



A Study on M/G/1 Queueing System with Extended Vacation, Random Breakdowns and General Repair

Research Article

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Abstract: This Paper study about a Non Markovian queueing model with general vacation, Random breakdown followed by a repair process .An additional aspect of Extended vacation is considered here. After a service completion, the server takes a vacation. The server has the option to take an extended vacation after general vacation or the server may continue to stay in the system to serve the customers. It is assumed that customers arrive at the system one by one. Vacation time, extended vacation time and repair time follows general distribution.Steady state results in explicit and closed form in terms of the probability generating functions for the number of customers in the queue, average number of customers, and the average waiting time in the queue are derived.

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1. Introduction and Preliminaries

Queueing systems with vacation time have been found to be useful in modeling the systems in which the server has additional tasks. Madan and Anabosi [6] have studied server vacations based on Bernoulli schedules and a single vacation policy. Madan and Choudhury [8] have studied a single server queue with two phase of heterogeneous service under Bernoulli schedule and a general vacation time. Thangaraj and Vanitha [2] have studied a single server M/G/1 feedback queue with two types of service having general distribution. Madan and Choudhury [7] proposed an queueing system with restricted admissibility of arriving batches. Igaki [12], Levi and Yechilai [13], Madan and Abu-Dayyeh [14] have studied vacation queues with different vacation policies.

Baba [1] discussed an M/G/1 system with variable vacations. Saravananarajan and Chandrasekaran [21] analysed M/G/1 feedback queueing system providing two types of services with vacations and breakdowns. Upon arrival to the system a customer can either choose type 1 service with probability p_1 or type 2 service with probability p_2 such that $p_1 + p_2 = 1$. After completion of each service he may join the tail of the queue with probability p , until he wishes for another service or leave the system with probability $q = 1 - p$. On each service completion, the server is allowed to take a vacation with probability θ or may continue service of a customer, if any, with probability $1 - \theta$. If no customer is found, it remains in the system until a customer arrives. The vacation times are exponentially distributed with parameter β . The system may breakdown at random and repair time follows exponential distribution with parameter η . Using supplementary variable

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technique, the Laplace transforms of time dependent probabilities of system state are derived. From this, the steady state results are deduced. Maragathasundari and Srinivasan [15], they studied about analysis of M/G/1 feedback queue with three stage multiple server vacation. Maragathasundari and Srinivasan [16], they studied about analysis of triple stage of service having compulsory vacation and service interruptions. Maragathasundari and Srinivasan [17], they studied about three phases M/G/1 queue with Bernoulli feedback and multiple server vacation.

In our study, we considered a M/G/1 queueing model with General vacation. After the completion of the service, the server goes for a vacation. A new assumption of optional Extended vacation is additionally considered in our model. Moreover the server may interrupt at any time during the service. Repair process follows immediately without any delay. Probability generating function of the queue size is derived. Performance measures are derived for the model. This paper is organized as follows. Model assumptions are given in section 2. Equations governing the system are given in section 3. The average queue size and the average waiting time are computed in section 4. Some special cases of the model are discussed in section 5. Conclusions are given in section 6.

2. Model Assumption

- a) Customers arrive at the system one by one in a compound Poisson process and they are provided one by one service on a 'first come-first served basis'. Let λ the mean arrival rate.
- b) There is a single server and the service time follows general(arbitrary)distribution with distribution function $K(s)$ and density function $k(s)$. Let $\mu(x)dx$ be the conditional probability density of service completion during the interval $(x, x + dx)$, given that the elapsed time is x , so that

$$\mu(x) = \frac{k(x)}{1 - K(x)} \quad (1)$$

and therefore

$$k(s) = \mu(s)e^{-\int_0^s \mu(x)dx} \quad (2)$$

- c) As soon as a service is completed, the server may take a vacation.
- d) The server's vacation time follows a general (arbitrary) distribution with distribution function $B(v)$ and density function $b(v)$. Let $\beta(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x + dx)$, so that

$$\beta(x) = \frac{m(x)}{1 - M(x)} \quad (3a)$$

and, therefore

$$m(v) = \beta(v)e^{-\int_0^v \beta(x)dx} \quad (3b)$$

- e) After general vacation, with probability r the server may take a extended vacation otherwise the server may join the system to serve the customers with probability $1-r$. The repair times follows a general(arbitrary) distribution with distribution function $G(x)$ and density function $g(x)$ Let $\theta(x)$ be the conditional probability of a extended vacation during the interval $(x, x + dx)$ so that

$$\theta(x) = \frac{g(x)}{1 - G(x)} \quad (4a)$$

and, therefore

$$g(v) = \theta(v)e^{-\int_0^v \theta(x)dx} \quad (4b)$$

- f) The server may breakdown at random, and breakdowns are assumed to occur according to Poisson stream with mean breakdown rate $\alpha > 0$. Further we assume that once the system breaks down, the customer whose service is interrupted comes back to the head of the queue.
- g) Once the system breaks down, it enters a repair process immediately. The repair times follows a general (arbitrary) distribution with distribution function $H(x)$ and density function $h(x)$. Let $\gamma(x)dx$ be the conditional probability of a completion of a repair during the interval $(x, x + dx)$, so that

$$\gamma(x) = \frac{b(x)}{1 - B(x)} \quad (5a)$$

and, therefore

$$b(v) = \gamma(v)e^{-\int_0^v \gamma(x)dx} \quad (5b)$$

- h) Various stochastic process involved in the system are assumed to be independent of each other.

3. Equations Governing the System

The equations governing the system are as follows:

$$\frac{\partial}{\partial x} P_n(x) + \frac{\partial}{\partial x} (\lambda + \mu(x) + \alpha) P_n(x) = \lambda P_{n-1}(x) \quad (6)$$

$$\frac{\partial}{\partial x} P_0(x) + \frac{\partial}{\partial x} (\lambda + \mu(x) + \alpha) P_0(x) = 0 \quad (7)$$

$$\frac{\partial}{\partial x} V_n(x) + \frac{\partial}{\partial x} (\lambda + \beta(x)) V_n(x) = \lambda V_{n-1}(x) \quad (8)$$

$$\frac{\partial}{\partial x} V_o(x) + (\lambda + \beta(x)) V_o(x) = 0 \quad (9)$$

$$\frac{\partial}{\partial x} E_n(x) + \frac{\partial}{\partial x} (\lambda + \theta(x)) E_n(x) = \lambda E_{n-1}(x) \quad (10)$$

$$\frac{\partial}{\partial x} E_0(x) + \frac{\partial}{\partial x} (\lambda + \theta(x)) E_0(x) = 0 \quad (11)$$

$$\frac{\partial}{\partial x} R_n(x) + (\lambda + \gamma(x)) R_n(x) = \lambda R_{n-1}(x) \quad (12)$$

$$\frac{\partial}{\partial x} R_o(x) + (\lambda + \gamma(x)) R_o(x) = 0 \quad (13)$$

$$\lambda Q = \int_0^\infty R_o(x) \gamma(x) dx + \int_0^\infty P_0(x) \mu(x) dx + \int_0^\infty V_o(x) \beta(x) dx + \int_0^\infty E_0(x) \theta(x) dx \quad (14)$$

The following boundary Conditions are used to solve the above equations:

$$P_n(0) = \int_0^\infty P_{n+1}(x) \mu(x) dx + (1 - r) \int_0^\infty V_{n+1}(x) \beta(x) dx + \int_0^\infty E_{n+1}(x) \theta(x) dx + \int_0^\infty R_{n+1}(x) \gamma(x) dx \quad (15)$$

$$V_n(0) = \int_0^\infty P_n(x) \mu(x) dx \quad (16)$$

$$E_n(0) = r \int_0^\infty V_n(x) \beta(x) dx \quad (17)$$

$$R_n(0) = \alpha \int_0^\infty P_{n-1}(x) dx = \alpha P_{n-1} \quad (18)$$

Multiplying Equation (6) by Z^n sum over n from 1 to ∞ and adding to (7), we obtain

$$\frac{\partial}{\partial x} P_q(x, z) + (\lambda - \lambda(z) + \mu(x) + \alpha) P_q(x, z) = 0 \quad (19)$$

Similarly,

$$\frac{\partial}{\partial x} V_q(x, z) + (\lambda - \lambda(z) + \beta(x))V_q(x, z) = 0 \quad (20)$$

$$\frac{\partial}{\partial x} E_q(x, z) + (\lambda - \lambda(z) + \theta(x))E_q(x, z) = 0 \quad (21)$$

$$\frac{\partial}{\partial x} R_q(x, z) + (\lambda - \lambda(z) + \gamma(x))R_q(x, z) = 0 \quad (22)$$

Multiplying Equation (15) by z^{n+1} , sum over n from 0 to ∞ ,

$$zP_q^{(1)}(0, z) = \int_0^\infty P_q(x, z)\mu(x)dx + (1-r) \int_0^\infty V_q(x, z)\beta(x)dx + \int_0^\infty E_q(x, z)\theta(x)dx + \int_0^\infty R_q(x, z)\gamma(x)dx - \lambda Q \quad (23)$$

$$V_q(0, z) = p \int_0^\infty P_q(x, z)\mu(x)dx \quad (24)$$

$$E_q(0, z) = r \int_0^\infty V_q(x, z)\beta(x)dx \quad (25)$$

$$R_q(0, z) = \alpha z \int_0^\infty P_q(x, z)dx = \alpha z P_q(z) \quad (26)$$

Integrating Equation (19) from 0 to ∞

$$P_q(x, z) = P_q(0, z)e^{-(\lambda - \lambda(z) + \alpha)x - \int_0^\infty \mu(t)dt} \quad (27)$$

Again Integrating the above equation, by parts with respect to x ,

$$P_q(z) = P_q(0, z) \left(\frac{1 - \bar{K}(s)}{S} \right) \quad (28)$$

Where $S = \lambda - \lambda z + \alpha \bar{K}(s) = e^{-(\lambda - \lambda(z) + \alpha)x}$ is Laplace Stieltjes Transform of the service time. Multiply both sides of Equation (27) by $\mu(x)$ integrating over x , we get

$$\int_0^\infty P_q(x, z)\mu(x)dx = P_q(0, z)\bar{K}(s) \quad (29)$$

Integrating Equation (20), we get

$$V_q(x, z) = V_q(0, z)e^{-(\lambda - \lambda(z) + \alpha)x - \int_0^x \beta(t)dt} \quad (30)$$

Again integrating the above equation by parts we get,

$$V_q(z) = V_q(0, z) \left[\frac{1 - \bar{M}(m)}{m} \right] \quad (31)$$

Also we have,

$$\int_0^\infty V_q(x, z)\beta(x)dx = V_q(0, z)\bar{M}(m) \quad (32)$$

Similarly for Extended vacation time & repair time we have,

$$E_q(x, z) = E_q(0, z)e^{-(\lambda - \lambda(z))x - \int_0^x \theta(t)dt} \quad (33)$$

$$E_q(z) = E_q(0, z) \left[\frac{1 - \bar{G}(m)}{m} \right] \quad (34)$$

Where $m = \lambda - \lambda z$. Also we have

$$\int_0^\infty E_q(x, z)\theta(x)dx = E_q(0, z)\bar{G}(m) \quad (35)$$

$$R_q(z) = R_q(0, z)\left[\frac{1 - \bar{B}(m)}{m}\right] \quad (36)$$

Also we have by similar process

$$\int_0^\infty R_q(x, z)\gamma(x)dx = R_q(0, z)\bar{B}(m) \quad (37)$$

Using all the above equations in eq (23) we get,

$$zP_q(0, z) = P_q(0, z)\bar{K}(s) + (1 - r)P_q(0, z)\bar{K}(s)\bar{M}(m) + \alpha zP_q(0, z)\bar{K}(s)\bar{B}(m) + P_q(0, z)\bar{K}(s)\bar{M}(m)\bar{G}(m) - \lambda Q \quad (38)$$

$$P_q(0, z) = \frac{-\lambda Q}{[z - \bar{K}(s) + (1 - r)\bar{K}(s)\bar{M}(m) - \bar{K}(s)\bar{M}(m)\bar{G}(m)] - \alpha z - \alpha z[1 - \bar{K}(s)]} \quad (39)$$

Let $S_q(z)$ be the probability generating function of the queue size

$$\begin{aligned} S_q(z) &= P_q(z) + V_q(z) + E_q(z) + R_q(z) \\ &= \frac{-\lambda Q[m(1 - \bar{K}(s)) + s[\bar{K}(s) - \bar{K}(s)\bar{M}(m)] + rs\bar{M}(m)\bar{K}(s)(1 - \bar{G}(m)) + \alpha z(1 - \bar{B}(m))(1 - \bar{K}(s))]}{s[z - \bar{K}(s) + (1 - r)\bar{K}(s)\bar{M}(m) - \bar{K}(s)\bar{M}(m)\bar{G}(m)] - \alpha z(1 - \bar{K}(s))} = \frac{N(z)}{D(z)} \end{aligned} \quad (40)$$

Where

$$N(z) = -\lambda Q[m(1 - \bar{K}(s)) + s[\bar{K}(s) - \bar{K}(s)\bar{M}(m)] + rs\bar{K}(s)\bar{M}(m)(1 - \bar{G}(m)) + \alpha z(1 - \bar{B}(m))(1 - \bar{K}(s))] \quad (41)$$

$$D(z) = m[s(z - \bar{K}(s)) + (1 - r)\bar{K}(s)\bar{M}(m) - \bar{K}(s)\bar{M}(m)\bar{G}(m) - \alpha z(1 - \bar{K}(s))] \quad (42)$$

Using the condition $S_q(1) + Q = 1$ the unknown factor Q can be determined. Also the utilization factor ρ can be determined.

4. The Average Queue Size and the Average Waiting Time

Let L_q denote the mean number of customers in the queue under the steady state. Then $L_q = \frac{d}{dz}S_q(z) |_{z=1}$. Since $S_q(z) = 0/0$ at $z = 1$, we use the result

$$\begin{aligned} L_q &= \lim_{z \rightarrow 1} \frac{d}{dz}P_q(z) \\ &= P_q(1) = \lim_{z \rightarrow 1} \frac{D'(z)N''(z) - N'(z)D''(z)}{2(D'(z))^2} \\ &= \frac{D'(1)N''(1) - N'(1)D''(1)}{2(D'(1))^2} \end{aligned} \quad (43)$$

$$N'(1) = \lambda^2[1 - \bar{K}(\alpha) + \alpha E(m)\bar{K}(\alpha) + r\alpha\bar{K}(\alpha)E(G) + \alpha E(B)(1 - \bar{K}(\alpha))]$$

$$D'(1) = r\alpha\lambda\bar{K}(\alpha)$$

$$\begin{aligned} N''(1) &= -\lambda[-2\lambda^2\bar{K}'(\alpha) + 2\lambda^2E(M)\bar{K}(\alpha) + 2\lambda^2r[E(G)\bar{K}(\alpha)[1 - \alpha E(m)]] + E(G^2)\alpha\lambda^2[-1 + \bar{K}(\alpha) + r\bar{K}(\alpha)] \\ &\quad - 2\alpha\lambda^2\bar{K}'(\alpha)[E(B) + E(m)] - \alpha\lambda^2\bar{K}(\alpha)E(M^2)] \end{aligned}$$

$$\begin{aligned} D''(1) &= (-\lambda)2\{(-\lambda)(1 - r\bar{K}(\alpha) - \bar{K}(\alpha) - (\alpha - \alpha\bar{K}(\alpha))) + \alpha[-\lambda\bar{K}'(\alpha) - r(\lambda\bar{K}'(\alpha) + E(m)) + (-\lambda)E(G)] \\ &\quad + \alpha\bar{K}(\alpha) + \alpha\lambda\bar{K}'(\alpha)\} \end{aligned}$$

Where primes and double primes in (43) denote first and second derivatives at $z = 1$, respectively. Carrying out the derivatives at $z = 1$ and if we substitute the values of $N'(1), N''(1), D'(1)$ and $D''(1)$ into (43) we obtain L_q in closed form.

Further, the mean waiting time of a customer could be found using $W_q = \frac{L_q}{\lambda}$.

5. Particular Cases

Case (i): Service time, Vacation time and Extended vacation time follows exponential distribution

Put

$$\begin{aligned} \bar{K}(\alpha) &= \frac{\mu}{\alpha + \mu} \bar{K}'(\alpha) = \frac{-\mu}{(\alpha + \mu)^2} E(m) = \frac{1}{\beta} E(m^2) = \frac{2}{\beta^2} E(G) = \frac{1}{\theta} E(G^2) = \frac{1}{\theta^2} \\ N'(1) &= \lambda^2 \left[1 - \frac{\mu}{\alpha + \mu} + \alpha \frac{1}{\beta} \frac{\mu}{\alpha + \mu} + r\alpha \frac{\mu}{\alpha + \mu} \frac{1}{\theta} + \alpha E(B) \left(1 - \frac{\mu}{\alpha + \mu} \right) \right] \\ D'(1) &= r\alpha \lambda \frac{\mu}{\alpha + \mu} \\ N''(1) &= -\lambda \left[-2\lambda^2 \frac{-\mu}{(\alpha + \mu)^2} + 2\lambda^2 \frac{1}{\beta} \frac{\mu}{\alpha + \mu} + 2\lambda^2 r \left[\frac{1}{\theta} \frac{\mu}{\alpha + \mu} \left[1 - \alpha \frac{1}{\beta} \right] + \frac{1}{\theta^2} \alpha \lambda^2 \left[-1 + \frac{\mu}{\alpha + \mu} + r \frac{\mu}{\alpha + \mu} \right] \right. \right. \\ &\quad \left. \left. - 2\alpha \lambda^2 \frac{-\mu}{(\alpha + \mu)^2} \left[E(B) + \frac{1}{\beta} \right] - \alpha \lambda^2 \frac{\mu}{\alpha + \mu} \frac{2}{\beta^2} \right] \right] \\ D''(1) &= (-\lambda) 2 \left\{ (-\lambda) \left(1 - r \frac{\mu}{\alpha + \mu} - \frac{\mu}{\alpha + \mu} - \left(\alpha - \alpha \frac{\mu}{\alpha + \mu} \right) \right) + \alpha \left[-\lambda \frac{-\mu}{(\alpha + \mu)^2} - r \left(\lambda \frac{-\mu}{(\alpha + \mu)^2} \right. \right. \right. \\ &\quad \left. \left. + \frac{1}{\beta} \right) + (-\lambda) \frac{1}{\theta} \right] + \alpha \frac{\mu}{\alpha + \mu} + \alpha \lambda \frac{-\mu}{(\alpha + \mu)^2} \left. \right\} \end{aligned}$$

Substituting the above in equation (43) we get L_q . By Little's formulae other performance measures are calculated.

Case (ii): No Extended vacation

Put $E(G) = 0$

$$\begin{aligned} N'(1) &= \lambda^2 \left[1 - \bar{K}(\alpha) + \alpha E(m) \bar{K}(\alpha) + \alpha E(B) (1 - \bar{K}(\alpha)) \right] \\ D'(1) &= r\alpha \lambda \bar{K}(\alpha) \\ N''(1) &= -\lambda \left[-2\lambda^2 \bar{K}'(\alpha) + 2\lambda^2 E(M) \bar{K}(\alpha) - 2\alpha \lambda^2 \bar{K}'(\alpha) [E(B) + E(m)] - \alpha \lambda^2 \bar{K}(\alpha) E(M^2) \right] \\ D''(1) &= (-\lambda) 2 \left\{ (-\lambda) \left(1 - r \bar{K}(\alpha) - \bar{K}(\alpha) - \left(\alpha - \alpha \bar{K}(\alpha) \right) \right) + \alpha \left[-\lambda \bar{K}'(\alpha) - r \left(\lambda \bar{K}'(\alpha) + E(m) \right) \right] + \alpha \bar{K}(\alpha) + \alpha \lambda \bar{K}'(\alpha) \right\} \end{aligned}$$

Substituting the above in equation (43) we get L_q . By Little's formulae other performance measures are calculated.

6. Conclusion

In this paper we have studied a Non Markovian queueing model with random breakdown followed by repair process, General vacation and Extended server vacation. This paper clearly analyses the Steady state results and some performance measures of the model. The result of the paper is useful for computer communication networks particularly in client-server communication, telephone systems and electronic mail services on Internet. As a future study, this work can be extended for multi stages with considering deterministic vacation as an additional parameter.

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