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New Generalized Continuous Functions

Research Article

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Abstract: We introduce some new generalized continuous functions and new generalized open sets like B-α-open, B-semi-open, B-preopen, B-β-open sets on simply extended topological spaces. We investigate characterizations and relationships among such functions and sets.

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Keywords: $(\tau(B), \sigma(B'))$-continuity, $(\alpha, \sigma(B'))$-continuity, $(\sigma, \sigma(B'))$-continuity, $(\pi, \sigma(B'))$-continuity, $(\beta, \sigma(B'))$-continuity.

1. Introduction

Levine [4] introduced the notion of simple extension of topologies in 1964. Quite recently, Abd El-Monsef et al. [2] introduced and studied the notion of B-open sets and B-closed sets on topological spaces and Vadivel et al. [12] introduced and studied B-generalized regular and B-generalized normal spaces.

In this paper, we introduce new generalized continuous functions and new generalized open sets like B-α-open sets, B-semi-open sets, B-preopen sets and B-β-open sets in simply extended topological spaces. We investigate characterizations and relationships among such functions and sets.

2. Preliminaries

Let X be a non-empty set and Levine [4] defined as $\tau(B) = \{O \cup (O' \cap B) : O, O' \in \tau\}$ and called it simple extension of $\tau\, by\, B$, where $B \notin \tau$. We call the pair $(X, \tau(B))$ a simply extended topological space (briefly SETS). The elements of $\tau(B)$ are called B-open sets [2] and the complements of B-open sets are called B-closed sets [2]. The B-closure of a subset S of X, denoted by $Bcl(S)$, is the intersection of B-closed sets of X including S [2]. The B-interior of S, denoted by $Bint(S)$, is the union of B-open sets of X contained in S [2].

Definition 2.1. *Let (X, τ) be a topological space and $A \subseteq X$. Then A is said to be*

(1) semi-open [5] if $A \subseteq cl(int(A))$,

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(2) preopen [7] if $A \subseteq int(cl(A))$*,*

(3) α*-open [9] if* $A \subseteq int(cl(int(A)))$*,*

(4) β*-open [1] if* $A \subseteq cl(int(cl(A)))$*.*

The complement of semi-open (resp. preopen, α*-open,* β*-open) is said to be semi-closed (resp. preclosed,* α*-closed,* β*-closed).*

Definition 2.2. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called:*

(1) a semi-continuous [5] if $f^{-1}(V)$ *is a semi-closed in* (X, τ) *for every closed set* V *in* (Y, σ)*,*

(2) a precontinuous [7] if $f^{-1}(V)$ *is a preclosed in* (X, τ) *for every closed set* V *in* (Y, σ)*,*

(3) an α*-continuous [8] if* $f^{-1}(V)$ *is an* α*-closed in* (X, τ) *for every closed set* V *in* (Y, σ)*,*

(4) a β*-continuous [1] if* $f^{-1}(V)$ *is a* β*-closed in* (X, τ) *for every closed set* V *in* (Y, σ)*.*

Definition 2.3. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is called:*

(1) α*-irresolute [6] if* $f^{-1}(V)$ *is* α*-closed in* X *for every* α*-closed subset* V *of* Y*.*

(2) irresolute [3] if $f^{-1}(V)$ *is semi-closed in* X *for every semi-closed subset* V *of* Y*.*

(3) preirresolute [11] if $f^{-1}(V)$ *is preclosed in* X *for every preclosed subset* V *of* Y*.*

(4) β*-irresolute [10] if* $f^{-1}(V)$ *is* β*-closed in* X *for every* β*-closed subset* V *of* Y*.*

3. New Generalized Open Sets

Theorem 3.1. *For an* $x \in X, x \in Bcl(A)$ *if and only if* $V \cap A \neq \phi$ *for every B-open set* V *containing* x*.*

Proof. Let $x \in X$ and $x \in Bcl(A)$. To prove $V \cap A \neq \phi$ for every B-open set V containing x. Suppose there exists a B-open set V containing x such that $V \cap A = \phi$. Then $A \subset X - V$and $X - V$ is B-closed. We have $Bcl(A) \subset X - V$. This shows that $x \notin Bcl(A)$, which is a contradiction. Hence $V \cap A \neq \phi$ for every B-open set V containing x.

Conversely, let $V \cap A \neq \phi$ for every B-open set V containing x. To prove $x \in Bcl(A)$. Suppose $x \notin Bcl(A)$. Then there exists a B-closed subset F containing A such that $x \notin F$. Then $x \in X - F$ and X − F is B-open. Also $(X - F) \cap A = \phi$ which is a contradiction. Hence $x \in Bcl(A)$. □

Theorem 3.2. *Let* $(X, \tau(B))$ *be SETS. Then*

(1) $Bcl(A) = X - Bint\ (X - A)$*,*

(2) $Bint(A) = X - Bcl(X - A)$*.*

Proof. (1) Let $x \in X - Bint(X - A)$. Then $x \notin Bint(X - A)$. That is, every B-open set U containing x is such that $U \nsubseteq X - A$. That is, every B-open set U containing x is such that $U \cap A \neq \phi$. By Theorem 3.1, $x \in Bcl(A)$ and therefore $X - Bint(X - A) \subset Bcl(A)$.

Conversely, let $x \in Bcl(A)$. Then by Theorem 3.1 every B-open set U containing x is such that $U \cap A \neq \phi$. That is every B-open set U containing x is such that $U \nsubseteq X - A$. This implies by definition of B-interior of $(X - A)$, $x \notin Bint(X - A)$. That is $x \in X - Bint(X - A)$ and we have $Bcl(A) \subset X - Bint(X - A)$. Thus $X - Bint(X - A) = Bcl(A)$.

(2) It is similar to the proof of (1). □

Definition 3.3. *Let $(X, \tau(B))$ be SETS and $A \subseteq X$. Then A is said to be*

(1) B-semi-open if $A \subseteq Bcl(Bint(A))$;

(2) B-preopen if $A \subseteq Bint(Bcl(A))$;

(3) B-α-open if $A \subseteq Bint(Bcl(Bint(A)))$;

(4) B-β-open if $A \subseteq Bcl(Bint(Bcl(A)))$.

The complement of B-semi-open (resp. B-preopen, B-α-open, B-β-open) is said to be B-semi-closed (resp. B-preclosed, B-α-closed, B-β-closed).
In this paper, let us denote by $\sigma(\tau(B))$ (or σ) the class of all B-semi-open sets on X, by $\pi(\tau(B))$ (or π) the class of all B-preopen sets on X, by $\alpha(\tau(B))$ (or α) the class of all B-α-open sets on X and by $\beta(\tau(B))$ (or β) the class of all B-β-open sets on X.

Remark 3.4. *From the above definition we have the following implications but none of the implications is reversible.*

$$\begin{array}{ccccc} \text{B-open} & \longrightarrow & \text{B-}\alpha\text{-open} & \longrightarrow & \text{B-preopen} \\ & & \downarrow & & \downarrow \\ & & \text{B-semi-open} & \longrightarrow & \text{B-}\beta\text{-open} \end{array}$$

Example 3.5. *Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}$ and $B = \{b\}$. Then $\tau(B) = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$ and B-closed sets are ϕ, X,$\{c\}$, $\{b, c\}$, $\{a, c\}$. Then $\{a, c\}$ is B-semi-open set but not B-α-open.*

Example 3.6. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a\}\}$ and $B = \{c\}$. Then $\tau(B) = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and B-closed sets are ϕ, X, $\{b\}$, $\{a, b\}$, $\{b, c\}$. Then $\{a, b\}$ is B-β-open set but not B-preopen.*

Example 3.7. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{b, c\}\}$ and $B = \{a\}$. Then $\tau(B) = \{\phi, X, \{a\},\{b, c\}\}$ and B-closed sets are ϕ, X, $\{a\}$, $\{b, c\}$. Then $\{a, b\}$ is B-preopen set but not B-α-open.*

Example 3.8. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X, \{a, c\}\}$ and $B = \{b\}$. Then $\tau(B) = \{\phi, X, \{b\}, \{a, c\}\}$ and B-closed sets are ϕ, X, $\{b\}$, $\{a, c\}$. Then $\{a, b\}$ is B-β-open set but not B-semi-open.*

Example 3.9. *Let $X = \{a, b, c\}$, $\tau = \{\phi, X\}$ and $B = \{c\}$. Then $\tau(B) = \{\phi, X, \{c\}\}$ and B-closed sets are ϕ, X, $\{a, b\}$. Then $\{a, c\}$ is B-α-open set but not B-open.*

4. Main Results

Definition 4.1. *Let $\tau(B_X)$ and $\sigma(B_Y)$ be simple extensions of τ and σ on X and Y respectively. Then a function defined between SETS $f : X \to Y$ is said to be $(\tau(B_X), \sigma(B_Y))$-continuous if $B' \in \sigma(B_Y)$ implies that $f^{-1}(B') \in \tau(B_X)$.*

Definition 4.2. *Let $(X, \tau(B_X))$ and $(Y, \sigma(B_Y))$ be SETS. Then the function defined between SETS $f : X \to Y$ is said to be*

(1) $(\alpha, \sigma(B_Y))$-continuous if for each B-open set U in Y, $f^{-1}(U)$ is B-α-open in X.

(2) $(\sigma, \sigma(B_Y))$-continuous if for each B-open set U in Y, $f^{-1}(U)$ is B-semi-open in X.

(3) $(\pi, \sigma(B_Y))$-continuous if for each B-open set U in Y, $f^{-1}(U)$ is B-preopen in X.

(4) $(\beta, \sigma(B_Y))$-continuous if for each B-open set U in Y, $f^{-1}(U)$ is B-β-open in X.

Remark 4.3. *Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be a function. Then we have the following implications but none of the implications is reversible.*

$$
\begin{array}{ccccc}
 & & (\tau(B_X), \sigma(B_Y))\text{-continuous} & & \\
 & & \downarrow & & \\
(\sigma, \sigma(B_Y))\text{-continuous} & \longleftarrow & (\alpha, \sigma(B_Y))\text{-continuous} & \longrightarrow & (\pi, \sigma(B_Y))\text{-continuous} \\
 & \searrow & \downarrow & \swarrow & \\
 & & (\beta, \sigma(B_Y))\text{-continuous} & &
\end{array}
$$

Example 4.4. *Let $X = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ and $B_X = \{a,\ b\}$. Then $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{a,\ b\}\}$. Let $Y = \{a,\ b,\ c\}$, $\sigma = \{\phi,\ Y\}$ and $B_Y = \{a,\ c\}$. Then $\sigma(B_Y) = \{\phi,\ Y,\ \{a,\ c\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is $(\alpha, \sigma(B_Y))$-continuous but not $(\tau(B_X), \sigma(B_Y))$-continuous, since $f^{-1}(\{a, c\}) = \{a, c\}$ is not B_X-open.*

Example 4.5. *Let $X = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a\}\}$ and $B_X = \{c\}$. Then $\tau(B_X) = \{\phi,\ X,\ \{a\},\ \{c\},\ \{a,\ c\}\}$. Let $Y = \{a,\ b,\ c\}$, $\sigma = \{\phi,\ Y\}$ and $B_Y = \{a,\ b\}$. Then $\sigma(B_Y) = \{\phi,\ Y,\ \{a,\ b\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is $(\sigma, \sigma(B_Y))$-continuous and $(\beta, \sigma(B_Y))$-continuous but not $(\alpha, \sigma(B_Y))$-continuous, since $f^{-1}(\{a, b\}) = \{a, b\}$ is not B_X-α-open.*

Example 4.6. *Let $X = \{a,\ b,\ c\}$, $\tau = \{\phi,\ X,\ \{a,\ c\}\}$ and $B_X = \{b\}$. Then $\tau(B_X) = \{\phi, X, \{b\}, \{a, c\}\}$. Let $Y = \{a, b, c\}, \sigma = \{\phi, Y\}$ and $B_Y = \{a, b\}$. Then $\sigma(B_Y) = \{\phi, Y, \{a, b\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity map. Then f is $(\beta, \sigma(B_Y))$-continuous but not $(\sigma, \sigma(B_Y))$-continuous, since $f^{-1}(\{a, b\}) = \{a, b\}$ is not B_X-semi-open.*

Example 4.7. *Let $X = \{a, b, c\}, \tau = \{\phi, X, \{b, c\}\}$ and $B_X = \{a\}$. Then $\tau(B_X) = \{\phi, X, \{a\}, \{b, c\}\}$. Let $Y = \{a, b, c\}, \sigma = \{\phi, Y\}$ and $B_Y = \{a, b\}$. Then $\sigma(B_Y) = \{\phi, Y, \{a, b\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity function. Then f is $(\pi, \sigma(B_Y))$-continuous but not $(\alpha, \sigma(B_Y))$-continuous, since $f^{-1}(\{a, b\}) = \{a, b\}$ is not B_X-α-open.*

Example 4.8. *Let $X = \{a, b, c\}, \tau = \{\phi, X, \{a\}\}$ and $B_X = \{c\}$. Then $\tau(B_X) = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$. Let $Y = \{a, b, c\}, \sigma = \{\phi, Y\}$ and $B_Y = \{b, c\}$. Then $\sigma(B_Y) = \{\phi, Y, \{b, c\}\}$. Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be an identity function. Then f is $(\beta, \sigma(B_Y))$-continuous but not $(\pi, \sigma(B_Y))$-continuous, since $f^{-1}(\{b, c\}) = \{b, c\}$ is not B_X-preopen.*

Theorem 4.9. *Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be a function. Then the following statements are equivalent.*

(1) f is $(\alpha, \sigma(B_Y))$-continuous.

(2) For every B-closed subset F in Y, $f^{-1}(F)$ is B-α-closed in X.

(3) $Bcl\,(Bint\,(Bcl(f^{-1}(G)))) \subseteq f^{-1}(Bcl(G))$ for every subset G in Y.

(4) $f^{-1}(Bint(G)) \subseteq Bint\,(Bcl\,(Bint\,(f^{-1}(G))))$ for every subset G in Y.

(5) $f(Bcl\,(Bint\,(Bcl(A)))) \subseteq Bcl(f(A))$ for every subset A in X.

Proof. (1)$\Leftrightarrow$ (2) It is obvious.

(2) $\Rightarrow$ (3) For $G \subseteq Y$, since $Bcl(G)$ is a B-closed set in Y by hypothesis $f^{-1}(Bcl(G))$ is B-α-closed in X. Hence $Bcl\,(Bint\,(Bcl(f^{-1}(G)))) \subseteq f^{-1}(Bcl(G))$.

(3) $\Rightarrow$ (4) It follows from Theorem 3.2.

(3) $\Rightarrow$ (5) For any subset A in X, from (3), it follows that $Bcl\,(Bint\,(Bcl(A))) \subseteq Bcl\,(Bint\,(Bcl(f^{-1}(f(A))))) \subseteq f^{-1}(Bcl(f(A)))$. Hence $f(Bcl\,(Bint\,(Bcl(A)))) \subseteq Bcl(f(A))$.

(5) $\Rightarrow$ (2) Let F be any B-closed set in Y. By (5), $f(Bcl\,(Bint\,(Bcl(f^{-1}(F))))) \subseteq Bcl(f(f^{-1}(F))) \subseteq Bcl(F)$. This implies $Bcl\,(Bint\,(Bcl\,(f^{-1}(F)))) \subseteq f^{-1}(Bcl(F)) = f^{-1}(F)$. Thus $f^{-1}(F)$ is B-α-closed in X. □

In this same manner, we have the following theorems.

Theorem 4.10. *Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be a function. Then the following statements are equivalent.*

(1) f is $(\sigma, \sigma(B_Y))$-continuous.

(2) For every B-closed subset F in Y, $f^{-1}(F)$ is B-semi-closed in X.

(3) $Bint\,(Bcl\,(f^{-1}(G))) \subseteq f^{-1}(Bcl(G))$ for every subset G in Y.

(4) $f^{-1}(Bint(G)) \subseteq Bcl\,(Bint\,(f^{-1}(G)))$ for every subset G in Y.

(5) $f(Bint\,(Bcl\,(A))) \subseteq Bcl(f(A))$ for every subset A in X.

Proof. It is similar to the proof of Theorem 4.9. □

Theorem 4.11. *Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be a function. Then the following statements are equivalent.*

(1) f is $(\pi, \sigma(B_Y))$-continuous.

(2) For every B-closed subset F in Y, $f^{-1}(F)$ is B-preclosed in X.

(3) $Bcl\,(Bint(f^{-1}(G))) \subseteq f^{-1}(Bcl(G))$ for every subset G in Y.

(4) $f^{-1}(Bint(G)) \subseteq Bint\,(Bcl\,(f^{-1}(G)))$ for every subset G in Y.

(5) $f(Bcl\,(Bint\,(A))) \subseteq Bcl(f(A))$ for every subset A in X.

Proof. It is similar to the proof of Theorem 4.9. □

Theorem 4.12. *Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be a function. Then the following statements are equivalent.*

(1) f is $(\beta, \sigma(B_Y))$-continuous.

(2) For every B-closed subset F in Y, $f^{-1}(F)$ is B-β-closed in X.

(3) $Bint\,(Bcl(Bint(f^{-1}(G)))) \subseteq f^{-1}(Bcl(G))$ for every subset G in Y.

(4) $f^{-1}(Bint(G)) \subseteq Bcl\,(Bint\,(Bcl\,(f^{-1}(G))))$ for every subset G in Y.

(5) $f(Bint\,(Bcl\,(Bint\,(A)))) \subseteq Bcl(f(A))$ for every subset A in X.

Proof. It is similar to the proof of Theorem 4.9. □

Remark 4.13. *Let $f : (X, \tau(B_X)) \to (Y, \sigma(B_Y))$ be a function. In the particular case $(X, \tau(B_X))$ and $(Y, \sigma(B_Y))$ are topological spaces, every $(\alpha, \sigma(B_Y))$-continuous (resp. $(\sigma, \sigma(B_Y))$-continuous, $(\pi, \sigma(B_Y))$-continuous, $(\beta, \sigma(B_Y))$-continuous) function is α-continuous (resp. semi-continuous, precontinuous, β-continuous).*
Let $\tau(B)$ be a simple extension of τ by B on a non-empty set X and $S \subseteq X$. The B-α-closure (resp. B-semi-closure, B-preclosure, B-β-closure) of a subset S of X, denoted by $Bcl_\alpha(S)$ (resp. $Bcl_\sigma(S), Bcl_\pi(S), Bcl_\beta(S)$), is the intersection of B-α-closed (resp. B-semi-closed, B-preclosed, B-β-closed) sets of X including S.
The B-α-interior (resp. B-semi-interior, B-preinterior, B-β-interior) of a subset S of X, denoted by $Bint_\alpha(S)$ (resp. $Bint_\sigma(S), Bint_\pi(S), Bint_\beta(S)$), is the union of B-$\alpha$-open (resp. B-semi-open, B-preopen, B-β-open) sets contained in S.

Lemma 4.14. *Let $\tau(B)$ be a simple extension of τ by B on a non-empty set X and $A \subseteq X$. Then we have*

(1) $Bint_\alpha(A) = A \cap Bint\,(Bcl\,(Bint(A)))$ *and* $Bcl_\alpha(A) = A \cup Bcl\,(Bint\,(Bcl(A)))$*;*

(2) $Bint_\sigma(A) = A \cap Bcl\,(Bint(A))$ *and* $Bcl_\sigma(A) = A \cup Bint\,(Bcl(A))$*.*

Lemma 4.15. *Let* $\tau(B)$ *be a simple extension of* τ *by* B *on a non-empty set* X *and* $A \subseteq X$*. Then for* $\mu = \alpha, \sigma, \pi, \beta$

(1) $Bint_\mu(A) = X - Bcl_\mu(X - A)$*;*

(2) $Bcl_\mu(A) = X - Bint_\mu(X - A)$*.*

Proof. We only show that $Bint_\alpha(A) = X - Bcl_\alpha(X - A)$. For any $A \subseteq X, X - Bcl_\alpha(X - A) = X - \cap\{F \subseteq X : X - A \subseteq F \text{ and } F^c \in \alpha\} = \cup\{X - F \subseteq X : X - F \subseteq A \text{ and } F^c \in \alpha\} = Bint_\alpha(A)$. □

Theorem 4.16. *Let* f *be a function between the SETS's* $(X, \tau(B_X))$ *and* $(Y, \sigma(B_Y))$*. Then the following statements are equivalent.*

(1) f *is* $(\alpha, \sigma(B_Y))$*-continuous.*

(2) $Bcl_\alpha(f^{-1}(G)) \subseteq f^{-1}(Bcl(G))$ *for every subset* G *in* Y*.*

(3) $f^{-1}(Bint(G)) \subseteq Bint_\alpha(f^{-1}(G))$ *for every subset* G *in* Y*.*

(4) $f(Bcl_\alpha(A)) \subseteq Bcl(f(A))$ *for every subset* A *in* X*.*

Proof. (1) $\Rightarrow$ (2) For $G \subseteq Y$, from Theorem 4.9 and Lemma 4.14 it follows: $f^{-1}(Bcl(G)) = f^{-1}(Bcl(G)) \cup f^{-1}(G) \supseteq Bcl(Bint(Bcl(f^{-1}(G)))) \cup f^{-1}(G) = Bcl_\alpha(f^{-1}(G))$. Hence $Bcl_\alpha(f^{-1}(G)) \subseteq f^{-1}(Bcl(G))$.
(2) $\Leftrightarrow$ (3) It follows from Lemma 4.15.
(2) $\Rightarrow$ (4) It is obvious.
(4) $\Rightarrow$ (1) Let F be any B-closed set in Y.
Then by (4), $f(Bcl_\alpha(f^{-1}(F))) \subseteq Bcl(f(f^{-1}(F))) \subseteq Bcl(F) = F$. This implies $Bcl_\alpha(f^{-1}(F)) \subseteq f^{-1}(F)$ and so $f^{-1}(F)$ is B-α-closed in X. Hence by Theorem 4.9, f is $(\alpha, \sigma(B_Y))$-continuous. □

Theorem 4.17. *Let* f *be a function between the SETS's* $(X, \tau(B_X))$ *and* $(Y, \sigma(B_Y))$*. Then the following statements are equivalent.*

(1) f *is* $(\sigma, \sigma(B_Y))$*-continuous.*

(2) $Bcl_\sigma(f^{-1}(G)) \subseteq f^{-1}(Bcl(G))$ *for every subset* G *in* Y*.*

(3) $f^{-1}(Bint(G)) \subseteq Bint_\alpha(f^{-1}(G))$ *for every subset* G *in* Y*.*

(4) $f(Bcl_\sigma(A)) \subseteq Bcl(f(A))$ *for every subset* A *in* X*.*

Proof. It is similar to the proof of Theorem 4.16. □

Theorem 4.18. *Let* f *be a function between the SETS's* $(X, \tau(B_X))$ *and* $(Y, \sigma(B_Y))$*. Then the following statements are equivalent.*

(1) f *is* $(\pi, \sigma(B_Y))$*-continuous.*

(2) $Bcl_\pi(f^{-1}(G)) \subseteq f^{-1}(Bcl(G))$ *for every subset* G *in* Y*.*

(3) $f^{-1}(Bint(G)) \subseteq Bint_\pi(f^{-1}(G))$ *for every subset* G *in* Y*.*

(4) $f(Bcl_\pi(A)) \subseteq Bcl(f(A))$ *for every subset* A *in* X*.*

Proof. (1) ⇒ (2) For $G \subseteq Y$, since Bcl(G) is B-closed in Y, by Theorem 4.11, $f^{-1}(Bcl(G))$ is B-preclosed in X. This implies $Bcl_\pi(f^{-1}(G)) \subseteq Bcl_\pi(f^{-1}(Bcl(G))) = f^{-1}(Bcl(G))$. So $Bcl_\pi(f^{-1}(G)) \subseteq f^{-1}(Bcl(G))$.

(2) ⇒ (1) Let F be any B-closed set in Y. Then from (2), $Bcl_\pi(f^{-1}(F)) \subseteq f^{-1}(Bcl(F)) = f^{-1}(F)$. So $f^{-1}(F)$ is B-preclosed and hence by Theorem 4.11(2), f is $(\pi, \sigma(B_Y))$-continuous.

(2) ⇔ (3) It follows from Lemma 4.15.

(2) ⇔ (4) It is obvious. □

Theorem 4.19. *Let f be a function between the SETS's* $(X, \tau(B_X))$ *and* $(Y, \sigma(B_Y))$*. Then the following statements are equivalent.*

(1) f is $(\beta, \sigma(B_Y))$*-continuous.*

(2) $Bcl_\beta(f^{-1}(G)) \subseteq f^{-1}(Bcl(G))$ *for every subset* G *in* Y*.*

(3) $f^{-1}(Bint(G)) \subseteq Bint_\beta(f^{-1}(G))$ *for every subset* G *in* Y*.*

(4) $f(Bcl_\beta(A)) \subseteq Bcl(f(A))$ *for every subset* A *in* X*.*

Proof. It is similar to the proof of Theorem 4.18. □

Theorem 4.20. *Let f be a function between the SETS's* $(X, \tau(B_X))$ *and* $(Y, \sigma(B_Y))$*. Then the following statements are equivalent.*

(1) f is $(\tau(B_X), \sigma(B_Y))$*-continuous.*

(2) $f^{-1}(Bint(G)) \subseteq Bint(f^{-1}(G))$ *for all* $G \subseteq Y$*.*

(3) $Bcl(f^{-1}(G)) \subseteq f^{-1}(Bcl(G))$ *for all* $G \subseteq Y$*.*

(4) $f(Bcl(A)) \subseteq Bcl(f(A))$ *for every subset* $A \subseteq X$*.*

(5) For every B-closed set F in Y, $f^{-1}(F)$ *is B-closed in X.*

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