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Switching of a Vertex in Path and b-coloring

Research Article

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Abstract: A proper coloring in which every color class has a vertex adjacent to at least one vertex in every other color classes is called b-coloring. The b-chromatic number of a graph is the largest integer for which graph admits a b-coloring. We investigate the b-chromatic number for the graphs obtained from path by means of switching of a vertex.

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1. Introduction

We begin with simple and finite graph G with vertex set $V(G)$ and edge set $E(G)$. A coloring of vertices of G is a mapping $f : V(G) \to \mathbb{N}$. For every vertex v, the integer $f(v)$ is called the color of v. If no two adjacent vertices have the same colors then f is called proper coloring. A graph G is k-colorable if it is colored by k integers. The chromatic number $\chi(G)$ is the minimum number k for which G is k-colorable. A b-coloring is a proper coloring of graph G such that each color class contains at least one vertex that has neighbour(s) in all the other color classes. Such vertex is called a color dominating vertex(cdv). If v is a color dominating vertex of color class c then we denote it as $cdv(c) = v$. The b-chromatic number $\varphi(G)$ is the largest integer for which G admits a b-coloring. The concept of b-coloring was originated by Irving and Manlove[6]. According to them every proper coloring of a graph G with $\chi(G)$ colors is obviously a b-coloring and the problem of determining $\varphi(G)$ is NP-hard in general but it is polynomial time solvable for trees. If the b-coloring exists for every integer k satisfying $\chi(G) \leqslant k \leqslant \varphi(G)$ then G is called b-continuous. The b-chromatic number of various graphs are investigated by Vaidya and Shukla[10, 11] as well as Vaidya and Rakhimol[9]. The lower bounds for the b-chromatic number of the Cartesian product of two graphs is obtained in Kouider and Mahéo [7] while Kouider and Zaker[8] have determined the bounds on b-chromatic number in terms of clique number. The b-coloring for bipartite graphs is explored by Blidia *et al.* [3]. The concept of b-coloring has been extensively studied by Faik [5], Alkhateeb [1] and Balakrishnan *et al.* [2].

Definition 1.1. *For a graph G, $\Delta(G) = \max\{d(v) : v \text{ is a vertex of } G\}$, where $d(v) = \deg ree$ of a vertex v.*

Definition 1.2. *The distance $d(u, v)$ between two vertices u and v in a connected graph G is the length of shortest $u - v$ path in G.*

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Definition 1.3. *The eccentricity $e(v)$ of v is the distance to a vertex farthest from v. Thus $e(v) = \max\{d(u, v) : u \in V(G)\}$.*

Definition 1.4. *The radius $r(G)$ is the minimum eccentricity of the vertices.*

Definition 1.5. *The vertex v is a central vertex if $e(v) = r(G)$. We note that every tree has either one or two central vertices.*

Definition 1.6 ([1]). *The m-degree of a graph G, denoted by $m(G)$, is the largest integer m such that G has m vertices of degree at least $m - 1$.*

Proposition 1.7 ([4]). *For any graph G, $\chi(G) \geqslant 3$ if and only if G has an odd cycle.*

Proposition 1.8 ([1]). *If graph G admits a b-coloring with m-colors, then G must have at least m vertices with degree at least $m - 1$.*

Proposition 1.9 ([1]). $\chi(G) \leqslant \varphi(G) \leqslant m(G)$

2. Switching of Vertex of P_n

Definition 2.1 ([12]). *The switching of a vertex v of G means removing all the edges incident to v and adding edges joining to every vertex which are not adjacent to v in G. We denote the resultant graph by $\widetilde{G}$.*

The graph $\widetilde{P_n}$ possesses following five types of vertices.

1. An isolated vertex;
2. Two pendant vertices;
3. Two vertices of degree two;
4. $n - 5$ vertices of degree 3;

5.• A vertex of degree $\Delta\left(\widetilde{P_n}\right) = n - 2$ when either of the pendant vertices of $\widetilde{P_n}$ is switched.

or

5.• A vertex of degree $\Delta\left(\widetilde{P_n}\right) = n - 3$ when either central vertex is switched or arbitrary internal vertex is switched.

We also note that the graphs obtained by switching of a vertex on either side of central vertex(vertices) will be isomorphic.

3. Main Results

Lemma 3.1. *If the graph $\widetilde{P_n}$ is obtained by switching of central vertex(vertices) then*

$$\chi\left(\widetilde{P_n}\right) = \begin{cases} 2, & n = 4, 5 \\ 3, & for\ n \geqslant 6,\ where\ n\ is\ even \\ 3, & for\ n > 6,\ where\ n\ is\ odd \end{cases}$$

Proof. Let $\{v_1, v_2, ..., v_n\}$ be the vertices of $\widetilde{P_n}$. Then $\left|V\left(\widetilde{P_n}\right)\right| = n$ and $\left|E\left(\widetilde{P_n}\right)\right| = 2n - 6$. To prove the result we consider following cases.

Case 1: For $n = 4, 5$

For the graph $\widetilde{P_4}$, $\chi\left(\widetilde{P_4}\right) = 2$ as the graph obtained by switching of central vertex v_2 or v_3 is bipartite and for the graph $\widetilde{P_5}$, $\chi\left(\widetilde{P_5}\right) = 2$ as the graph obtained by switching of central vertex v_3 is bipartite.

Case 2: For $n \geq 6$, where n is even

Without loss of generality we switch either of the central vertices $v_{\frac{n}{2}}$ or $v_{\frac{n+2}{2}}$. Then $\widetilde{P_n}$ contains at least one odd cycle. According to Proposition 1.7, $\chi\left(\widetilde{P_n}\right) \geqslant 3$. But $\widetilde{P_n} - v_1$ is bipartite, it is 2-colorable, which implies that one more color is required to color $\widetilde{P_n}$. Hence $\chi\left(\widetilde{P_n}\right) = 3$.

Case 3: For $n > 6$, where n is odd

In this case the graph $\widetilde{P_n}$ is obtained by switching of a central vertex $v_{\lfloor \frac{n}{2} \rfloor +1}$ which contains at least one odd cycle. Then by Proposition 1.7, $\chi\left(\widetilde{P_n}\right) \geqslant 3$. But $\widetilde{P_n} - v_1$ is 2-colorable as it is bipartite, which implies that one more color is required to color $\widetilde{P_n}$. Hence $\chi\left(\widetilde{P_n}\right) = 3$.

□

Theorem 3.2. *For odd $n \neq 3$ let $\widetilde{P_n}$ be the graph obtained by switching of a central vertex then*
$$\varphi\left(\widetilde{P_n}\right) = \begin{cases} 3, & n = 5,\ 7 \\ 4, & n \geqslant 9 \end{cases}$$

Proof. We continue with the convention stated in Lemma-3.1 and consider the following cases. Without loss of generality we switch the central vertex $v_{\lfloor \frac{n}{2} \rfloor +1}$, for odd n.

Case 1: For $n = 5, 7$

When $n = 5$, the graph $\widetilde{P_5}$ has three vertices of degree at least 2. Then by Proposition 1.9, $\varphi\left(\widetilde{P_5}\right) \leqslant 3$. Consider the color class $c = \{1,\ 2,\ 3\}$. To assign the proper coloring to the vertices define the color function $f : V \to \{1,\ 2,\ 3\}$ as $f(v_1) = 3,\ f(v_2) = 2,\ f(v_3) = 1,\ f(v_4) = 3,\ f(v_5) = 2$. This proper coloring gives the color dominating vertices as $cdv(1) = v_3,\ cdv(2) = v_5,\ cdv(3) = v_1$. Hence, $\varphi\left(\widetilde{P_5}\right) = 3$.

Similarly, for $n = 7$ the graph $\widetilde{P_7}$ has m-degree 3. Then by Proposition 1.9, $\varphi\left(\widetilde{P_7}\right) \leqslant 3$. Consider the color class $c = \{1,\ 2,\ 3\}$. To assign the proper coloring to the vertices define the color function $f : V \to \{1,\ 2,\ 3\}$ as $f(v_1) = 3,\ f(v_2) = 2,\ f(v_3) = 1,\ f(v_4) = 1,\ f(v_5) = 1,\ f(v_6) = 3,\ f(v_7) = 2$. This proper coloring gives the color dominating vertices as $cdv(1) = v_4,\ cdv(2) = v_2,\ cdv(3) = v_1$. Hence, $\varphi\left(\widetilde{P_7}\right) = 3$.

Case 2: For $n \geq 9$

In this case the graph $\widetilde{P_9}$ has m-degree 4. Then according to Proposition 1.9, $\varphi\left(\widetilde{P_9}\right) \leqslant 4$.

If $\varphi\left(\widetilde{P_9}\right) = 4$, then by Proposition 1.8, $\widetilde{P_9}$ must have 4 vertices of degree at least three which is possible. Consider the color class $c = \{1,\ 2,\ 3,\ 4\}$. To define the color function $f : V \to \{1,\ 2,\ 3,\ 4\}$ as $f(v_1) = 3,\ f(v_2) = 4,\ f(v_3) = 2,\ f(v_4) = 3,\ f(v_5) = 1,\ f(v_6) = 2,\ f(v_7) = 3,\ f(v_8) = 4,\ f(v_9) = 2$..

Thus, $\varphi\left(\widetilde{P_9}\right) = 4$.

For $n > 9$ The graph $\widetilde{P_n}$ has m-degree 4. Then by Proposition 1.9, $\varphi\left(\widetilde{P_n}\right) \leqslant 4$. Consider the color class $c = \{1,\ 2,\ 3,\ 4\}$ and assign color 1 to the vertex of degree $\Delta\left(\widetilde{P_n}\right)$, assign the colors 2 and 3 to the pendant vertices and assign three different colors $2, 3$ and 4 repeatedly to the remaining two vertices of degree 2 and $n - 5$ vertices of degree 3. This proper coloring gives the color dominating vertices for the color classes $1, 2, 3$ and 4 respectively. Thus, $\varphi\left(\widetilde{P_n}\right) = 4,\ n \geqslant 9$.

□

Theorem 3.3. *For even n let $\widetilde{P_n}$ be the graph obtained by switching of a central vertices then*
$$\varphi\left(\widetilde{P_n}\right) = \begin{cases} 2, & n = 4 \\ 3, & n = 6 \\ 4, & n \geqslant 8 \end{cases}$$

Proof. We continue with the terminology and notations introduced in Lemma 3.1 and Theorem 3.2. Without loss of generality we switch the central vertices $v_{\frac{n}{2}}$ and $v_{\frac{n+2}{2}}$ for even n and consider the following cases.

Case 1: For $n = 4$

The graph $\widetilde{P_4}$ is bipartite. Hence $\varphi\left(\widetilde{P_4}\right) = 2$.

Case 2: For $n = 6$

The graph $\widetilde{P_6}$ has three vertices of degree at least 2 by switching the vertex v_3. Then by Proposition 1.9, $\varphi\left(\widetilde{P_6}\right) \leqslant 3$. Consider the color class $c = \{1,\ 2,\ 3\}$. To assign the proper coloring to the vertices define the color function $f : V \to \{1,\ 2,\ 3\}$ as $f(v_1) = 3,\ f(v_2) = 2,\ f(v_3) = 1,\ f(v_4) = 3,\ f(v_5) = 2,\ f(v_6) = 3$. This proper coloring gives the color dominating vertices as $cdv(1) = v_3,\ cdv(2) = v_5,\ cdv(3) = v_6$. Hence, $\varphi\left(\widetilde{P_6}\right) = 3$.

Similarly, if we switch the vertex v_4 the graph $\widetilde{P_6}$ has m-degree 3. Then according to Proposition 1.9, $\varphi\left(\widetilde{P_6}\right) \leqslant 3$. Consider the color class $c = \{1,\ 2,\ 3\}$. To assign the proper coloring to the vertices define the color function $f : V \to \{1,\ 2,\ 3\}$ as $f(v_1) = 3,\ f(v_2) = 2,\ f(v_3) = 3,\ f(v_4) = 1,\ f(v_5) = 2,\ f(v_6) = 3$. This proper coloring gives the color dominating vertices as $cdv(1) = v_4,\ cdv(2) = v_2,\ cdv(3) = v_6$. Hence, $\varphi\left(\widetilde{P_6}\right) = 3$.

Case 3: For $n \geq 8$

By switching the vertex v_4 the graph $\widetilde{P_8}$ has m-degree 4. Then according to Proposition 1.9, $\varphi\left(\widetilde{P_8}\right) \leqslant 4$. Consider the color class $c = \{1,\ 2,\ 3,\ 4\}$. To assign proper coloring to the vertices define the color function $f : V \to \{1,\ 2,\ 3,\ 4\}$ as $f(v_1) = 3,\ f(v_2) = 4,\ f(v_3) = 2,\ f(v_4) = 1,\ f(v_5) = 4,\ f(v_6) = 2,\ f(v_7) = 3,\ f(v_8) = 4$. This proper coloring gives the color dominating vertices as $cdv(1) = v_4,\ cdv(2) = v_6,\ cdv(3) = v_7,\ cdv(4) = v_2$. Hence, $\varphi\left(\widetilde{P_8}\right) = 4$.

Similarly, when we switch the vertex v_5 of $\widetilde{P_8}$ the graph has m-degree 4. Then by Proposition 1.9, $\varphi\left(\widetilde{P_8}\right) \leqslant 4$. Consider the color class $c = \{1,\ 2,\ 3,\ 4\}$ and assign the proper coloring. Define the color function $f : V \to \{1,\ 2,\ 3,\ 4\}$ as $f(v_1) = 2,\ f(v_2) = 4,\ f(v_3) = 3,\ f(v_4) = 2,\ f(v_5) = 1,\ f(v_6) = 4,\ f(v_7) = 2,\ f(v_8) = 3$. This proper coloring gives the color dominating vertices as $cdv(1) = v_5,\ cdv(2) = v_7,\ cdv(3) = v_3,\ cdv(4) = v_2$. Hence, $\varphi\left(\widetilde{P_8}\right) = 4$.

For $n > 8$, the graph $\widetilde{P_n}$ has m-degree 4. Then by Proposition 1.9, $\varphi\left(\widetilde{P_n}\right) \leqslant 4$. Consider the color class $c = \{1,\ 2,\ 3,\ 4\}$ and assign color 1 to the vertex of degree $\Delta\left(\widetilde{P_n}\right)$, color 2 and 4 to the pendant vertices and assign the proper coloring to the two vertices of degree 2 and remaining $n - 5$ vertices of degree 3 with three different colors $2, 3$ and 4 respectively. This proper coloring gives the color dominating vertices for color classes $1, 2, 3$ and 4 respectively. Hence, $\varphi\left(\widetilde{P_n}\right) = 4$. □

Lemma 3.4. *If the graph $\widetilde{P_n}$ is obtained by switching of internal vertex other than central vertex(vertices) which is on the left side of central vertex(vertices) then* $\chi\left(\widetilde{P_n}\right) = \begin{cases} 2, & n = 4 \\ 3, & n \geqslant 5 \end{cases}$

Proof. The graph $\widetilde{P_n}$ has $\left|V(\widetilde{P_n})\right| = n$ and $\left|E(\widetilde{P_n})\right| = 2n - 6$. We consider the following cases.

Case 1: For $n = 4$

The graph $\widetilde{P_4}$ is bipartite. Thus, $\chi\left(\widetilde{P_4}\right) = 2$.

Case 2: For $n \geq 5$

The graph $\widetilde{P_n}$ contains at least an odd cycle. Then by Proposition 1.7, $\chi\left(\widetilde{P_4}\right) \geqslant 3$. For proper coloring assign color to only vertex v_1. As $\widetilde{P_n} - v_1$ is bipartite, it is 2-colorable. Thus, $\chi\left(\widetilde{P_n}\right) = 3$.

□

Theorem 3.5. *Let $\widetilde{P_n}$ be the graph obtained by switching of arbitrary internal vertex*

then $\varphi\left(\widetilde{P_n}\right) = \begin{cases} 2, & n = 4 \\ 3, & n = 5, 6 \\ 3, & n = 7,\ switching\ of\ v_3 \\ 4, & n = 7,\ switching\ of\ v_2 \\ 4, & n \geqslant 8 \end{cases}$

Proof. We continue with the convention stated in Lemma 3.4 and consider the following cases.

Case 1: For $n = 4$

The graph $\widetilde{P_4}$ is bipartite. Hence, $\varphi\left(\widetilde{P_4}\right) = 2$.

Case 2: For $n = 5, 6$

The graphs $\widetilde{P_5}$ and $\widetilde{P_6}$ has m-degree 3. Then according to Proposition 1.9, $\varphi\left(\widetilde{P_4}\right) \leqslant 3$. Consider the color class $c = \{1,\ 2,\ 3\}$. To assign the proper coloring to the vertices define the color function as $f : V \to \{1,\ 2,\ 3\}$ as $f(v_1) = 1,\ f(v_2) = 1,\ f(v_3) = 1,\ f(v_4) = 2,\ f(v_5) = 3,\ f(v_6) = 2$. This proper coloring gives the color dominating vertices as $cdv(1) = v_2,\ cdv(2) = v_4,\ cdv(3) = v_5$. Thus, $\varphi\left(\widetilde{P_5}\right) = 3 = \varphi\left(\widetilde{P_6}\right)$.

Case 3: For $n = 7$, switching the vertex v_3

The graph $\widetilde{P_7}$ has 3 vertices of degree at least 2. Then by Proposition 1.9, $\varphi\left(\widetilde{P_7}\right) \leqslant 3$. Consider the color class $c = \{1,\ 2,\ 3\}$. To assign the proper coloring to the vertices define the color function as $f : V \to \{1,\ 2,\ 3\}$ as $f(v_1) = 2,\ f(v_2) = f(v_3) = 1 = f(v_4),\ f(v_5) = 2,\ f(v_6) = 3,\ f(v_7) = 2$. This proper coloring gives the color dominating vertices as $cdv(1) = v_3,\ cdv(2) = v_5,\ cdv(3) = v_6$. Hence, $\widetilde{P_7} = 3$.

Case 4: For $n = 7$, switching the vertex v_2

In this case the graph $\widetilde{P_7}$ has m-degree 4. Then by Proposition 1.9, $\varphi\left(\widetilde{P_4}\right) \leqslant 4$. Consider the color class $c = \{1,\ 2,\ 3,\ 4\}$. To assign the proper coloring to the vertices define the color function $f : V \to \{1,\ 2,\ 3,\ 4\}$ as $f(v_1) = 1 = f(v_2),\ f(v_3) = 4,\ f(v_4) = 2,\ f(v_5) = 3,\ f(v_6) = 4,\ f(v_7) = 2$. This proper coloring gives the color dominating vertices as $cdv(1) = v_2,\ cdv(2) = v_4,\ cdv(3) = v_5,\ cdv(4) = v_6$. Hence, $\varphi\left(\widetilde{P_7}\right) = 4$.

Case 5: For $n \geq 8$

The graph $\widetilde{P_8}$ has m-degree 4. Then according to Proposition 1.9, $\varphi\left(\widetilde{P_8}\right) \leqslant 4$. Consider the color class $c = \{1,\ 2,\ 3,\ 4\}$ and assign color 1 to the vertex of degree $\Delta\left(\widetilde{P_n}\right)$, assign any color to the isolated vertex, assign colors 2 and 4 to two pendant vertices, assign three different colors from 2, 3 and 4 to the two vertices of degree two and remaining $n - 5$ vertices of degree 3. Such proper coloring gives the color dominating vertices for color classes 1, 2, 3 and 4 respectively.

When $n > 8$, repeat the same colors assignment as in case of $n = 8$. Also the color dominating vertices are same. Thus, $\varphi\left(\widetilde{P_n}\right) = 4$ for all $n \geq 8$.

□

Lemma 3.6. *If the graph $\widetilde{P_n}$ is obtained by switching of pendant vertex which is on left side of central vertex(vertices) then*
$$\chi\left(\widetilde{P_n}\right) = \begin{cases} 2, & n = 3 \\ 3, & n \geqslant 4 \end{cases}$$

Proof. The graph $\widetilde{P_n}$ has $\left|V\left(\widetilde{P_n}\right)\right| = n$ and $\left|E\left(\widetilde{P_n}\right)\right| = 2n - 4$. We consider the following cases.

Case 1: For $n = 3$

The graph $\widetilde{P_3}$ is bipartite. Hence, $\varphi\left(\widetilde{P_3}\right) = 2$.

Case 2: For $n \geq 4$

The graph $\widetilde{P_n}$ contains at least an odd cycle. Then according to Proposition 1.7, $\chi\left(\widetilde{P_n}\right) \geqslant 3$. For proper coloring of $\widetilde{P_n}$ assign color to only vertex v_1. As $\widetilde{P_n} - v_1$ is bipartite, it is 2-colorable. Thus, $\chi\left(\widetilde{P_n}\right) = 3$.

□

Theorem 3.7. *Let $\widetilde{P_n}$ be the graph obtained by switching of pendant vertex which is on left side of central vertex(vertices) then*
$$\varphi\left(\widetilde{P_n}\right) = \begin{cases} 2, & n = 3 \\ 3, & n = 4, 5 \\ 4, & n \geqslant 6 \end{cases}$$

Proof. We continue with the convention stated in Lemma 3.6 and consider the following cases.

Case 1: For $n = 3$

The graph $\widetilde{P_3}$ is bipartite. Hence, $\varphi\left(\widetilde{P_3}\right) = 2$.

Case 2: For $n = 4, 5$

The graph $\widetilde{P_4}$ has three vertices of degree at least 2. Then according to Proposition 1.9, $\varphi\left(\widetilde{P_4}\right) \leqslant 3$. Consider the color class $c = \{1,\ 2,\ 3\}$. To assign the proper coloring to the vertices define the color function $f : V \to \{1,\ 2,\ 3\}$ as $f(v_1) = 2,\ f(v_2) = 1 = f(v_3),\ f(v_4) = 3,\ f(v_5) = 2$. this proper coloring gives the color dominating vertices as $cdv(1) = v_3,\ cdv(2) = v_1,\ cdv(3) = v_4$. Thus, $\varphi\left(\widetilde{P_4}\right) = 3$. Similarly with the same color assignment and color dominating vertices we have $\varphi\left(\widetilde{P_5}\right) = 3$.

Case 3: For $n \geq 6$

In this case $\widetilde{P_6}$ has m-degree 4. Then according to Proposition 1.9, $\varphi\left(\widetilde{P_4}\right) \leqslant 4$. Consider the color class $c = \{1,\ 2,\ 3,\ 4\}$ and define the color function as $f : V \to \{1,\ 2,\ 3,\ 4\}$ as $f(v_1) = 2,\ f(v_2) = 4,\ f(v_3) = 1,\ f(v_4) = 3,\ f(v_5) = 4,\ f(v_6) = 1$. This proper coloring gives the color dominating vertices as $cdv(1) = v_3,\ cdv(2) = v_1,\ cdv(3) = v_4,\ cdv(4) = v_5$. Thus, $\varphi\left(\widetilde{P_7}\right) = 4$.

For $n > 6$, the graph $\widetilde{P_7}$ is obtained from $\widetilde{P_6}$ by adding a vertex v_7 and making v_7 adjacent to v_6 and v_1. The addition of a vertex and two edges will not change the m-degree as $m\left(\widetilde{P_n}\right) = 4$. By recursive constructions of graphs $\widetilde{P_7}, \widetilde{P_8}, ..., \widetilde{P_n}$ each graphs has m-degree 4 and repeat the proper coloring after assigning the colors used $\widetilde{P_6}$. We have $\varphi\left(\widetilde{P_n}\right) = 4,\ for\ all\ n \geqslant 6$. □

Remark 3.8. *The graph $\widetilde{P_n}$ obtained by switching of a vertex is obviously b-continuous as $\chi\left(\widetilde{P_n}\right) = \varphi\left(\widetilde{P_n}\right)$ and every proper coloring of a graph with $\chi\left(\widetilde{P_n}\right)$ colors is obviously a b-coloring which implies b-continuity of $\widetilde{P_n}$.*

4. Concluding Remarks

In any network when the existing link(s) between vertices is(are) failed then the concepts of switching of a vertex come to rescue. By switching of a vertex the existing network will switch to new one and new link(s) will be established. This concept is widely used for fault detection and fault tolerance. We explore the concept of b-coloring in the context of switching of a vertex in path P_n.

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