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On Weakly Concircular Symmetries of Three-Dimensional $\epsilon-$Trans-Sasakian Manifolds

Research Article

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Abstract: The object of the present paper is to study weakly concircular symmetric and weakly concircular Ricci symmetric three-dimensional $\epsilon-$ Trans-Sasakian manifolds. Futher, non existence of weakly concircular symmetric and weakly concircular Ricci symmetric three dimensional $\epsilon-$Trans-Sasakian manifolds has been proved under certain conditions.

MSC: 53C15, 53C25.

Keywords: Weakly symmetric, weakly concircular symmetric, weakly Ricci symmetric, weakly concircular Ricci symmetric manifold, ϵ-trans Sasakian manifold.

1. Introduction

In mathematics, a weakly symmetric space is a notion introduced by the Norwegian mathematician Atle Selberg in the 1950s as a generalization of a symmetric space, due to Elie Cartan. Geometrically, the spaces are defined as a complete Riemannian manifolds such that any two points can be exchanged by an isometry, the symmetric case being when the isometry is required to have period two. The classification of weakly symmetric space relies on that of periodic automorphism of complex bi-semi simple Lie algebras.

As a proper generalization of Pseudo symmetric manifolds by Chaki [13] in 1989. The notions of weakly symmetric and weakly projective symmetric manifolds were introduced by Tamassy and Binh [11] and later Binh [12] studied decomposable weakly symmetric manifolds. The notion of weakly symmetric manifolds was introduced by Tamassy and Binh [12]. A non flat Riemannian Manifold (M^n, g) $(n > 2)$ is called weakly symmetric if its curvature tensor R of type $(0, 4)$ satisfies the condition

$$\begin{aligned}(\nabla_X R)(Y, Z, U, V) \;=\;& A(X)R(Y, Z, U, V) + B(Y)R(X, Z, U, V) + H(Z)R(Y, X, U, V) \\ &+D(U)R(Y, Z, X, V) + E(V)R(Y, Z, U, X)\end{aligned} \tag{1}$$

for all vector fields $X, Y, Z, U, V \in \chi(M^n)$; $\chi(M)$ being the Lie Algebra of the smooth vector fields of M. Where A, B, H, D and E are 1-forms (not simultaneously zero) and ∇ denotes the operator of covariant differentiation with respect to the Riemannian metric g. The 1-forms are called the associated 1-forms of the manifold and an n-dimensional manifold of this

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kind is denoted by $(WS)_n$. If in (1) the 1-form A is replaced by $2A$ and E is replaced by A, then a $(WS)_n$ reduces to the notion of generalized pseudo symmetric manifold by Chaki [13]. In 1999, De and Bandyopadhyay [19] studied a $(WS)_n$ and proved that in such a manifold the associated 1-forms $B = H$ and $D = E$. Hence (1) reduces to the following form.

$$\begin{aligned}(\nabla_X R)(Y,Z,U,V) &= A(X)R(Y,Z,U,V) + B(Y)R(X,Z,U,V) + B(Z)R(Y,X,U,V) \\ &\quad + D(U)R(Y,Z,X,V) + D(V)R(Y,Z,U,X)\end{aligned} \tag{2}$$

A transformation of 3-dimensional Riemannian manifold M, which transforms every geodesic circle of M into a geodesic circle, is called a concircular transformation [10] and is defined by

$$C(Y,Z)U = R(Y,Z)U - \frac{r}{6}[g(Z,U)Y - g(Y,U)Z] \tag{3}$$

where r is the scalar curvature of the manifold.

Recently Shaikh and Hui [1] introduced the notion of weakly concircular symmetric manifolds. A Riemannian manifold $(M^n, g)(n > 2)$ is called weakly concircular symmetric manifold if its concircular curvature tensor C satisfies the condition

$$(\nabla_X C)(Y,Z,U) = A(X)C(Y,Z,U) + B(Y)C(X,Z,U) + H(Z)C(Y,X.U) + D(U)R(Y,Z,X) \tag{4}$$

for all vector fields $X, Y, Z, U \in X(M^3)$, where A, B, H and D are 1-forms (not simultaneously zero) and 3-dimensional manifold of this kind is denoted by $(WCS)_3$. Also it is shown that in a $(WCS)_3$ the associated 1-forms $B = H$, and hence the defining condition (4) of a $(WCS)_3$ reduces to the following form:

$$(\nabla_X C)(Y,Z,U) = A(X)C(Y,Z,U) + B(Y)C(X,Z,U) + B(Z)C(Y,X,U) + D(U)R(Y,Z,X) \tag{5}$$

A, B and D are 1-forms (not simultaneously zero).

Again Tamassy and Binh [12] introduced the notion of weakly Ricci symmetric manifolds. A Riemannian manifold (M^n, g) $(n > 2)$ is called weakly Ricci symmetric manifold if its Ricci tensor S of type $(0, 2)$ is not identically zero and satisfies the condition

$$(\nabla_X S)(Y,Z) = A(X)S(Y,Z) + B(Y)S(X,Z) + D(Z)C(Y,X) \tag{6}$$

where A, B and D are three non-zero 1-forms, called the associated 1-forms of the manifolds, and ∇ denotes the operator of covariant differentiation with respect to the metric g. Such 3-dimensional manifold is denoted by $(WCS)_3$.

Let $\{e_1,\ e_{2,}\ e_3\}$ be an orthonormal basis of the tangent space at each point of the manifold and let

$$P(Y,V) = \sum_{i=1}^{3} C(Y,e_i,e_i,V) \tag{7}$$

then from (3), we get

$$P(Y,V) = S(Y,V) - \frac{r}{3}g(Y,V) \tag{8}$$

The tensor P is called the concircular Ricci symmetric tensor [17], which is a symmetric tensor of type $(0, 2)$. In [17], De and Ghosh introduced the notion of weakly concircular Ricci symmetric manifolds. A Riemannian manifold (M^n, g) $(n > 2)$ is called weakly concircular Ricci symmetric manifold [17], if its concircular Ricci tensor P of type $(0,2)$ is not identically zero and satisfies the condition

$$(\nabla_X P)(Y,Z) = A(X)P(Y,Z) + B(Y)P(X,Z) + D(Z)P(Y,X) \tag{9}$$

where A, B and D are 1-forms (not simultaneously zero).

In [3], A. Bejancu and K. L. Duggal introduced the notion of ϵ-Sasakian manifolds with indefinite metric. In 1998, Xu. Xufeng and Chao Xiaoli proved that every ϵ-Sasakian manifold is a hypersurface of an indefinite Khalerian manifold and established a necessary and sufficient condition for an odd dimensional Riemannian manifold to be an ϵ-Sasakian manifolds [21]. In [16], U.C. De and Avijit Sarkar introduced and studied the notion of ϵ-Kenmotsu manifolds with indefinite mertic with an example.

In this paper we consider the three dimensional ϵ-trans- Sasakian manifold. The purpose of this paper is to introduce a new concept such as weakly concircular symmetries of three-dimensional ϵ-trans- Sasakian manifold, and studying some properties. Section 2 is devoted to the preliminary results of ϵ-trans- Sasakian manifold that are needed in the rest of sections. Recently S.K.Hui [14] studied weak concircular symmetries of the trans-Sasakian manifolds, However, In section 3 of this paper we have obtained all the 1-forms of weakly concircular symmetric three dimensional ϵ-trans- Sasakian manifold and hence such a structure exist. In the last section we study weakly concircular Ricci symmetric three dimensional ϵ-trans- Sasakian manifolds and obtained all the 1-forms of weakly concircular Ricci symmetric three dimensional ϵ-trans- Sasakian manifold and consequently such a structure exist.

2. Preliminaries

In this section, we list the basic definitions and known results of ϵ-trans- Sasakian manifolds.

Definition 2.1. *A $(2n+1)$-dimensional differentiable manifold (M, g) is said to be an ϵ-almost contact metric manifold, if it admits a $(1,1)$ tensor field ϕ, a structure vector field ξ, a 1-form η an indefinite metric g such that*

$$\phi^2 = -I + \eta \otimes \xi, \qquad \eta(\xi) = 1 \tag{10}$$

$$g(\xi,\xi) = \epsilon, \qquad \eta(X) = \epsilon g(X,\xi) \tag{11}$$

$$g(\phi X, \phi Y) = g(X,Y) - \epsilon\eta(X)\eta(Y) \tag{12}$$

for all vector fields X, Y on M, where ϵ is 1 or -1 according as ξ is space like or timelike and rank of ϕ is $2n$. From the above equations, one can deduce that

$$\phi\xi = 0, \qquad \eta(\phi X) = 0 \tag{13}$$

Definition 2.2. *An ϵ-almost contact metric manifold is called an ϵ-Trans- Sasakian manifold if*

$$(\nabla_X \phi)Y = \alpha\{g(X,Y)\xi - \epsilon\eta(Y)X\} + \beta\{g(\phi X, Y)\xi - \epsilon\eta(Y)\phi X\} \tag{14}$$

for any X, Y on M where ∇ is the Lie-Civita connection with respect to g.

we note that if $\epsilon = 1$, i.e. structure vector field ξ is space like, and then an ϵ-Trans-Sasakian manifold is usual trans-sasakian manifold [5]. A Trans-Sasakian manifold of type $(0,0)$, $(0,\beta)$, $(\alpha,0)$ are the cosympletic, β-Kenmotsu and α-Sasakian manifolds recpectively. In particular if $\alpha = 1, \beta = 0$ and $\alpha = 0, \beta = 1$, then trans-Sasakian manifold reduces to Sasakian and Kenmotsu manifold, we have [17].

$$(\nabla_X \xi) = \epsilon\{-\alpha\phi X + \beta(X - \eta(X)\xi)\} \tag{15}$$

$$(\nabla_X \eta)Y = -\alpha g(\phi X, Y) + \beta\{g(X,Y) - \epsilon\eta(X)\eta(Y)\} \tag{16}$$

In a ϵ-Trans-Sasakian manifold $M^3(\phi,\xi,\eta,g)$ the following relations hold

$$\begin{aligned} R(X,Y)\xi &= (\alpha^2-\beta^2)\{\eta(Y)X - \eta(X)Y\} + 2\alpha\beta\{\eta(Y)\phi X - \eta(X)\phi Y\} \\ &\quad +\epsilon\{(Y\alpha)\phi X - (X\alpha)\phi Y + (Y\beta)\phi^2 X - (X\beta)\phi^2 Y\} \end{aligned} \tag{17}$$

$$\begin{aligned} \eta(R(X,Y)Z) &= \epsilon(\alpha^2-\beta^2)\{g(Y,Z)\eta(X) - g(X,Z)\eta(Y)\} + 2\epsilon\alpha\beta\{\eta(X)g(\phi Y,Z) - \eta(Y)g(\phi X,Z)\} \\ &\quad +\{(X\beta)g(\phi^2 Y,Z) - (Y\beta)g(\phi^2 X,Z)\} + \{(X\alpha)g(\phi Y,Z) - (Y\alpha)g(\phi X,Z)\} \end{aligned} \tag{18}$$

$$S(X,\xi) = \{2(\alpha^2-\beta^2) - \epsilon(\xi\beta)\}\eta(X) - \epsilon(\phi X)\alpha - \epsilon(X\beta) \tag{19}$$

$$R(\xi,X)\xi = \{\alpha^2 - \beta^2 - \epsilon(\xi\beta)\}\{-X + \eta(X)\xi\} - \{2\alpha\beta + \epsilon(\xi\alpha)\}(\phi X) \tag{20}$$

$$S(\xi,\xi) = 2\{\alpha^2 - \beta^2 - \epsilon(\xi\beta)\} \tag{21}$$

$$2\alpha\beta + \epsilon(\xi\alpha) = 0 \tag{22}$$

where R is the curvature tensor of type $(1,3)$ of the manifold and Q is the symmetric endomorphism of the tangent space at each point of the manifold corresponding to the Ricci tensor S, that is, $g(QX,Y) = S(X,Y)$ for any vector fields X, Y on M.

3. Weakly Concircular Symmetric Three-Dimensional $\epsilon-$Trans-Sasakian Manifolds

Definition 3.1. *An ϵ-trans-Sasakian manifold $M^3(\phi,\xi,\eta,g)$ is said to be weakly concircular symmetric if its concircular curvature tensor C satisfies* (5)

Putting $Y = V = e_i$ in (5) *and taking summation over i, $1 \le i \le 3$, we obtain*

$$\begin{aligned} (\nabla_X S)(Z,U) - \frac{dr(X)}{3}g(Z,U) &= A(X)[S(Z,U) - \frac{r}{3}g(Z,U)] + B(Z)[S(X,U) - \frac{r}{3}g(X,U)] \\ &\quad +D(U)[S(X,Z) - \frac{r}{3}g(X,Z)] - \frac{r}{6}[\{B(X)+D(X)\}g(Z,U) \\ &\quad -B(Z)g(X,U) - D(U)g(Z,X)] + B(R(X,Z)U) + D(R(X,U)Z) \end{aligned} \tag{23}$$

Putting $X = Y = U = \xi$ in (23) and then using (16) and (20) we get

$$A(\xi) + B(\xi) + D(\xi) = \frac{grad\ F.\xi}{F} \tag{24}$$

where $F = 6(\alpha^2 - \beta^2 - \epsilon\xi\beta) - \epsilon r$. We can see that if $grad\ F$ is orthogonal to ξ then

$$A(\xi) + B(\xi) + D(\xi) = 0 \tag{25}$$

since $A(X) = g(X, \rho), A(\xi) = B(\xi) = D(\xi) = g(\rho, \xi)$. In view of (25) we get $A(\xi) = B(\xi) = D(\xi) = 0$. If $grad\ F$ and ξ are not inclined orthogonal then $grad\ F.\xi \neq 0$. Hence

$$A(\xi) + B(\xi) + D(\xi) \neq 0 \tag{26}$$

that is $A(\xi) = B(\xi) = D(\xi) \neq 0$. We can state the following theorem:

Theorem 3.2. *In a weakly concircular ϵ-trans-Sasakian manifold $M^3(\phi, \xi, \eta, g)$ the relation (24) holds.*

Next putting X and Z by ξ in (23) and then using (18) and (21) we get

$$\begin{aligned} 6(\nabla_\xi S)(\xi, U) - 2\epsilon dr(\xi)\eta(U) &= 6[A(\xi) + B(\xi)][S(U, \xi) - \frac{r}{3}\epsilon\eta(U)] \\ &\quad + [6(\alpha^2 - \beta^2 - \epsilon\xi\beta) - \epsilon r][D(U) + \eta(U)D(\xi)] \end{aligned} \tag{27}$$

Again we have

$$\begin{aligned} (\nabla_\xi S)(\xi, U) &= \nabla_\xi S(\xi, U) - S(\nabla_\xi \xi, U) - S(\xi, \nabla_\xi U) \\ &= [2\{2\alpha(\xi\alpha) - 2\beta(\xi\beta)\} - \epsilon\xi(\xi\beta)]\eta(U) - \epsilon(U(\xi\beta)) - \epsilon(\phi U(\xi\alpha)) \end{aligned} \tag{28}$$

where (18) has been used. In view of (27) and (28) we get from (24) that

$$\begin{aligned} D(U) &= \frac{[12\{2\alpha(\xi\alpha) - 2\beta(\xi\beta)\} - 6\epsilon\xi(\xi\beta)]\eta(U)}{F} \\ &\quad - \frac{6\epsilon U[\xi\beta] + 6\epsilon\phi U[\xi\alpha] + 2\epsilon\xi[r]\eta(U)}{F} - \left\{\frac{\xi[F]}{F^2}\right\} \\ &\quad \{\{6[2(\alpha^2 - \beta^2) - \epsilon\xi\beta] - 2\epsilon r\}\eta(U) - 6\{\epsilon U[\beta] + \epsilon\phi U[\alpha]\}\} \\ &\quad + D(\xi)\left\{\frac{6\{(\alpha^2 - \beta^2)\eta(U) - \epsilon U[\beta]\} - 6\epsilon\phi U[\alpha] - \epsilon r\eta(U)}{F}\right\} \end{aligned} \tag{29}$$

for any vector field U. If $grad\ F$ and ξ are orthogonal then by virtue of (24) we obtain

$$\begin{aligned} D(U) &= \frac{[12\{2\alpha(\xi\alpha) - 2\beta(\xi\beta)\} - 6\epsilon\xi(\xi\beta)]\eta(U)}{F} - \frac{6U[\xi\beta] + 6\epsilon\phi U[\xi\alpha] + 2\epsilon\xi[r]\eta(U)}{F} \\ &\neq 0 \end{aligned}$$

If $grad\ F$ and ξ are not orthogonal then by virtue of (26) we obtain $D(U) \neq 0$.

Next Putting $X = Y = \xi$ in (23) and proceeding in a similar manner as above we obtain

$$\begin{aligned} B(Z) &= \frac{[12\{2\alpha(\xi\alpha) - 2\beta(\xi\beta)\} - 6\epsilon\xi(\xi\beta)]\eta(Z)}{F} \\ &\quad - \frac{6\epsilon Z[\xi\beta] + 6\epsilon\phi Z[\xi\alpha] + 2\epsilon\xi[r]\eta(Z)}{F} - \left\{\frac{\xi[F]}{F^2}\right\} \\ &\quad \{\{6[2(\alpha^2 - \beta^2) - \epsilon\xi\beta] - 2\epsilon r\}\eta(Z) - 6\{\epsilon Z[\beta] + \epsilon\phi Z[\alpha]\}\} \\ &\quad + B(\xi)\left\{\frac{6\{(\alpha^2 - \beta^2)\eta(Z) - \epsilon Z[\beta]\} - 6\epsilon\phi Z[\alpha] - \epsilon r\eta(Z)}{F}\right\} \end{aligned} \tag{30}$$

for any vector field Z. If *grad* F and ξ are orthogonal then by virtue of (24) and (25) we obtain

$$\begin{aligned} B(Z) &= \frac{[12\{2\alpha(\xi\alpha) - 2\beta(\xi\beta)\} - 6\epsilon\xi(\xi\beta)]\eta(Z)}{F} - \frac{6\epsilon Z[\xi\beta] + 6\epsilon\phi Z[\xi\alpha] + 2\epsilon\xi[r]\eta(Z)}{F} \\ &\neq 0 \end{aligned}$$

If *grad* F and ξ are not orthogonal then by virtue of (26) we obtain $B(Z) \neq 0$. Again putting $Z = U = \xi$ in (23) we obtain

$$\begin{aligned} (\nabla_X S)(\xi,\xi) - \epsilon\frac{dr(X)}{3} &= A(X)[S(\xi,\xi) - \frac{r}{3}\epsilon] + B(R(X,\xi)\xi) + D(R(X,\xi)\xi) \\ &+[B(\xi) + D(\xi)]\ [S(X,\xi) - \frac{r}{3}\epsilon\eta(X)] - \frac{r}{6}[\{B(X) + D(X)\}\epsilon - \epsilon B(\xi)\eta(X) - \epsilon D(\xi)\eta(X)] \end{aligned} \tag{31}$$

Now we have

$$(\nabla_X S)(\xi, U) = \nabla_X S(\xi,\xi) - 2S(\nabla_X\xi,\xi)$$

which yields by using (14) and (18) that

$$\begin{aligned} (\nabla_X S)(\xi, U) &= 2[2\alpha(X\alpha) - 2\beta(X\beta) - \epsilon(X(\xi\beta))] \\ &+2\alpha[(X\alpha) - \eta(X)(\xi\alpha) - (\phi X)\beta] + 2\beta[(\phi X)\alpha + \{X\beta - (\xi\beta)\eta(X)\}] \end{aligned} \tag{32}$$

using (19), (20) and (32) in (31) we get

$$\begin{aligned} A(X) &= \frac{X(F+\epsilon r)}{F} + \frac{6\alpha\{(X\alpha) - \eta(X)(\xi\alpha) - (\phi X)\beta\}}{F} \\ &+\frac{6\beta[(\phi X)[\alpha] + \{X[\beta] - \xi\beta\eta(X)\}] - \epsilon X[r]}{F} - [B(\xi) + D(\xi)] \\ &\frac{[3\{(\alpha^2 - \beta^2)\eta(X) - \epsilon X[\beta]\} - 3\epsilon\phi X[\alpha] - \frac{r}{2}\epsilon\eta(X)]}{F} \\ &-\frac{[B(X) + D(X)][3(\alpha^2 - \beta^2) - \frac{r}{2}\epsilon\xi\beta]}{F} \end{aligned} \tag{33}$$

for any vector X. If *grad* F and ξ are orthogonal then by virtue of (24) and (25) we obtain

$$\begin{aligned} A(X) &= \frac{X(F+\epsilon r)}{F} + \frac{6\alpha\{(X\alpha) - \eta(X)(\xi\alpha) - (\phi X)\beta\}}{F} \\ &+\frac{6\beta[(\phi X)[\alpha] + \{X[\beta] - \xi\beta\eta(X)\}] - \epsilon X[r]}{F} \\ &-\frac{[B(X) + D(X)][3(\alpha^2 - \beta^2) - \frac{r}{2}\epsilon\xi\beta]}{F} \\ &\neq 0 \end{aligned}$$

If *grad* F and ξ are not orthogonal then by virtue of (26) we obtain

$$A(X) \neq 0.$$

We can state the following theorem:

Theorem 3.3. *There exists no weakly concircular symmetric ϵ-trans-Sasakian manifold M^3, if $A+B+D$ is not everywhere zero.*

4. Weakly Concircular Ricci Symmetric Three-Dimensional $\epsilon-$ Trans-Sasakian Manifolds

Definition 4.1. *An ϵ-trans-Sasakian manifold $M^3(\phi,\xi,\eta,g)$ is said to be weakly concircular Ricci symmetric if its concircular Ricci tensor P of type $(0,2)$ satisfies (9).*

In view of (8), (9) yields

$$\begin{aligned}(\nabla_X S)(Y,Z) - \frac{dr(X)}{3}g(Y,Z) &= A(X)[S(Y,Z) - \frac{r}{3}g(Y,Z)] \\ &\quad + B(Y)[S(X,Z) - \frac{r}{3}g(X,Z)] + D(Z)[S(X,Y) - \frac{r}{3}g(X,Y)]\end{aligned} \tag{34}$$

Putting $X = Y = Z = \xi$ in (34), we get the relation (24) and hence we can state the following:

Theorem 4.2. *In a weakly concircular Ricci symmetric ϵ-trans-Sasakian manifold $M^3(\phi,\xi,\eta,g)$, the relation (24) holds*

Next, putting X and Y by ξ in (34) and using (18) and (24), we get

$$\begin{aligned}D(Z) &= \frac{\{6\xi[\alpha^2-\beta^2] - 3\epsilon\xi[\xi\beta]\}\eta(Z)}{F} - \frac{3\epsilon Z\xi[\beta] + 3\epsilon\phi Z[\xi\alpha] + \epsilon\xi[r]\eta(Z)}{F} \\ &\quad + D(\xi)\frac{\{[6(\alpha^2-\beta^2) - 3\epsilon\xi\beta - \epsilon r]\eta(Z) - 3\epsilon\phi Z[\alpha] - 3\epsilon Z[\beta]\}}{F} \\ &\quad - \left\{\frac{\xi[F]}{F^2}\right\}\{[2n(\alpha^2-\beta^2) - \epsilon(\xi\beta)]\eta(Z) - \epsilon Z[\beta] - \epsilon\phi Z[\alpha] - \frac{r}{3}\epsilon\eta(Z)\}\end{aligned} \tag{35}$$

for any vector Z. If *grad* F and ξ are orthogonal then by virtue of (24) and (25) we obtain

$$\begin{aligned}D(Z) &= \frac{\{6\xi[\alpha^2-\beta^2] - 3\epsilon\xi[\xi\beta]\}\eta(Z)}{F} - \frac{3\epsilon Z\xi[\beta] + 3\epsilon\phi Z[\xi\alpha] + \epsilon\xi[r]\eta(Z)}{F} \\ &\neq 0\end{aligned}$$

If *grad* F and ξ are not orthogonal then by virtue of (26) we obtain $D(Z) \neq 0$. Again putting $X = Z = \xi$ in (34) and proceeding in a similar manner as above. we obtain

$$\begin{aligned}B(Y) &= \frac{\{6\xi[\alpha^2-\beta^2] - 3\epsilon\xi[\xi\beta]\}\eta(Y)}{F} \\ &\quad - \frac{3\epsilon Y\xi[\beta] + 3\epsilon\phi Y[\xi\alpha] + \epsilon\xi[r]\eta(Y)}{F} \\ &\quad + B(\xi)\frac{\{[6(\alpha^2-\beta^2) - 3\epsilon\xi\beta - \epsilon r]\eta(Y) - 3\epsilon\phi Y[\alpha] - 3\epsilon Y[\beta]\}}{F} \\ &\quad - \left\{\frac{\xi[F]}{F^2}\right\}\{[2n(\alpha^2-\beta^2) - \epsilon(\xi\beta)]\eta(Y) - \epsilon Y[\beta] - \epsilon\phi Y[\alpha] - \frac{r}{3}\epsilon\eta(Y)\}\end{aligned} \tag{36}$$

for any vector Y. If *grad* F and ξ are orthogonal then by virtue of (24) and (25) we obtain

$$\begin{aligned}B(Y) &= \frac{\{6\xi[\alpha^2-\beta^2] - 3\epsilon\xi[\xi\beta]\}\eta(Y)}{F} - \frac{3\epsilon Y\xi[\beta] + 3\epsilon\phi Y[\xi\alpha] + \epsilon\xi[r]\eta(Y)}{F} \\ &\neq 0\end{aligned}$$

If *grad* F and ξ are not orthogonal then by virtue of (26) we obtain $B(Y) \neq 0$. Again putting $Y = Z = \xi$ in (34) and using (20) and (24), we obtain

$$\begin{aligned} A(X) &= \frac{X(F+\epsilon r)}{F} + \frac{6\alpha\{(X\alpha) - \eta(X)(\xi\alpha) - (\phi X)\beta\}}{F} \\ &+ \frac{6\beta[(\phi X)[\alpha] + \{X[\beta] - \xi\beta\eta(X)\}] - \epsilon X[r]}{F} \\ &+ A(\xi)\frac{\{[6(\alpha^2-\beta^2) - 3\epsilon\xi\beta - \epsilon r]\eta(X) - 3\epsilon\phi X[\alpha] - 3\epsilon X[\beta]\}}{F} \\ &- \left\{\frac{\xi[F]}{F^2}\right\}\{[2n(\alpha^2-\beta^2) - \epsilon(\xi\beta)]\eta(X) - \epsilon X[\beta] - \epsilon\phi X[\alpha] - \frac{r}{3}\epsilon\eta(X)\} \end{aligned} \tag{37}$$

for any vector X. If *grad* F and ξ are orthogonal then by virtue of (24) and (25) we obtain

$$\begin{aligned} A(X) &= \frac{X(F+\epsilon r)}{F} + \frac{6\alpha\{(X\alpha) - \eta(X)(\xi\alpha) - (\phi X)\beta\}}{F} + \frac{6\beta[(\phi X)[\alpha] + \{X[\beta] - \xi\beta\eta(X)\}] - \epsilon X[r]}{F} \\ &\neq 0 \end{aligned}$$

If *grad* F and ξ are not orthogonal then by virtue of (26) we obtain $A(X) \neq 0$.

We can state the following theorem:

Theorem 4.3. *There exists no weakly concircular Ricci symmetric ϵ-trans-Sasakian manifold $M^3(\phi,\xi,\eta,g)$, if sum of the associated 1-forms D, B and A is not everywhere zero.*

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