International Journal of Mathematics And its Applications
Volume 3, Issue 3–C (2015), 61–73.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

(1,2)*-Generalized Continuous Map in Fuzzy Bitopological Spaces

Research Article

P.Saravanaperumal[1] and S.Murugesan[2]*

1 Research Scholar, Reg. No. 7357, Manonmaniam Sundaranar University, Tirunelveli, Tamil Nadu, India.

2 Department of Mathematics, Sri.S.R.Naidu Memorial College, Sattur, Tamil Nadu, India.

Abstract: In this paper, we introduce (1,2)*-Fuzzy generalized continuous maps and (1,2)*-Fuzzy$\ddot{g}$ continuous maps and study their relations with existing generalized (1,2)*-Fuzzy continuous maps.

Keywords: Fuzzy bitopological space,(1,2)*-fuzzy g- continuous maps, (1,2)*-fuzzy$\ddot{g}$-continuous maps.

1. Introduction

Levine [9] introduced the concept of Generalized closed sets in topological spaces. Sundaram et al [25] introduced and studied the concept of a class maps namely g- continuous maps which included the continuous maps and a class of gc irresolute maps. Recently Ravi et al [17] Ravi and Thivagar [16] and Duszynski et al [5] introduced (1,2)*-g-closed sets, (1,2)*-sg-closed sets and (1,2)*-f$\ddot{g}$-closed sets in bitopological space respectively. In this paper, we introduce (1,2)*-Fuzzy generalized continuous maps and (1,2)*-Fuzzy -f$\ddot{g}$continuous maps and study their relations with various generalized (1,2)*-fuzzy continuous maps.

2. Preliminaries

In this section, we list out the definitions and the results which are needed in sequel. Fuzzy sub sets in (X, τ_1, τ_2) will be denoted byλ, μ,γ and F(X) denotes set of all fuzzy sub sets of (X, τ_1, τ_2) .

Definition 2.1. *A fuzzy subset set λ of (X,τ) is called:*

(i) Fuzzy semi open (briefly, fs-open) if $\lambda \leq Cl(Int(\lambda))$ and a fuzzy semi closed (briefly, fs-closed) if $Int(Cl(\lambda)) \leq \lambda$[1] ;

(ii) Fuzzy pre open (briefly, fp-open) if $A \leq Int(Cl(\lambda))$ and a fuzzy pre closed (briefly, fp-closed) if $Cl(Int(\lambda)) \leq \lambda$[4];

(iii) Fuzzy α-open (briefly, fα-open) if $\lambda \leq IntCl(Int(\lambda))$ and a fuzzy α-closed (briefly, fα-closed) if $ClInt(Cl(\lambda) \leq \lambda$ [4];

(iv) Fuzzy semi-preopen (briefly, fsp-open) if $\lambda \leq ClInt(Cl(\lambda))$ and a fuzzy semi-pre closed (briefly, fsp-closed) if $IntCl(Int(\lambda)) \leq \lambda$ [26]. By FSPO (X,τ), we denote the family of all fuzzy semi pre open sets of fts X . The semi closure (resp. α-closure, semi-pre closure) of a fuzzy set A of (X,τ) is the intersection of all fs-closed (resp. fα-closed, fsp-closed) sets that contain A and is denoted by $sCl(\lambda)$ (resp. $\alpha Cl(\lambda)$ and $spCl(\lambda)$).

* E-mail: satturmuruges@gmail.com

The following lemma is well-known,

Lemma 2.2. *Let λ be a fuzzy set in a fuzzy topological space (X,τ). Then*

(i) $\alpha Cl\lambda = \lambda \vee\ ClIntCl\lambda$.

(ii) $sCl\lambda = \lambda \vee\ IntCl\lambda$.

(ii) $pCl\lambda \geq \lambda \vee\ ClInt\lambda$.

(iii) $spCl\lambda \geq \lambda \vee\ IntClInt\lambda$.

Definition 2.3. *A fuzzy set λ in a fuzzy topological space (X,τ) is called:*

(i) [2] Fuzzy generalized closed set if $Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fg-closed set.

(ii) [11] Fuzzy semi-generalized closed set if $sCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semi open. We briefly denote it as fsg-closed set.

(iii) [8] Fuzzy generalized almost strongly semi-closed set if $\alpha\ Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy α-open. We briefly denote it as fαg-closed set.

(iv) [23]Fuzzy semi-pre-generalized closed set if $spCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semi-pre-open. We briefly denote it as fspg-closed set.

(v) [3] Fuzzy generalized strongly closed set if $\alpha\ Cl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fgα-closed set.

(vi) [6] Fuzzy $\ddot{g}$ -closed set (briefly f$\ddot{g}$-closed set) if $cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fsg-open.

(vii) [7] Fuzzy αgs-closed set if $\alpha\ cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy semi-open. We briefly denote it as fαs-closed set.

(viii) [22] Fuzzy generalized semi-closed set if $sCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fgs-closed set.

(ix) [22] Fuzzy generalized semi-pre-closed set if $spCl\lambda \leq \mu$ whenever $\lambda \leq \mu$ and μ is fuzzy open. We briefly denote it as fgsp-closed set.

Definition 2.4 ([19])**.** *Let S be a fuzzy subset of a bitopological space X. Then S is said to be $\tau_{1,2}$-fuzzy open if $S = \lambda \cup \mu$, where $\lambda \in \tau_1$ and $\mu \in \tau_2$. The complement of $\tau_{1,2}$-fuzzy open set is called $\tau_{1,2}$-fuzzy-closed.*

Definition 2.5. *[19] Let S be a subset of a bitopological space X. Then*

(i) the $\tau_{1,2}$-closure of S, denoted by $\tau_{1,2}$-cl(S), is defined as $\cap\{F : S \subset F$ and F is $\tau_{1,2}$-closed$\}$.

(ii) the $\tau_{1,2}$-interior of S, denoted by $\tau_{1,2}$-int(S), is defined as $\cup\{F : F \subset S$ and F is $\tau_{1,2}$-open$\}$.

Definition 2.6. *A fuzzy subset λ of a bitopological space (X, τ_1, τ_2) is called.*

(i) (1,2)-sg-closed set[16] if (1,2)*-scl(λ) $\subset \mu$ whenever $\lambda \subset \mu$ and μ is (1,2)*- semi open in X. The complement of (1,2)*-sg-closed set is called (1,2)*-sg-open set;*

(ii) (1,2)-gs-closed set[16] if (1,2)*-scl (λ) $\subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-gs-closed set is called (1,2)*-gs-open set;*

(iii) *(1,2)*-αg-closed set[18] if (1,2)*-$\alpha cl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-αg-closed set is called (1,2)*-αg-open set;*

(iv) *(1,2)*-$\hat{g}$-closed set or fω-closed set [5] if $\tau_{1,2}$-$cl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is (1,2)*-semi open in X. The complement of-$\hat{g}$-closed (resp. ω-closed) set is called (1,2)*-$\hat{g}$-open (resp. ω-open)set;*

(v) *(1,2)*-ψ-closed set [13] if (1,2)*-$scl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is (1,2)*-sg-open in X. The complement of (1,2)*-ψ-closed set is called (1,2)*-ψ-open set;*

(vi) *(1,2)*-$\ddot{g}\alpha$-closed set [13] if (1,2)*-$\alpha cl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is (1,2)*-sg-open in X. The complement of (1,2)*-$\ddot{g}\alpha$-closed set is called (1,2)*–$\ddot{g}\alpha$-open set;*

(vii) *(1,2)*-gsp-closed set [21] if (1,2)*- spcl$(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-gsp-closed set is called (1,2)*-gsp-open set;*

(viii) *(1,2)*- g-closed set [17] if (1,2)*-$\tau_{1,2}$-$cl(\lambda) \subset \mu$ whenever $\lambda \subset \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-αg-closed set is called (1,2)*-αg-open set.*

Definition 2.7. *A map f : (X, τ_1,τ_2) $\rightarrow$ (Y, σ_1,σ_2) is called;*

(i) *(1,2)*-g_α-continuous[12] if $f^{-1}(V)$ is a (1,2)*-g_α-closed set of X for every $\sigma_{1,2}$-closed set V of Y.*

(ii) *(1,2)*-ψ-continuous [14] if $f^{-1}(V)$ is a (1,2)*-ψ-closed set of X for every $\sigma_{1,2}$-closed set V of Y.*

(iii) *(1,2)*-$\hat{g}$-continuous[5] if $f^{-1}(V)$ is a (1,2)*-$\hat{g}$-closed set of X for every $\sigma_{1,2}$-closed set V of Y.*

(iv) *(1,2)*-g-continuous [22] if $f^{-1}(V)$ is a (1,2)*-g-closed set of X for every $\sigma_{1,2}$-closed set V of Y.*

(v) *(1,2)*-αgs-continuous [20] if $f^{-1}(V)$ is an (1,2)*-αgs-closed set of X for every $\sigma_{1,2}$closed set V of Y.*

(vi) *(1,2)*-αg-continuous [10] if $f^{-1}(V)$ is an (1,2)*-αg-closed set of X for every $\sigma_{1,2}$-closed set V of Y.*

(vii) *(1,2)*-gs-continuous[15] if $f^{-1}(V)$ is a (1,2)*-gs-closed set of X for every $\sigma_{1,2}$-closed set V of Y.*

(viii) *(1,2)*-gsp-continuous[15] if $f^{-1}(V)$ is a (1,2)*-gsp-closed set of X for every $\sigma_{1,2}$-closed set V of Y.*

(ix) *(1,2)*-sg-continuous[16] if $f^{-1}(V)$ is a (1,2)*-sg-closed set of X for every $\sigma_{1,2}$-closed set V of Y.*

(x) *(1,2)*-semi-continuous[16] if $f^{-1}(V)$ is a (1,2)*-semi-open set of X for every $\sigma_{1,2}$-open set V of Y.*

(xi) *(1,2)*-α-continuous [19] if $f^{-1}(V)$ is an (1,2)*- α-closed set of X for every $\sigma_{1,2}$-closed set V of Y.*

(xii) *(1,2)*-continuous[21] if $f^{-1}(V)$ is a (1,2)*-closed set of X for every $\sigma_{1,2}$-closed set V of Y.*

3. (1,2)*-Fuzzy generalized closed set

Definition 3.1 ([24]). *A fuzzy subset λ of a fbts (X, τ_1, τ_2)called.*

(i) *(1,2)*-fsg-closed set if (1,2)*-$scl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is (1,2)*-fuzzy semi open in X. The complement of (1,2)*-fsg-closed set is called (1,2)*-fsg-open set;*

(ii) *(1,2)*-fgs-closed set if (1,2)*-scl $(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-fgs-closed set is called (1,2)*-fgs-open set;*

(iii) *(1,2)*-fαg-closed set if (1,2)*-$\alpha cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-fαg-closed set is called (1,2)*-fαg-open set;*

(iv) *(1,2)*-f$\hat{g}$-closed set or fω-closed set if $\tau_{1,2}$-cl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is (1,2)*-fs-open in X. The complement of-f$\hat{g}$-closed (resp. ω-closed) set is called (1,2)*-f$\hat{g}$-open (resp. ω-open)set;*

(v) *(1,2)*-fψ-closed set if (1,2)*-scl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is (1,2)*-fsg-open in X. The complement of (1,2)*-fψ-closed set is called (1,2)*-fψ-open set;*

(vi) *(1,2)*-f$\ddot{g}\alpha$-closed set if (1,2)*-$\alpha cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is (1,2)*-fsg-open in X. The complement of (1,2)*-f$\ddot{g}\alpha$-closed set is called (1,2)*-Fg_α-open set;*

(vii) *(1,2)*-fgsp-closed set if (1,2)*- spcl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $\tau_{1,2}$-open in X. The complement of (1,2)*-fgsp-closed set is called (1,2)*-fgsp-open set;*

(viii) *(1,2)*-fαgs-closed set if (1,2)*-$\alpha cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is (1,2)*-fs-open in X. The complement of (1,2)*-fαgs-closed set is called (1,2)*-fαgs-open set.*

Definition 3.2 ([24]). *A fuzzy subset λ of a fbts X is called (1,2)*-fg closed set if $\tau_{1,2}-cl(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is $\tau_{1,2}-$open subset in X . The complement of (1,2)*-fg-closed set is called (1,2)*-fg-open set.*

Definition 3.3 ([24]). *A fuzzy subset λ of a fbts X is called (1,2)*-f$\ddot{g}$-closed set if $\tau_{1,2}$-cl$(\lambda) \leq \mu$ whenever $\lambda \leq \mu$ and μ is (1,2)*-fsg-open in X. The complement of (1,2)*-f$\ddot{g}$-closed set is called (1,2)*-f$\ddot{g}$-open set.*

Proposition 3.4. *Every (1,2)*-fα -closed set is (1,2)*-f$\ddot{g}\alpha$-closed.*

Proof. If λ is a (1,2)*-f α closed and μ is any (1,2)*-fsg-open set Containing λ , we have $\mu \geq \tau_{1,2}$-cl$(\lambda) \geq \alpha$cl(λ). Hence λ is (1,2)*-f$\ddot{g}\alpha$-closed in X. ☐

Proposition 3.5. *Every (1,2)*-f$\ddot{g}\alpha$-closed is (1,2)*-fαgs-closed*

Proof. If λ is a (1,2)*-f$\ddot{g}\alpha$ -closed subset of X and μ is any (1,2)* -fs-open set containing λ, since every(1,2)*-fs-open set is (1,2)* -fsg-open,we have (1,2)*-α cl$(\lambda) \leq \mu$. Hence λ is (1,2)*-fαgs-closed in X. ☐

Proposition 3.6. *Every(1,2)*-fαgs-closed set is(1,2)*-fαg-closed.*

Proof. If λ is (1,2)*-f_α gs-closed subset of X and μ is any $\tau_{1,2}$-open set containing λ, since every $\tau_{1,2}$-open set is (1,2)*-fs-open,we have α cl$(\lambda) \leq \mu$. Hence λ is (1,2)*-fα g-closed in X. ☐

Proposition 3.7. *Every (1,2)*-fuzzy closed set is(1,2)*-f$\ddot{g}$-closed.*

Proof. If λ is a (1,2)*- fuzzy closed subset set of X and μ is any (1,2)*-fsg open set such that $\mu \geq \lambda = cl(\lambda)$. Hence λ is (1,2)*- f$\ddot{g}$- closed. ☐

Proposition 3.8. *Every (1,2)*-f$\ddot{g}$- closed set is(1,2)*-fψ-closed.*

Proof. If λ is a (1,2)*-f$\ddot{g}$-closed subset of X and μ is any (1,2)*-fs open set such that $\mu \geq \lambda$, since every (1,2)*-fs open set is(1,2)*- fsg open, we have $\mu \geq scl(\lambda)$. Hence λ is (1,2)*-fψ-closed . ☐

Proposition 3.9. *Every (1,2)*-f$\ddot{g}$ -closed set is (1,2)*-f$\ddot{g}\alpha$ closed.*

Proof. If λ is a (1,2)*-f$\ddot{g}$-closed subset of X and μ is any (1,2)*-fsg−open set such that $\lambda \leq \mu$, then $\mu \geq cl(\lambda) \geq \alpha$ cl(λ). Hence λ is (1,2)*- f$\ddot{g}\alpha$ closed. □

Proposition 3.10. *Every (1, 2)*-fω-closed set is (1,2)*-fg-closed.*

Proof. If λ is a (1,2)*- fω-closed subset of X and μ is any (1,2)*- fuzzy open set containing λ, then $\mu \geq \tau_{1,2}-$cl(λ). Hence λ is (1,2)*-fg-closed in X. □

Proposition 3.11. *Every (1,2)*-fψ-closed set is (1,2)*-fsg closed.*

Proof. If λ is a (1,2)*- fψ -closed subset of X and μ is any (1,2)*-fs-open set containing λ, we have (1,2)*-scl(λ) $\leq \mu$. Hence λ is (1,2)*-fsg-closed. □

Proposition 3.12. *Every (1,2)*-fg-closed set is (1,2)*-fgs-closed.*

Proof. If λ is a (1,2)*- fg-closed subset of X and μ is any $\tau_{1,2}$-open set containing λ, since every $\tau_{1,2}$-open set is (1,2)*-fsg-open, we have $\mu \geq \tau_{1,2}$-cl(λ) $\geq$ (1,2)*-scl(λ). Hence λ is (1,2)*-fgs-closed in X. □

Proposition 3.13. *Every (1,2)*-fg-closed set is (1,2)*-fαg-closed.*

Proof. If λ is a (1,2)*- fg-closed subset of X and μ is any $\tau_{1,2}$-open set containing λ. we have $\mu \geq \tau_{1,2}$-cl(λ) $\geq$ (1,2)*-αcl(λ). Hence λ is (1,2)*- fαg-closed in X. □

Proposition 3.14. *Every (1,2)*-fω-closed set is (1,2)*-fsg-closed.*

Proof. If λ is a (1,2)*- fω-closed subset of X and μ is any (1,2)*-fs open set containing λ, we have $\mu \geq \tau_{1,2}-$cl(λ) $\geq$ (1,2)*-scl(λ). Hence λ is (1,2)*- fsg-closed in X. □

Proposition 3.15. *Every (1,2)*-fuzzy sg-closed set is (1,2)*-fuzzy gs-closed.*

Proof. If λ is (1,2)*-fuzzy sg-closed subset of X and μ is any $\tau_{1,2}-$open set Containing λ, since every $\tau_{1,2}-$ open set is (1,2)*-fs open, we have scl(λ) $\leq \mu$. Hence λ is (1,2)*-fuzzy gs-closed. □

Proposition 3.16. *Every (1,2)*-fuzzyω- closed set is (1,2)*-fuzzy αgs-closed.*

Proof. If λ is (1,2)*-fuzzy ω-closed and μ is any (1,2)*-fuzzy semi open set containing λ, we have αcl(λ) $\leq \tau_{1,2}-$cl(λ) $\leq \mu$. Hence λ is (1,2)*-fuzzy αgs-closed. □

Proposition 3.17. *Every(1,2)*-fuzzy semi-pre-closed set is (1,2)*-fuzzy pre-semi-closed.*

Proof. If λ is a (1,2)*-fuzzy semi pre closed and μ is any (1,2)*-fg-open set such that $\lambda \leq \mu$, we have spCl(λ) $= \lambda$, it follows that spCl (λ) $= \lambda \leq \mu$. Hence λ is fuzzy pre-semi-closed in X. □

Proposition 3.18. *Every fuzzy pre-semi-closed set is fgsp-closed.*

Proof. If λ is a (1,2)*-fuzzy pre semi closed and μ is any (1,2)*-fg-open set such that $\lambda \leq \mu$, we have spCl (λ) $= \lambda \leq \mu$. Hence λ is fgsp-closed in X. □

4. (1,2)*- fuzzy g-continuous Maps and (1,2)*-fuzzy$\ddot{g}$-continuous Maps.

In this section we introduce the following definitions.

Definition 4.1. *A map $f : (X, \tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ is called;*

(i) (1,2)-fuzzy$\ddot{g}\alpha$ continuous if $f^{-1}(V)$ is a (1,2)*- fuzzy$\ddot{g}\alpha$ closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

(ii) (1,2)-fuzzy ψ-continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy ψ-closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

(iii) (1,2)-fuzzy $\hat{g}$-continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy $\hat{g}$-closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

(iv) (1,2)-fuzzy αgs-continuous if $f^{-1}(V)$ is an (1,2)*- fuzzy αgs-closed set of X for every $\sigma_{1,2}$ -fuzzy closed set V of Y.*

(v) (1,2)-fuzzy αg-continuous if $f^{-1}(V)$ is an (1,2)*-fuzzy αg-closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

(vi) (1,2) fuzzy -gs-continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy gs-closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

(vii) (1,2)-fuzzy gsp-continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy gsp-closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

(viii) (1,2)-fuzzy sg-continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy sg-closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

(ix) (1,2)-fuzzy semi-continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy semi-open set of X for every $\sigma_{1,2}$-fuzzy open set V of Y.*

(x) (1,2)-fuzzy α-continuous if $f^{-1}(V)$ is an (1,2)*-fuzzy α-closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

(xi) (1,2)-fuzzy continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

(xii) (1,2)-fuzzy pre semi continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy pre semi closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

(xiii) (1,2)-fuzzy semi pre continuous if $f^{-1}(V)$ is a (1,2)*-fuzzy semi pre closed set of X for every $\sigma_{1,2}$-fuzzy closed set V of Y.*

Definition 4.2. *A map f : (X,τ_1,τ_2) $\rightarrow$ (Y,σ_1,σ_2) is called (1,2)*-fuzzy g continuous if every $f^{-1}(\lambda)$ is (1,2)* -fuzzy g -closed in (X,τ_1,τ_2) for every $\sigma_{1,2}$ closed set λ of (Y,σ_1,σ_2).*

Definition 4.3. *A map f : (X,τ_1,τ_2) $\rightarrow$ (Y,σ_1,σ_2) is called (1,2)*-fuzzy g- irresolute if every $f^{-1}(\lambda)$ is (1,2)* -fuzzy g -closed in (X,τ_1,τ_2) for every (1,2)*-fuzzy g-closed set λ of (Y,σ_1,σ_2).*

Definition 4.4. *A map f : (X,τ_1,τ_2) $\rightarrow$ (Y,σ_1,σ_2) is called (1,2)*-fuzzy $\ddot{g}$ continuous if every $f^{-1}(\lambda)$ is (1,2)* -fuzzy$\ddot{g}$ closed in (X,τ_1,τ_2) for every $\sigma_{1,2}$ closed set λ of (Y,σ_1,σ_2).*

Proposition 4.5. *Every -fuzzy continuous is (1,2)*-fuzzy g-continuous but not conversely.*

Proof. It is obvious. The converse of the proposition 4.5 is not true as seen from the following example. □

Example 4.6. *Let $X = Y = \{a, b, c\}$, $\tau_1 = \{0, 1, \lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{0.8}{c}\}$, $\tau_2 = \{0, 1, \mu = \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.7}{c}\}$. $\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let $f : (X,\tau_1,\tau_2) \rightarrow (Y,\sigma_1,\sigma_2)$ be the identity map. Then f is (1,2)*-fuzzy g-continuous but not (1,2)*-fuzzy continuous, since $f^{-1}(\{\gamma = \frac{0.8}{a} + \frac{0.8}{b} + \frac{0.8}{c}\}) = \{\gamma\}$ is not $\tau_{1,2}-$closed in X.*

Proposition 4.7. *Every (1,2)*-fuzzy α-continuous map is (1,2)*-fuzzy $\ddot{g}\alpha$-continuous but not conversely.*

Proof. The proof follows from Proposition 3.4. The converse of the proposition 4.7 is not true as seen from the following example. □

Example 4.8. *Let $X = Y = \{a,b\}$, $\tau_1 = \left\{0,1,\mu = \frac{0.3}{a} + \frac{0.4}{b}\right\}, \tau_2 = \{0,1\}, \sigma_1 = F(x)$ and $\sigma_2 = \tau_2$ Let f : $(X, \tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be the identity map. Then f is (1,2)*-fuzzy $\ddot{g}\alpha$-continuous but not (1,2)*-fuzzy α-continuous, since $f^{-1}\left(\{\gamma = \frac{0.4}{a} + \frac{0.5}{b}\}\right) = \{\gamma\}$ is not (1,2)*-fα-closed in X.*

Proposition 4.9. *Every (1,2)*-fuzzy $\ddot{g}\alpha$-continuous map is (1,2)*- fuzzy αgs-continuous but not conversely.*

Proof. The proof follows from Proposition 3.5. The converse of the proposition 4.9 is not true as seen from the following example. □

Example 4.10. *Let X=Y=$\{a,\ b\}$, $\tau_1 = \left\{0,1,\mu = \frac{0.4}{a} + \frac{0.6}{b}\right\}$,$\tau_2 = \{0,1\}$,$\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let f : $(X, \tau_1,\tau_2) \to (Y, \sigma_1,\sigma_2)$ be the identity map. Then f is (1,2)*-fuzzy αgs-continuous but not (1,2)*-fuzzy $\ddot{g}\alpha$-continuous, since $f^{-1}(\{\gamma = \frac{0.4}{a} + \frac{0.5}{b}\}) = \{\gamma\}$is not (1,2)*-f$\ddot{g}\alpha$-closed in X.*

Proposition 4.11. *Every (1,2)*- fuzzy g-continuous map is (1,2)*- fuzzy gs-continuous but not conversely.*

Proof. The proof follows from Proposition 3.12. The converse of the proposition 4.11 is not true as seen from the following example. □

Example 4.12. *Let $X = Y = \{a,b\}$, $\tau_1 = \left\{0,1,\mu = \frac{0.4}{a} + \frac{0.6}{b}\right\}, \tau_2 \{0,1\}, \sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let f : $(X, \tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be the identity map. Then f is (1,2)*-fuzzy-gs continuous but not (1,2)*-fuzzy g-continuous, since $f^{-1}(\{\gamma = \frac{0.4}{a} + \frac{0.4}{b}\}) = \{\gamma\}$ is not (1,2)*-fuzzy g-closed in X.*

Proposition 4.13. *Every (1,2)*-fuzzy αgs-continuous is (1,2)*- fuzzy αg-continuous but not conversely.*

Proof. The proof follows from Proposition 3.6. The converse of the proposition 4.13 is not true as seen from the following example. □

Example 4.14. *Let $X = Y = \{a,b\}$, $\tau_1 = \left\{0,1,\mu = \frac{0.3}{a} + \frac{0.6}{b}\right\}; \tau_2 \{0,1\}$,$\sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let f : $(X, \tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be the identity map. Then f is (1,2)*-fuzzy αg-continuous but not (1,2)*-fuzzy αgs-continuous, since $f^{-1}(\{\lambda = \frac{0.4}{a} + \frac{0.5}{b}\}) = \{\lambda\}$ is not (1,2)*-fuzzy αgs-closed in X.*

Proposition 4.15. *Every (1,2)*- fuzzy continuous is (1,2)*- fuzzy$\ddot{g}$-continuous but not conversely.*

Proof. The proof follows from proposition 3.7. The converse of the proposition 4.15 is not true as seen from the following example. □

Example 4.16. *Let $X = Y = \{a,b,c\}\tau_1 = \left\{0,1,\lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{0.8}{c}\right\}, \tau_2 = \left\{0,1,\mu = \frac{0.3}{a} + \frac{0.5}{b} + \frac{0.7}{c}\right\} \sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let f : $(X, \tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be the identity map. Then f is (1,2)*-fuzzy$\ddot{g}$continuous but not (1,2)*-fuzzy continuous, since $f^{-1}(\{\lambda = \frac{0.8}{a} + \frac{0.9}{b} + \frac{0.8}{c}\}) = \{\lambda\}$ is not (1,2)*-fuzzy closed set in X.*

Proposition 4.17. *Every (1,2)*- fuzzy $\ddot{g}$- continuous is (1,2)*- fuzzy ψ-continuous but not conversely.*

Proof. The proof follows from proposition 3.8. The converse of the proposition 4.17 is not true as seen from the following example. □

Example 4.18. *Let X=Y=$\{\ a,\ b\}$,$\tau_1 = \left\{0,1,\mu = \frac{0.4}{a} + \frac{0.7}{b}\right\}; \tau_2 \{0,1\}, \sigma_1 = F(x)$ and $\sigma_2 = \tau_2$. Let f : $(X, \tau_1,\tau_2) \to (Y, \sigma_1,\sigma_2)$ be the identity map. Then f is (1, 2)*-fuzzy ψ-continuous but not (1,2)*-fuzzy $\ddot{g}$-continuous, since $f^{-1}(\{\ \lambda = \frac{0.4}{a} + \frac{0.3}{b}\}) = \{\ \lambda\}$ is not (1,2)*-fuzzy $\ddot{g}$-closed set in X.*

Proposition 4.19. *Every (1,2)*- fuzzy ψ-continuous is (1,2)*- fuzzy sg continuous but not conversely.*

Proof. The proof follows from proposition 3.11. The converse of the proposition 4.19 is not true as seen from the following example. □

Example 4.20. *Let X=Y={ a,b}* $\tau_1 = \left\{0, 1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\right\}$; $\tau_2\,\{0,1\}$ $\sigma_1 = F\,(x)$ *and* $\sigma_2 = \tau_2$. *Let f : (X,* τ_1,τ_2) $\rightarrow$ *(Y,* σ_1,σ_2) *be the identity map. Then f is (1,2)*-fuzzy sg-continuous but not (1,2)*-fuzzy* ψ*-continuous, since* f^{-1}({ $\lambda = \frac{0.4}{a} + \frac{0.3}{b}$}) = { λ} *is not (1,2)*-fuzzy* ψ *closed in X*

Proposition 4.21. *Every (1, 2)*-fuzzy ω-continuous is (1,2)*- fuzzy g-continuous but not conversely.*

Proof. The proof follows from proposition 3.10. □

The converse of the proposition 4.21 is not true as seen from the following example.

Example 4.22. *Let X=Y={ a, b}* $\tau_1 = \left\{0, 1, \mu = \frac{0.5}{a} + \frac{0.5}{b}\right\}$; $\tau_2\,\{0,1\}$ $\sigma_1 = F\,(x)$ *and* $\sigma_2 = \tau_2$ *.Let f : (X,* τ_1,τ_2) $\rightarrow$ *(Y,* σ_1,σ_2) *be the identity map. Then f is (1,2)*-fuzzy -g continuous but not (1,2)*-fuzzy -*ω*continuous, since* f^{-1}({ $\lambda = \frac{0.4}{a} + \frac{0.4}{b}$}) = { λ} *is not (1,2)*-fuzzy* ω*closed in X.*

Proposition 4.23. *Every (1,2)*-fuzzy $\ddot{g}$-continuous is (1,2)*- fuzzy $\ddot{g}\alpha$-continuous but not conversely.*

Proof. The proof follows from proposition 3.9 □

. The converse of the proposition 4.23 is not true as seen from the following example.

Example 4.24. *Let X=Y={ a,b} ,* $\tau_1 = \left\{0, 1\mu = \frac{0.4}{a} + \frac{0.7}{b}\right\}$;$\tau_2 = \{0,1\}$, $\sigma_1 = F(x)$ *and* $\sigma_2 = \tau_2$. *Let f : (X,* τ_1,τ_2) $\rightarrow$ *(Y,* σ_1,σ_2) *be the identity map. Then f is (1,2)*-fuzzy* $\ddot{g}\alpha$*-continuous but not (1,2)*-fuzzy* $\ddot{g}$*-continuous, since* $f^{-1}\left\{\lambda = \frac{0.4}{a} + \frac{0.3}{b}\right\} = \{\lambda\}$ *is not (1,2)*-fuzzy* $\ddot{g}$*-closed.*

Proposition 4.25. *Every (1,2)*-fuzzy g-continuous is(1,2)*-fuzzy αg-continuous but not conversely.*

Proof. The proof follows from Proposition 3.13. The converse of the proposition 4.25 is not true as seen from the following example. □

Example 4.26. *Let X=Y={ a,b} and* $\tau_1 = \left\{0, 1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\right\}$;$\tau_2 = F(x)$*and* $\sigma_2 = \tau_2$. *Let f : (X,* τ_1,τ_2) $\rightarrow$ *(Y,* σ_1,σ_2) *be the identity map. Then f is (1,2)*-fuzzy* α*g-continuous but not fuzzy (1,2)*-fuzzy g -continuous, since* $f^{-1}\left\{\lambda = \frac{0.4}{a} + \frac{0.3}{b}\right\} = \{\lambda\}$ *is not (1,2)*-fuzzy g –closed in X*

Proposition 4.27. *Every (1,2)*-fuzzy ω-continuous is(1,2)*-fuzzy sg-continuous but not conversely.*

Proof. The proof follows from Proposition 3.14. The converse of the proposition 4.27 is not true as seen from the following example. □

Example 4.28. *Let X=Y={ a,b} and* $\tau_1 = \left\{0, 1, \mu = \frac{0.4}{a} + \frac{0.6}{b}\right\}$;$\tau_2 = F(x)$*and* $\sigma_2 = \tau_2$. *Let f : (X,* τ_1,τ_2) $\rightarrow$ *(Y,* σ_1,σ_2) *be the identity map. Then f is (1,2)*-fuzzy sg-continuous but not fuzzy (1,2)*-fuzzy* ω *-continuous, since* $f^{-1}\left\{\lambda = \frac{0.4}{a} + \frac{0.4}{b}\right\} = \{\lambda\}$ *is not (1,2)*-fuzzy* ω *-closed X.*

Proposition 4.29. *Every (1,2)*-fuzzy sg-continuous is(1,2)*-fuzzy gs-continuous but not conversely.*

Proof. The proof follows from Proposition 3.15. The converse of the proposition 4.29 is not true as seen from the following example. □

Example 4.30. *Let X=Y={ a,b} and* $\tau_1 = \{0, 1, \mu = \frac{0.3}{a} + \frac{0.6}{b}\}$;$\tau_2 = F(x)$*and* $\sigma_2 = \tau_2$. *Let f : (X,* τ_1,τ_2*)* $\to$ *(Y,* σ_1,σ_2*) be the identity map. Then f is (1,2)*-fuzzy gs-continuous but not fuzzy (1,2)*-fuzzy sg -continuous, since* $f^{-1}\{\lambda = \frac{0.3}{a} + \frac{0.4}{b}\} = \{\lambda\}$ *is not (1,2)*-fuzzy sg -closed X.*

Proposition 4.31. *Every (1,2)*-fuzzy* ω*-continuous is(1,2)*-fuzzy* α*gs-continuous but not conversely.*

Proof. The proof follows from Proposition 3.16. The converse of the proposition 4.31 is not true as seen from the following example. □

Example 4.32. *Let X=Y={ a,b} and* $\tau_1 = \{0, 1, \mu = \frac{0.4}{a} + \frac{0.7}{b}\}$;$\tau_2 = F(x)$*and* $\sigma_2 = \tau_2$.
Let f : (X, τ_1,τ_2*)* $\to$ *(Y,* σ_1,σ_2*) be the identity map. Then f is (1,2)*-fuzzy* α*gs-continuous but not fuzzy (1,2)*-fuzzy* ω *-continuous, since* $f^{-1}\{\lambda = \frac{0.4}{a} + \frac{0.3}{b}\} = \{\lambda\}$ *is not (1,2)*-fuzzy* ω*-closed X.*

Proposition 4.33. *Every (1,2)*-fuzzy semi pre continuous is(1,2)*-fuzzy pre semi continuous but not conversely.*

Proof. The proof follows from Proposition 3.17. □

The converse of the proposition 4.33 is not true as seen from the following example.

Example 4.34. *Let X=Y= { a, b ,c} ,* $\tau_1 = \{0, 1, \lambda = \frac{0.7}{a} + \frac{0.3}{b} + \frac{1}{c}\}$, $\tau_2 = \{0, 1, \mu = \frac{0.7}{a} + \frac{0}{b} + \frac{0}{c}\}$. $\sigma_1 = F(x)$ *and* $\sigma_2 = \tau_2$. *Let f : (X,* τ_1,τ_2*)* $\to$ *(Y,* σ_1,σ_2*) be the identity map. Then f is (1,2)*-fuzzy pre semi continuous but not (1,2)*-fuzzy semi pre continuous, since* $f^{-1}(\{\ \gamma = \frac{1}{a} + \frac{0.3}{b} + \frac{0}{c}\}\ ) = \{\ \gamma\}$ *is not fuzzy semi pre closed in X.*

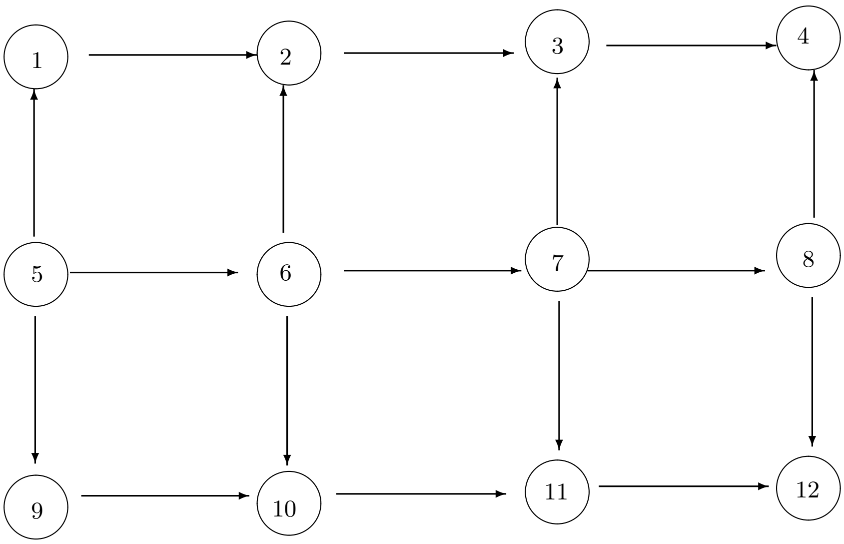
Proposition 4.35. *Every (1,2)*-fuzzy pre semi continuous is fgsp continuous but not conversely.*

Proof. The proof follows from Proposition 3.18. □

The converse of the proposition 4.35 is not true as seen from the following example.

Example 4.36. *Let X=Y= { a, b ,c} ,* $\tau_1 = \{0, 1, \lambda = \frac{1}{a} + \frac{0}{b} + \frac{0}{c}\}$, $\tau_2 = \{0, 1\}$. $\sigma_1 = F(x)$ *and* $\sigma_2 = \tau_2$. *Let f : (X,* τ_1,τ_2*)* $\to$ *(Y,* σ_1,σ_2*) be the identity map. Then f is (1,2)*-fgsp continuous but not (1,2)*-fuzzy pre semi continuous, since* $f^{-1}(\{\gamma = \frac{1}{a} + \frac{1}{b} + \frac{0}{c}\}) = \{\gamma\}$ *is not fuzzy pre semi closed in X.*

Remark 4.37. *From the above Examples 4.6, 4.8, 4.10, 4.12, 4.14, 4.16, 4.18, 4.20, 4.22, 4.24, 4.26, 4.28, 4.30, 4.32, 4.34, 4.36 and propositions4.5, 4.7, 4.9, 4.11, 4.13, 4.15, 4.17, 4.19, 4.21, 4.23, 4.25, 4.27, 4.29, 4.31, 4.33, 4.35 we obtain the following diagram, where A* $\to$ *B (resp. A* $\nleftrightarrow$ *B) represents A implies B but not conversely (resp. A and B are independent of each other)*



Where

(1) (1,2)*-fα-continuous
(2) (1,2)*- f$\ddot{g}\alpha$-continuous
(3) (1,2)*- fαgs-continuous
(4) (1,2)*- fαg-continuous
(5) (1,2)*- fuzzy -continuous
(6) (1,2)*-f$\ddot{g}$-continuous
(7) (1,2)*-fω- continuous
(8) (1,2)*-f g-continuous
(9) (1,2)*-fs-continuous
(10) (1,2)*-fψ-continuous
(11) (1,2)*-fsg-continuous
(12) (1,2)*-fgs-continuous

Definition 4.38. *For every set $A \leq X$, we define the (1,2)*-g-closure of A to be the intersection of all (1,2)*-fg-closed sets containing A. In symbols, (1,2)*-g-cl(A) $= \cap$ { $F : A \leq F \in$ (1,2)*-FGC(X)} .*

Proposition 4.39. *A map $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*-fg-continuous if and only if $f^{-1}(U)$ is (1,2)*-fg-open in X for every $\sigma_{1,2}$-open set U in Y.*

Proof. Let f : (X, τ_1, τ_2) $\to$ (Y, σ_1, σ_2) be (1,2)*-fg-continuous and U be an $\sigma_{1,2}$-open set in Y. Then U^c is $\sigma_{1,2}$-closed in Y and since f is (1,2)*-fg-continuous, f^{-1}(U^c) is (1,2)*-fg-closed in X. But f^{-1}(U^c) = (f^{-1}(U))^c and so f^{-1}(U) is (1,2)*-fg-open in X.

Conversely, assume that f^{-1}(U) is (1,2)*-fg-open in X for each $\sigma_{1,2}$-open set U in Y. Let F be a $\sigma_{1,2}$-closed set in Y. Then F^c is $\sigma_{1,2}$-open in Y and by assumption, f^{-1}(F^c) is (1,2)*-fg-open in X. Since $f^{-1}(F^c) = (f^{-1}(F))^c$, we have $f^{-1}(F)$ is (1,2)*-fg-closed in X and so f is (1,2)*-fg-continuous. □

Remark 4.40. *The composition of two (1,2)*-fg-continuous maps need not be (1,2)*-fg-continuous and this is shown by the following example.*

Example 4.41. *Let $X = Y = Z$={ a, b} ,$\tau_1 = 0, 1, \lambda = \frac{0.4}{a} + \frac{0.6}{b}, \mu = \frac{0.2}{a} + \frac{0.3}{b}$,$\tau_2 = 0, 1.$ Then τ_{12}-open sets are $0, 1, \lambda = \frac{0.4}{a} + \frac{0.6}{b}, \mu = \frac{0.2}{a} + \frac{0.3}{b}$ and*
$\tau_{12}-$closed sets are $0, 1, \lambda' = \frac{0.6}{a} + \frac{0.4}{b}, \mu' = \frac{0.8}{a} + \frac{0.7}{b}$.
Let $\sigma_1 = 0, 1, \eta = \frac{0.4}{a} + \frac{0.5}{b}$ .Thenσ_{12}-open sets are$0, 1, \eta = \frac{0.4}{a} + \frac{0.5}{b}$
and σ_{12}-closed sets are $0, 1, \eta' = \frac{0.6}{a} + \frac{0.5}{b}$. Let $\gamma_1 = 0, 1, \phi = \frac{0.7}{a} + \frac{0.4}{b}$,
$\gamma_2 = 0, 1.$Then $\gamma_{12}-$open sets are $0, 1, \phi = \frac{0.7}{a} + \frac{0.4}{b}$and γ_{12}-closed sets are$0, 1, \phi' = \frac{0.3}{a} + \frac{0.6}{b}$. Let $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$
be the identity maps. Then f and g are (1,2)-fg-continuous but their*
g o f : $(X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is not (1,2)-fg-continuous, because*
$V = \frac{0.3}{a} + \frac{0.6}{b}$ is $\eta_{1,2}$-closed in Z but (g o f)$^{-1}$(V) =$f^{-1}(g^{-1}(V)) = f^{-1}(V) =$ V= $\frac{0.3}{a} + \frac{0.6}{b}$ which is not (1,2)-fg-closed in X.*

Proposition 4.42. *If $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*-fg-continuous and $g : (Y, \sigma_1, \sigma_2) \to (Z, \eta_1, \eta_2)$ is (1,2)*-continuous, then their composition g o f : $(X, \tau_1, \tau_2) \to (Z, \eta_1, \eta_2)$ is (1,2)*-fg-continuous.*

Proof. Let F be any $\eta_{1,2}$-closed set in (Z, η_1, η_2). Since g : (Y, σ_1, σ_2) $\to$ (Z, η_1, η_2) is (1,2)*-continuous, g^{-1}(F) is $\sigma_{1,2}$-closed in (Y, σ_1, σ_2). Since f : (X,τ_1, τ_2) $\to$ (Y, σ_1, σ_2) is (1,2)*-fg-continuous, f^{-1}(g^{-1}(F)) = (g o f)$^{-1}$(F) is (1,2)*-fg-closed in X and so g o f is (1,2)*-fg-continuous. □

Proposition 4.43. *If A is (1,2)*-fg-closed in X and if $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is (1,2)*-fsg-irresolute and (1,2)*-fuzzy closed, then f(A) is (1,2)*-fg-closed in Y.*

Proof. Let U be any (1,2)*-fsg-open in Y such that f(A) $\leq$ U. Then A $\leq$ f^{-1}(U) and by hypothesis, $\tau_{1,2}$-cl(A) $\leq$ f^{-1}(U). Thus f($\tau_{1,2}$-cl(A)) $\leq$ U and f($\tau_{1,2}$-cl(A)) is a $\sigma_{1,2}$-closed set. Now, $\sigma_{1,2}$-cl(f(A)) $\leq$ $\sigma_{1,2}$-cl(f($\tau_{1,2}$-cl(A)))= f($\tau_{1,2}$-cl(A)) $\leq$ U. That is $\sigma_{1,2}$-cl(f(A))$\leq$ U and so f(A) is (1,2)*-fg-closed. ☐

Definition 4.44. *Let x be a point of X and A be a subset of a fuzzy bitopological space X. Then A is called an (1,2)*-fg-neighbourhood of x (briefly, (1,2)*-fg-nbhd of x) in X if there exists an (1,2)*-fg-open set U of X such that $x \in U \leq A$.*

Definition 4.45. *Let A be a subset of a fuzzy bitopological space X. Then $x \in$ (1,2)*-g-cl(A) if and only if for any (1,2)*-fg–nbhd G_x of x in X, $A \cap G_x \neq \varphi$.*

Proof. Necessity. Assume x $\in$ (1,2)*-g-cl(A). Suppose that there is an (1,2)*-fg-nbhd G of the point x in X such that G $\cap$ A $= \varphi$. Since G is (1,2)*-fg-nbhd of x in X, by Definition 4.44, there exists an (1,2)*-fg-open set U_x such that x $\in U_x \leq$ G. Therefore, we have $U_x \cap$ A $= \varphi$ and so A $\leq (U_x)^c$. Since $(U_x)^c$ is an (1,2)*-fg-closed set containing A, we have by Definition 4.38 (1,2)*-g-cl(A) $\leq (U_x)^c$ and therefore x $\notin$ (1,2)*-g-cl(A), which is a contradiction.
Sufficiency. Assume for each (1,2)*-fg-nbhd G_x of x in X, A $\cap$ $G_x \neq \varphi$. Suppose x $\notin$ (1,2)*-g-cl(A). Then by Definition 4.38, there exists a (1,2)*-fg-closed set F of X such that A $\leq$ F and x $\notin$ F. Thus x $\in F^c$ and F^c is (1,2)*-fg-open in X and hence F^c is a (1,2)*-fg-nbhd of x in X. But A $\cap F^c = \varphi$, which is a contradiction. ☐

In the next theorem we explore certain characterizations of (1,2)*-fg-continuous functions.

Theorem 4.46. *Suppose the collection of all (1,2)*-fg-open sets of X is closed under arbitrary unions. Let f : $(X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ be a map from a fuzzy bitopological space (X, τ_1, τ_2) into a fuzzy bitopological space (Y, σ_1, σ_2). Then the following statements are equivalent.*

(i) The function f is (1,2)-fg-continuous.*

(ii) The inverse of each $\sigma_{1,2}$-open set is (1,2)-fg-open.*

(iii) For each point x in X and each $\sigma_{1,2}$-open set V in Y with f(x) $\in$ V, there is an (1,2)-fg-open set U in X such that x $\in$ U, f(U) $\leq$ V.*

(iv) The inverse of each $\sigma_{1,2}$-closed set is (1,2)-fg-closed.*

(v) For each x in X, the inverse of every neighborhood of f(x) is an (1,2)-fg-nbhd of x.*

(vi) For each x in X and each neighborhood N of f(x), there is an (1,2)-fg-nbhd G of x such that f(G) $\leq$ N.*

(vii) For each subset A of X, f((1,2)-g-cl(A)) $\leq$ $\sigma_{1,2}$-cl(f(A)).*

(viii) For each subset B of Y, (1,2)-g-cl(f^{-1}(B)) $\leq$ f^{-1}($\sigma_{1,2}$-cl(B)).*

Proof. (i) $\Leftrightarrow$ (ii). This follows from Definition 4.2

(i) $\Leftrightarrow$ (iii). Suppose that (iii) holds and let V be an $\sigma_{1,2}$-open set in Y and let x $\in$ f^{-1}(V). Then f(x) $\in$ V and thus there exists an (1,2)*-fg-open set U_x such that x $\in U_x$ and f(U_x) $\leq$ V. Now, x $\in U_x \leq$ f^{-1}(V) and f^{-1}(V) $= \cup_{x \in f-1(V)} U_x$. Then f^{-1}(V) is (1,2)*-fg-open in X and therefore f is (1,2)*-fg-continuous.

Conversely, suppose that (i) holds and let f(x) $\in$ V where V is $\sigma_{1,2}$-open in Y. Then f^{-1}(V) $\in$ (1,2)*-GO(X), since f is (1,2)*-fg-continuous. Let U = f^{-1}(V). Then x $\in$ U and f(U) $\leq$ V.

(ii) $\Leftrightarrow$ (iv). This result follows from the fact if A is a subset of Y, then $f^{-1}(A^c) = (f^{-1}(A))^c$.

(ii) $\Rightarrow$ (v). For x in X, let N be a neighborhood of f(x). Then there exists an $\sigma_{1,2}$-open set U in Y such that $f(x) \in U \leq N$. Consequently, $f^{-1}(U)$ is an (1,2)*-fg-open set in X and $x \in f^{-1}(U) \leq f^{-1}(N)$. Thus $f^{-1}(N)$ is an (1,2)*-fg-nbhd of x.

(v) $\Rightarrow$ (vi). Let $x \in X$ and let N be a neighborhood of f(x). Then by assumption, $G = f^{-1}(N)$ is an (1,2)*-fg-nbhd of x and $f(G) = f(f^{-1}(N)) \leq N$.

(vi) $\Rightarrow$ (iii). For x in X, let V be an $\sigma_{1,2}$-open set containing f(x). Then V is a neighborhood of f(x). So by assumption, there exists an (1,2)*-fg-nbhd G of x such that $f(G) \leq V$. Hence there exists an (1,2)*-fg-open set U in X such that $x \in U \leq G$ and so $f(U) \leq f(G) \leq V$.

(vii) $\Leftrightarrow$ (iv). Suppose that (iv) holds and let A be a subset of X. Since $A \leq f^{-1}(f(A))$, we have $A \leq f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$. Since $\sigma_{1,2}$-cl(f(A)) is a $\sigma_{1,2}$-closed set in Y, by assumption $f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$ is an (1,2)*-fg-closed set containing A. Consequently, $(1,2)^*\text{-g-cl}(A) \leq f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$. Thus $f((1,2)^*\text{-g-cl}(A)) \leq f(f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))) \leq \sigma_{1,2}\text{-cl}(f(A))$.

Conversely, suppose that (vii) holds for any subset A of X. Let F be a $\sigma_{1,2}$-closed subset of Y. Then by assumption, $f((1,2)^*\text{-g-cl}(f^{-1}(F))) \leq \sigma_{1,2}\text{-cl}(f(f^{-1}(F))) \leq \sigma_{1,2}\text{-cl}(F) = F$. That is $(1,2)^*\text{-g-cl}(f^{-1}(F)) \leq f^{-1}(F)$ and so $f^{-1}(F)$ is (1,2)*-fg-closed.

(vii) $\Leftrightarrow$ (viii). Suppose that (vii) holds and B be any subset of Y. Then replacing A by $f^{-1}(B)$ in (vii), we obtain $f((1,2)^*\text{-g-cl}(f^{-1}(B))) \leq \sigma_{1,2}\text{-cl}(f(f^{-1}(B))) \leq \sigma_{1,2}\text{-cl}(B)$. That is $(1,2)^*\text{-g-cl}(f^{-1}(B)) \leq f^{-1}(\sigma_{1,2}\text{-cl}(B))$.

Conversely, suppose that (viii) holds. Let $B = f(A)$ where A is a subset of X. Then we have, $(1,2)^*\text{-g-cl}(A) \leq (1,2)^*\text{-g-cl}(f^{-1}(B)) \leq f^{-1}(\sigma_{1,2}\text{-cl}(f(A)))$ and so $f((1,2)^*\text{-g-cl}(A)) \leq \sigma_{1,2}\text{-cl}(f(A))$. This completes the proof of the theorem.

□

Acknowledgment

The authors would like to thank the reviewers for their valuable comments and helpful suggestions for improvement of the original manuscript.

References

[1] K.K.Azad, *On fuzzy semicontinuity, fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. Appl., 82(1)(1981), 14-32.

[2] G.Balasubramanian and P.Sundaram, *On some generalizations of fuzzy discontinuous functions*, Fuzzy Sets and Systems, 86(1)(1997), 93-100.

[3] O.Bedre Ozbakir, *On generalized fuzzy strongly semi closed sets in fuzzy topological spaces*, Int. J. Math. Math. Sci., 30(11)(2002), 651-657.

[4] A.S.Bin Shahna, *On fuzzy strong semicontinuity and fuzzy precontinuity*, Fuzzy Sets and Systems, 44(2)(1991), 303-308.

[5] Z.Duszynski, M.Joseph Israel and O.Ravi, *A new generalization of closed sets in bitopology*, International Mathematical Forum (Toappear).

[6] M.Jeyaraman, S.Vijayalakshmi and O.Ravi, *New Notion Of Closed Sets In Fuzzy Topology*, International Journal of Mathematical Archive, 4(8)(2013), 67-73.

[7] M.Jeyaraman, J.Rajalakshmi and O.Ravi, *Another Generalization Of Closed Sets In Fuzzy Topological Spaces*, International Journal of Mathematical Archive, 4(8)(2013), 187-192.

[8] Kandil, *Biproximities and fuzzy bitopologicalspaces*, Simon Stevin, 63(1989), 45-66.

[9] N.Levine, *Generalized closed sets in topology*, Rend. Circ. Math.Palermo, 19(2)(1970), 89-96.

[10] M.Lellis Thivagar, O.Ravi and M.E.Abd El-Monsef, *Remarks on bitopological (1,2)*-quotient mappings*, J. Egypt Math. Soc., 16(1) (2008), 17-25.

[11] H.Maki et al., *Generalized closed sets in fuzzy topological space*, Meetings on Topological Spaces Theory and its Applications (1998), 23-36.

[12] O.Ravi, J.Antony Rex Rodrigo, S.Ganesan and A.Kumaradhas, *On-continuous functions in topological spaces (submitted)*.

[13] O.Ravi and S.Ganesan, *(1,2)*-$\ddot{g}\alpha$-closed sets in bitopology (submitted)*.

[14] O.Ravi, M.Kamaraj, A.Pandi and K.Kumaresan, *(1,2)*- -closed and (1,2)*- -open maps in bitopological spaces*, International Journal of Mathematical Archive, 3(2)(2012), 586-594.

[15] O.Ravi, S.Pious Missier and T.Salai Parkunan, *On bitopological (1,2)*-generalized homeomorphisms*, Int J. Contemp. Math. Sciences, 5(11)(2010), 543-557.

[16] O.Ravi and M.L.Thivagar, *A bitopological (1,2)*-semi-generalized continuous maps*, Bull.Malaysian Math. Sci. Soc., 2(29)(1)(2006), 76-88.

[17] O.Ravi, M.L.Thivagar and Jinjinli, *Remarks on extensions of (1,2)*-g-closed maps*, Archimedes J. Math., 1(2)(2011), 177-187.

[18] O.Ravi, M.L.Thivagar and A.Nagarajan, *(1,2)*-αg-closed sets and (1,2)*-gα-closed sets* (submitted).

[19] O.Ravi and M.Lellis Thivagar, *On stronger forms of (1,2)*-quotient mappings in bitopological spaces*, Internat. J. Math. Game Theory and Algebra, 14(6)(2004), 481-492.

[20] O.Ravi, C.M.Santhana, A.Pandi and J.Antony Rex Rodrigo, *On (1,2)*- -closed sets in bitopological spaces (submitted)*.

[21] O.Ravi and M.L.Thivagar, *Remarks on λ-irresolute functions via (1,2)*-sets*, Advances in App. Math. Analysis, 5(1)(2010), 1-15.

[22] R.K.Saraf and M.Khana, *Fuzzy generalized semi preclosed sets*, J. Tripura Math. Soc., 3(2001), 59-68.

[23] R.K.Saraf, G.Navalagi and Meena Khanna, *On fuzzy semi-pre-generalized closed sets*, Bull. Mal. Mat. Sci. Soc., 28(1)(2005), 19-30.

[24] P.Saravanaperumal and S.Murugesan, *(1,2)*-Generalized closed sets in fuzzy bitopological spaces* (communicated).

[25] P.Sundaram, H.Maki and K.Balachandran, *Semi-generalized continuous maps and semi- T1/2 spaces*, Bull. Fukuoka Univ. Ed. III 40(1991), 3340.

[26] M.K.R.S.Veerakumar, *$\hat{g}$-closed sets in topological spaces*, Bull. Allah. Math. Soc., 18(2003), 99-112.