,2)*-wg^*-closed. Its complement X \S is also (1,2)*-wg^*-open and (1,2)*-wg^*-closed. Then $X = S \cup (X \setminus S)$ is a disjoint union of two non-empty (1,2)*-wg^*-open sets which is a contradiction with the fact that X is (1,2)*-wg^*-connected. Hence, $S = \phi$ or X.
(2) ⇒ (1). Let $X = A \cup B$ where $A \cap B = \phi$, $A \neq \phi$, $B \neq \phi$ and A, B are (1,2)*-wg^*-open. Since $A = X \setminus B$, A is (1,2)*-wg^*-closed. According to the assumption $A = \phi$, which is a contradiction.
(2) ⇒ (3). Let $f : X \to Y$ be a (1,2)*-wg^*-continuous function where Y is a discrete bitopological space with at least two points. Then $f^{-1}(\{y\})$ is (1,2)*-wg^*-closed and (1,2)*-wg^*-open for each $y \in Y$ and $X = \cup \{ f^{-1}(\{y\}) : y \in Y\}$. According to the assumption, $f^{-1}(\{y\}) = \phi$ or $f^{-1}(\{y\}) = X$. If $f^{-1}(\{y\}) = \phi$ for all $y \in Y$, f will not be a function. Also there is no exist more than one $y \in Y$ such that $f^{-1}(\{y\}) = X$. Hence, there exists only one $y \in Y$ such that $f^{-1}(\{y\}) = X$ and $f^{-1}(\{y_1\}) = \phi$ where $y \neq y_1 \in Y$. This shows that f is a constant function.
(3) ⇒ (2). Let $S \neq \phi$ be both (1,2)*-wg^*-open and (1,2)*-wg^*-closed in X. Let $f : X \to Y$ be a (1,2)*-wg^*-continuous function defined by $f(S) = \{a\}$ and $f(X \setminus S) = \{b\}$ where $a \neq b$. Since f is constant function we get $S = X$. □

Theorem 4.19. *Let $f : X \to Y$ be a (1,2)*-wg*-continuous surjective function. If X is (1,2)*-wg*-connected, then Y is (1,2)*-connected.*

Proof. We suppose that Y is not (1,2)*-connected. Then Y = A ∪ B where A ∩ B = ϕ, A ≠ ϕ, B ≠ ϕ and A, B are $\sigma_{1,2}$-open sets in Y. Since f is (1,2)*-wg^*-continuous surjective function, X = f^{-1}(A) ∪ f^{-1}(B) are disjoint union of two non-empty (1,2)*-wg^*-open subsets. This is contradiction with the fact that X is (1,2)*-wg^*-connected. □

5. Weakly (1,2)*-g^*-open and Weakly (1,2)*-g^*-closed Functions

Definition 5.1. *Let X and Y be bitopological spaces. A function f : X → Y is called weakly (1,2)*-g^*-open (briefly, (1,2)*-wg^*-open) if f(V) is a (1,2)*-wg^*-open set in Y for each $\tau_{1,2}$-open set V of X.*

Remark 5.2. *Every (1,2)*-g^*-open function is (1,2)*-wg^*-open but not conversely.*

Example 5.3. *Let X = {a, b, c, d}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ, {a, b, d}, X}. Then the sets in {ϕ, {a}, {a, b, d}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, {c}, {b, c, d}, X} are called $\tau_{1,2}$-closed. Let Y = {a, b, c, d}, σ_1 = {ϕ, {a}, Y} and σ_2 = {ϕ, {b, c}, {a, b, c}, Y}. Then the sets in {ϕ, {a}, {b, c}, {a, b, c}, Y} are called $\sigma_{1,2}$-open and the sets in {ϕ, {d}, {a, d}, {b, c, d}, Y} are called $\sigma_{1,2}$-closed. Let f : X → Y be the identity function. Then f is (1,2)*-wg^*-open but not (1,2)*-g^*-open.*

Definition 5.4. *Let X and Y be bitopological spaces. A function f : X → Y is called weakly (1,2)*-g^*-closed (briefly, (1,2)*-wg^*-closed) if f(V) is a (1,2)*-wg^*-closed set in Y for each $\tau_{1,2}$-closed set V of X. It is clear that an (1,2)*-open function is (1,2)*-wg^*-open and a (1,2)*-closed function is (1,2)*-wg^*-closed.*

Theorem 5.5. *Let X and Y be bitopological spaces. A function f : X → Y is (1,2)*-wg^*-closed if and only if for each subset B of Y and for each $\tau_{1,2}$-open set G containing f^{-1}(B) there exists a (1,2)*-wg^*-open set F of Y such that B ⊆ F and f^{-1}(F) ⊆ G.*

Proof. Let B be any subset of Y and let G be an $\tau_{1,2}$-open subset of X such that f^{-1}(B) ⊆ G. Then F = Y \f(X \G) is (1,2)*-wg^*-open set containing B and f^{-1}(F) ⊆ G.

Conversely, let U be any $\tau_{1,2}$-closed subset of X. Then f^{-1}(Y \f(U)) ⊆ X \U and X \U is $\tau_{1,2}$-open. According to the assumption, there exists a (1,2)*-wg^*-open set F of Y such that Y \f(U) ⊆ F and f^{-1}(F) ⊆ X \U. Then U ⊆ X \$f^{-1}$(F). From Y \F ⊆ f(U) ⊆ f(X \$f^{-1}$(F)) ⊆ Y \F it follows that f(U) = Y \F, so f(U) is (1,2)*-wg^*-closed in Y. Therefore f is a (1,2)*-wg^*-closed function. □

Remark 5.6. *The composition of two (1,2)*-wg^*-closed functions need not be a (1,2)*-wg^*-closed as we can see from the following example.*

Example 5.7. *Let X = {a, b, c}, τ_1 = {ϕ, {a}, X} and τ_2 = {ϕ, {a, b}, X}. Then the sets in {ϕ, {a}, {a, b}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, {c}, {b, c}, X} are called $\tau_{1,2}$-closed. Let Y = {a, b, c}, σ_1 = {ϕ, {a}, Y} and σ_2 = {ϕ, {b, c}, Y}. Then the sets in {ϕ, {a}, {b, c}, Y} are called $\sigma_{1,2}$-open and $\sigma_{1,2}$-closed. Let Z = {a, b, c}, η_1 = {ϕ, Z} and η_2 = {ϕ, {a, b}, Z}. Then the sets in {ϕ, {a, b}, Z} are called $\eta_{1,2}$-open and the sets in {ϕ, {c}, Z} are called $\eta_{1,2}$-closed. We define f : X → Y by f(a) = c, f(b) = b and f(c) = a and let g : Y → Z be the identity function. Hence both f and g are (1,2)*-wg^*-closed functions. For a $\tau_{1,2}$-closed set U = {b, c}, (g o f)(U) = g(f(U)) = g({a, b}) = {a, b} which is not (1,2)*-wg^*-closed in Z. Hence the composition of two (1,2)*-wg^*-closed functions need not be a (1,2)*-wg^*-closed.*

Theorem 5.8. *Let X, Y and Z be bitopological spaces. If f : X → Y is a (1,2)*-closed function and g : Y → Z is a (1,2)*-wg^*-closed function, then g ∘ f : X → Z is a (1,2)*-wg^*-closed function.*

Definition 5.9. *A function $f : X \to Y$ is called a weakly (1,2)*-g^*-irresolute (briefly, (1,2)*-wg^*-irresolute) if $f^{-1}(U)$ is a (1,2)*-wg^*-open set in X for each (1,2)*-wg^*-open set U of Y.*

Example 5.10. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{b\}, X\}$ and $\tau_2 = \{\phi, \{a, c\}, X\}$. Then the sets in $\{\phi, \{b\}, \{a, c\}, X\}$ are called $\tau_{1,2}$-open and $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, Y\}$ and $\sigma_2 = \{\phi, \{b\}, Y\}$. Then the sets in $\{\phi, \{b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{a, c\}, Y\}$ are called $\sigma_{1,2}$-closed. Let $f : X \to Y$ be the identity function. Then f is (1,2)*-wg^*-irresolute.*

Remark 5.11. *Every (1,2)*-g^*-irresolute function is (1,2)*-wg^*-continuous but not conversely. Also, the concepts of (1,2)*-g^*-irresoluteness and (1,2)*-wg^*-irresoluteness are independent of each other.*

Example 5.12. *Let $X = \{a, b, c, d\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, \{a, b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, \{a, b, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{d\}, \{a, d\}, \{b, c, d\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c, d\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ and $\sigma_2 = \{\phi, \{a, b, d\}, Y\}$. Then the sets in $\{\phi, \{a\}, \{a, b, d\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, \{b, c, d\}, Y\}$ are called $\sigma_{1,2}$-closed. Let $f : X \to Y$ be the identity function. Then f is (1,2)*-wg^*-continuous but not (1,2)*-g^*-irresolute.*

Example 5.13. *Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, \{a\}, X\}$ and $\tau_2 = \{\phi, \{b, c\}, X\}$. Then the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-open and the sets in $\{\phi, \{a\}, \{b, c\}, X\}$ are called $\tau_{1,2}$-closed. Let $Y = \{a, b, c\}$, $\sigma_1 = \{\phi, \{a\}, Y\}$ and $\sigma_2 = \{\phi, \{a, b\}, Y\}$. Then the sets in $\{\phi, \{a\}, \{a, b\}, Y\}$ are called $\sigma_{1,2}$-open and the sets in $\{\phi, \{c\}, \{b, c\}, Y\}$ are called $\sigma_{1,2}$-closed. Let $f : X \to Y$ be the identity function. Then f is (1,2)*-wg^*-irresolute but not (1,2)*-g^*-irresolute.*

Example 5.14. *Let X, τ_1 and τ_2 be as in Example 3.19. Let Y, σ_1 and σ_2 be as in Example 3.3. Let f be the identity function, then f is (1,2)*-g^*-irresolute but not (1,2)*-wg^*-irresolute.*

Theorem 5.15. *The composition of two (1,2)*-wg^*-irresolute functions is also (1,2)*-wg^*-irresolute.*

Theorem 5.16. *Let $f : X \to Y$ and $g : Y \to Z$ be functions such that $g \circ f : X \to Z$ is (1,2)*-wg^*-closed function. Then the following statements hold:*

1. *if f is (1,2)*-continuous and injective, then g is (1,2)*-wg^*-closed.*

2. *if g is (1,2)*-wg^*-irresolute and surjective, then f is (1,2)*-wg^*-closed.*

Proof.

1. Let F be a $\sigma_{1,2}$-closed set of Y. Since $f^{-1}(F)$ is $\tau_{1,2}$-closed in X, we can conclude that $(g \circ f)(f^{-1}(F))$ is (1,2)*-wg^*-closed in Z. Hence g(F) is (1,2)*-wg^*-closed in Z. Thus g is a (1,2)*-wg^*-closed function.

2. It can be proved in a similar manner as (1).

□

Theorem 5.17. *If $f : X \to Y$ is an (1,2)*-wg^*-irresolute function, then it is (1,2)*-wg^*-continuous.*

Remark 5.18. *The converse of the above need not be true in general. The function $f : X \to Y$ in the Example 5.14 is (1,2)*-wg^*-continuous but not (1,2)*-wg^*-irresolute.*

Theorem 5.19. *If $f : X \to Y$ is surjective (1,2)*-wg^*-irresolute function and X is (1,2)*-wg^*-compact, then Y is (1,2)*-wg^*-compact.*

Theorem 5.20. *If $f : X \to Y$ is surjective (1,2)*-wg^*-irresolute function and X is (1,2)*-wg^*-connected, then Y is (1,2)*-wg^*-connected.*

References

[1] M.E.Abd El-Monsef, S.N. El-Deeb and R.A.Mahmoud, *β-open sets and β-continuous mappings*, Bull. Fac. Sci. Assiut Univ., 12(1983), 77-90.

[2] D.Andrijevic, *Semi-preopen sets*, Mat. Vesnik, 38(1986), 24-32.

[3] J.Antony Rex Rodrigo, O.Ravi, A.Pandi and C.M.Santhana, *On (1,2)*-s-normal spaces and pre-(1,2)*-gs-closed functions*, International Journal of Algorithms, Computing and Mathematics, 4(1) (2011), 29-42.

[4] I.Arockiarani, K.Balachandran and M.Ganster, *Regular-generalized locally closed sets and RGL-continuous functions*, Indian J. Pure Appl. Math., 28(1997), 661-669.

[5] K.Balachandran, P.Sundaram and H.Maki, *Generalized locally closed sets and GLC-continuous functions*, Indian J. Pure Appl. Math., 27(3)(1996), 235-244.

[6] K.Balachandran, P.Sundaram and H.Maki, *On generalized continuous maps in topological spaces*, Mem. Fac. Sci. Kochi Univ. Math., 12(1991), 5-13.

[7] P.Bhattacharyya and B.K.Lahiri, *Semi-generalized closed sets in topology*, Indian J. Math., 29(3)(1987), 375-382.

[8] N.Bourbaki, *General topology*, Part I, Addison-Wesley, Reading, Mass., (1966).

[9] M.Caldas, *Semi-generalized continuous maps in topological spaces*, Portugaliae Mathematica., 52 Fasc. 4(1995), 339-407.

[10] D.E.Cameron, *Topology atlas*, http://gozips. uakron. deu/.

[11] J.Cao, M.Ganster and I.Reilly, On sg-closed sets and gα-closed sets, Mem. Fac. Sci. Kochi Univ. Ser. A. Math., 20(1999), 1-5.

[12] Z.Duszynski, M.Jeyaraman, M.Sajan Joseph, M.Lellis Thivagar and O.Ravi, *A new generalization of closed sets in bitopology*, South Asian Journal of Mathematics, 4(5)(2014), 215-224.

[13] S.Jafari, M.Lellis Thivagar and Nirmala Mariappan, *On (1,2)*-$\alpha\hat{g}$-closed sets*, J. Adv. Math. Studies, 2(2)(2009), 25-34.

[14] M.Lellis Thivagar and Nirmala Mariappan, *On weak separation axioms associated with (1,2)*-sg-closed sets*, Int. Journal of Math. Analysis, 4(13)(2010), 631-644.

[15] O.Ravi, M.L.Thivagar and A.Nagarajan, *(1,2)*-αg-closed sets and (1,2)*-gα-closed sets*, (submitted).

[16] O.Ravi and M.Lellis Thivagar, *On stronger forms of (1,2)*-quotient mappings in bitopological spaces*, Internat. J. Math. Game Theory and Algebra., 14(6)(2004), 481-492.

[17] O.Ravi, M.L.Thivagar and Jinjinli, *Remarks on extensions of (1,2)*-g-closed maps*, Archimedes J. Math., 1(2)(2011), 177-187.

[18] O.Ravi, M.L.Thivagar and E.Hatir, *Decomposition of (1,2)*-continuity and (1,2)*-α-continuity*, Miskolc Mathematical Notes., 10(2)(2009), 163-171.

[19] O.Ravi, E.Ekici and M.Lellis Thivagar, *On (1,2)*-sets and decompositions of bitopological (1,2)*-continuous mappings*, Kochi J. Math., 3(2008), 181-189.

[20] O.Ravi, K.Kayathri, M.L.Thivagar and M.Joseph Israel, *Mildly (1,2)*-normal spaces and some bitopological functions*, Mathematica Bohemica, 135(1)(2010), 1-15.

[21] A.Thamilisai, *Studies on new bitopological separation axioms*, Ph. D Thesis, Madurai Kamaraj University, Madurai,(2015).