

A New Chain Ratio-Ratio-Type Exponential Estimator Using Auxiliary Information in Sample Surveys

Research Article

Housila P.Singh¹ and Surya K.Pal^{1*}¹ School of studies in Statistics, Vikram University, Ujjain, M.P, India.

Abstract: This paper advocates the problem of estimating the finite population mean using auxiliary information in sample surveys. We have suggested a new chain ratio-ratio- type exponential estimator and its properties are studied up to first degree of approximation. It has been shown that the proposed estimator is more efficient than the usual unbiased estimator, classical ratio estimator, Bahl and Tuteja [1] ratio-type exponential estimator and Kadilar and Cingi [3] chain ratio-type estimator under very realistic condition. Generalized version of the suggested chain ratio-ratio-type estimator is also given along with its properties. An empirical study is given in support of the present study.

MSC: 62D05.

Keywords: Study variate, Auxiliary variate, Chain ratio-ratio-type exponential estimator, Bias, Mean squared error.

© JS Publication.

1. Introduction

In survey sampling, the use of auxiliary information at the estimation stage has been discussed by various statisticians in order to improve the efficiency of their formulated estimators for estimating the population mean. Out of many ratio, product and regression methods of estimation are good examples in this context. The use of auxiliary information dates back to the year (1934), when Neyman used it for stratification of the finite population. Cochran [2] used auxiliary information in estimation procedure and envisaged ratio method of estimation to provide more efficient estimator of the population mean or total compared to the simple mean per unit estimator under certain conditions when the correlation between the study variable and the auxiliary variable is positive (high). On the other hand, if the correlation is negative (high), Robson [6] and Murthy [5] suggested the use of product method of estimation. Recent developments in ratio and product methods of estimation alongwith their variety of modified forms are lucidly described in detail by Singh [19], Singh [9] and Solanki et al. [20].

Let $U = \{U_1, \dots, U_N\}$ be finite population of size N . To each unit U_i ($i = 1, \dots, N$) in the population paired values (y_i, x_i) corresponding to the study variable y and an auxiliary variable x , correlated with the study variable y are attached. Let (\bar{Y}, \bar{X}) be the population means of the study variable y and the auxiliary variable x respectively. Let there be a positive (high) correlation between y and x . It is assumed that the population mean \bar{X} of the auxiliary variable x is known. For estimating the population mean \bar{Y} the study variable y , a simple random sample of size n is drawn without replacement.

* E-mail: suryakantpal6676@gmail.com

Let (\bar{y}, \bar{x}) be the sample means of (y, x) respectively. It is well known that sample mean \bar{y} is an unbiased estimator of the population mean \bar{Y} and its variance under simple random sampling without replacement (SRSWOR) is given by

$$MSE(\bar{y}) = Var(\bar{y}) = \frac{(1-f)}{n} S_y^2 = \frac{(1-f)}{n} \bar{Y}^2 C_y^2, \quad (1)$$

where $f = \frac{n}{N}$ is the sampling fraction, $S_y^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{Y})^2$ and $C_y = S_y/\bar{Y}$. The classical ratio estimator of the population mean \bar{Y} using auxiliary information on the auxiliary variable x is given by

$$\bar{y}_R = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right), \quad \bar{x} \neq 0. \quad (2)$$

The bias and mean squared error of the ratio estimator \bar{y}_R to the first degree of approximation are respectively given by

$$B(\bar{y}_R) = \frac{(1-f)}{n} \bar{Y} C_x^2 (1-C), \quad (3)$$

$$MSE(\bar{y}_R) = \frac{(1-f)}{n} \bar{Y}^2 [C_y^2 + C_x^2 (1-2C)], \quad (4)$$

where $C = \rho \left(\frac{C_y}{C_x} \right)$, $\rho = \left(\frac{S_{yx}}{S_y S_x} \right)$, $C_x = \left(\frac{S_x}{\bar{X}} \right)$, $S_{yx} = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{Y})(x_i - \bar{X})$ and $S_x^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{X})^2$.

It follows from (1) and (4) that the ratio estimator \bar{y}_R is more efficient than the sample mean \bar{y} if

$$C > \frac{1}{2}. \quad (5)$$

Kadilar and Cingi [3] suggested a chain ratio estimator for estimating the population mean \bar{Y} as

$$\bar{y}_{CR} = \bar{y} \left(\frac{\bar{X}^2}{\bar{x}^2} \right). \quad (6)$$

To the first degree of approximation, the bias and MSE of \bar{y}_{CR} are respectively given by

$$B(\bar{y}_{CR}) = \frac{(1-f)}{n} \bar{Y} C_x^2 (3-2C), \quad (7)$$

$$MSE(\bar{y}_{CR}) = \frac{(1-f)}{n} \bar{Y}^2 [C_y^2 + 4C_x^2 (1-C)]. \quad (8)$$

It can be easily shown that the chain ratio estimator \bar{y}_{CR} is more efficient than the usual unbiased estimator \bar{y} and the usual ratio estimator \bar{y}_R as long as the conditions

$$C > 1 \quad (9)$$

and

$$C > \frac{3}{2} = 1.5 \quad (10)$$

are respectively satisfied. Bahl and Tuteja [1] have suggested a ratio type exponential estimator for population mean \bar{Y} as

$$\bar{y}_{Re} = \bar{y} \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right). \quad (11)$$

To the first degree of approximation, the bias and MSE of \bar{y}_{Re} are respectively given by

$$B(\bar{y}_{Re}) = \frac{(1-f)}{8n} \bar{Y} C_x^2 (3-4C) \quad (12)$$

and

$$MSE(\bar{y}_{Re}) = \frac{(1-f)}{n} \bar{Y}^2 [C_y^2 + \frac{C_x^2}{4}(1-4C)]. \tag{13}$$

Recently using square root transformation Swain [14] suggested a ratio type estimator for population mean \bar{Y} as

$$\bar{y}_{SQR} = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)^{\frac{1}{2}} \tag{14}$$

whose bias and MSE to the first degree of approximation are same as given by (12) and (13) respectively i.e. $B(\bar{y}_{SQR}) = B(\bar{y}_{Re})$ and $MSE(\bar{y}_{SQR}) = MSE(\bar{y}_{Re})$. It can be easily shown from (1), (4) and (13) that the ratio type exponential estimator \bar{Y}_{Re} due to Bahl and Tuteja [1] is more efficient than the usual unbiased estimator \bar{y} and the ordinary ratio estimator \bar{y}_R if

$$\frac{1}{4} < C < \frac{3}{4} \tag{15}$$

Again from (8) and (13) we have

$$MSE(\bar{y}_{Re}) - MSE(\bar{y}_{CR}) = \frac{3(1-f)}{4n} \bar{Y}^2 C_x^2 (4C - 5)$$

which is positive if

$$C > \frac{5}{4} = 1.25 \tag{16}$$

Thus, the chain ratio estimator \bar{y}_{CR} due to Kadilar and Cingi [3] is more efficient than the ratio type exponential estimator \bar{y}_{Re} if the condition (15) is satisfied. It is observed from (9), (10) and (15) that the condition (10) is sufficient for the chain ratio estimator \bar{y}_{CR} due to Kadilar and Cingi [3] to be more efficient than the sample mean estimator \bar{y} , the classical ratio estimator \bar{y}_R and the ratio type exponential estimator \bar{y}_{Re} due to Bahl and Tuteja [1].

In this paper we have made an effort towards developing new chain ratio-type exponential and ratio-ratio-type exponential estimators and the study their properties. Numerical illustration is given in support of the present study.

2. The Proposed Chain Estimator

Adopting the same procedure as outlined in Kadilar and Cingi [3], if sample mean \bar{y} in (11) is replaced with \bar{y}_{Re} , the chain ratio-type exponential is obtained as

$$\bar{y}_{CRe} = \bar{y}_{Re} \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \tag{17}$$

We can re-write (17) using (11) as

$$\bar{y}_{CRe} = \bar{y} \exp \left\{ \frac{2(\bar{X} - \bar{x})}{(\bar{X} + \bar{x})} \right\} \tag{18}$$

If we replace \bar{y} in (11) by $\bar{y}_R = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right)$, then we get another chain ratio-ratio type exponential estimator for population mean \bar{y} as

$$\bar{y}_{CRR} = \bar{y}_R \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right)$$

or

$$\bar{y}_{CRR} = \bar{y} \left(\frac{\bar{X}}{\bar{x}} \right) \exp \left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}} \right) \tag{19}$$

To the first degree of approximation, the biases and mean squared errors (MSEs) of the proposed estimators \bar{y}_{CRRe} and \bar{y}_{CRRRe} are respectively given by

$$B(\bar{y}_{CRRe}) = \frac{(1-f)}{n} \bar{Y} C_x^2 (1-C) \quad (20)$$

$$= B(\bar{y}_R),$$

$$B(\bar{y}_{CRRRe}) = \frac{3(1-f)}{8n} \bar{Y} C_x^2 (5-4C), \quad (21)$$

$$MSE(\bar{y}_{CRRe}) = \frac{(1-f)}{n} \bar{Y}^2 [C_y^2 + C_x^2 (1-2C)], \quad (22)$$

$$= MSE(\bar{y}_R),$$

$$MSE(\bar{y}_{CRRRe}) = \frac{(1-f)}{n} \bar{Y}^2 [C_y^2 + \frac{3}{4} C_x^2 (3-4C)]. \quad (23)$$

where $B(\bar{y}_R)$ and $MSE(\bar{y}_R)$ are given by (3) and (4) respectively. It is to be noted from (3), (4), (20) and (22) that $B(\bar{y}_{CRRe}) = B(\bar{y}_R)$ and $MSE(\bar{y}_{CRRe}) = MSE(\bar{y}_R)$. So the comparison of classical ratio estimator \bar{y}_R and the chain ratio-type exponential estimator \bar{y}_{CRRe} with the chain ratio estimator \bar{y}_{CR} due to Kadilar and Cingi [3] are same as given by (10). Now, from (1) and (23) we have

$$MSE(\bar{y}_{CRRRe}) - MSE(\bar{y}) = \frac{(1-f)}{n} \bar{Y}^2 C_x^2 (3-4C) \quad (24)$$

which is negative if $(3-4C) < 0$. i.e if

$$C > \frac{3}{4} = 0.75. \quad (25)$$

From (4), (22) and (23) we have

$$MSE(\bar{y}_{CRRRe}) - MSE(\bar{y}_R) \{= MSE(\bar{y}_{CRRe})\} = \frac{(1-f)}{n} \bar{Y}^2 C_x^2 \left(\frac{5}{4} - C \right)$$

which is negative if $\frac{5}{4} - C < 0$. i.e if

$$C > \frac{5}{4} = 1.25 \quad (26)$$

From (13) and (23) we have

$$MSE(\bar{y}_{CRRRe}) - MSE(\bar{y}_{Re}) \{= MSE(\bar{y}_{SQRe})\} = \frac{2(1-f)}{n} \bar{Y}^2 C_x^2 (1-C) \quad (27)$$

which is negative if $(1-C) < 0$. i.e if

$$C > 1. \quad (28)$$

Further from (8) and (23) we have

$$MSE(\bar{y}_{CRRRe}) - MSE(\bar{y}_{CR}) = \frac{(1-f)}{n} \bar{Y}^2 C_x^2 \left(C - \frac{7}{4} \right) \quad (29)$$

which is negative if $(C - \frac{7}{4}) < 0$. i.e if

$$C < \frac{7}{4} = 1.75 \quad (30)$$

It is observed from (25), (26), (27) and (30) that the proposed chain ratio- ratio-type estimator exponential estimator $MSE(\bar{y}_{CRRRe})$ is better than:

- (i) the usual unbiased estimator \bar{y} if $C > \frac{3}{4} = 0.75$.

- (ii) the classical ratio estimator \bar{y}_R and the proposed chain ratio-type exponential estimator $MSE(\bar{y}_{CRRe})$ if $C > \frac{5}{4} = 1.25$.
- (iii) the ratio-type exponential estimator \bar{y}_{Re} due to Bahl and Tuteja [1] and the ratio-type estimator \bar{y}_{SQR} based on square root transformation due to Swain's [14] if $C > 1$.
- (iv) the chain ratio-type estimator \bar{y}_{CR} due to Kadilar and Cingi [3] if $C > \frac{7}{4} = 1.75$.

Thus from (i) to (iv) we conclude that the proposed chain ratio-ratio- type exponential estimator \bar{y}_{CRRe} is more efficient than the usual unbiased estimator \bar{y} , classical ratio type estimator \bar{y}_R , Bahl and Tuteja [1] ratio-type exponential estimator \bar{y}_{Re} and Swain's [14] ratio-type estimator \bar{y}_{SQR} , chain ratio-type exponential estimator \bar{y}_{CRe} and the chain ratio-type estimator \bar{y}_{CR} due to Kadilar and Cingi [3] if

$$1.25 = \frac{5}{4} < C < \frac{7}{4} = 1.75 \tag{31}$$

3. A Generalized Chain Ratio-Ratio-Type Exponential Estimator

Keeping the estimators due to Sisodia and Dwivedi [18], Upadhyaya and Singh [16], Singh [9], Singh and Tailor [8], Singh [19], Kadilar and Cingi [4], Subramani and Kumarapandiyan [10–12] and Yan and Tian [17] in view , we define a general class of chain ratio-ratio-type exponential estimator for population mean \bar{y} as

$$\bar{y}_{gCRRe} = \bar{y} \left(\frac{a\bar{X} + b}{a\bar{x} + b} \right) \exp \left\{ \frac{a(\bar{X} - \bar{x})}{a(\bar{X} + \bar{x}) + 2b} \right\}, \tag{32}$$

where (a, b) are real constants or the functions the parameters such as coefficient of variation C_x , coefficient of skewness $\beta_1(x)$, coefficients of kurtosis $\beta_2(x)$, standard deviation S_x , $\Delta = \beta_2(x)\beta_1(x) - 1$, quartiles Q_i ($i = 1, 2, 3$) and deciles D_i ($i = 1 \dots 10$) of the auxiliary variable x , coefficient of variation C_y , and coefficients of kurtosis $\beta_2(y)$, of the auxiliary variable x and ρ , the correlation coefficient between the auxiliary variable y and the auxiliary variable x . Some estimators belonging to the class of estimators \bar{y}_{gCRRe} for the convenience to the readers are given in the Table 3.1.

Table 1. Some members of the class of estimators \bar{y}_{gCRRe}

S.No.	Estimator	Values of Constants	
		a	b
1	$\bar{y}_{g1} = \bar{y} \left(\frac{\bar{X} + C_x}{\bar{x} + C_x} \right) \exp \left\{ \frac{(\bar{X} - \bar{x})}{(\bar{X} + \bar{x}) + 2C_x} \right\}$	1	C_x
2	$\bar{y}_{g2} = \bar{y} \left(\frac{\bar{X} + \beta_2(x)}{\bar{x} + \beta_2(x)} \right) \exp \left\{ \frac{(\bar{X} - \bar{x})}{(\bar{X} + \bar{x}) + 2\beta_2(x)} \right\}$	1	$\beta_2(x)$
3	$\bar{y}_{g3} = \bar{y} \left(\frac{\bar{X}\beta_2(x) + C_x}{\bar{x}\beta_2(x) + C_x} \right) \exp \left\{ \frac{\beta_2(x)(\bar{X} - \bar{x})}{\beta_2(x)(\bar{X} + \bar{x}) + 2C_x} \right\}$	$\beta_2(x)$	C_x
4	$\bar{y}_{g4} = \bar{y} \left(\frac{\bar{X}C_x + \beta_2(x)}{\bar{x}C_x + \beta_2(x)} \right) \exp \left\{ \frac{C_x(\bar{X} - \bar{x})}{C_x(\bar{X} + \bar{x}) + 2\beta_2(x)} \right\}$	C_x	$\beta_2(x)$
5	$\bar{y}_{g5} = \bar{y} \left(\frac{\bar{X} + \rho}{\bar{x} + \rho} \right) \exp \left\{ \frac{(\bar{X} - \bar{x})}{(\bar{X} + \bar{x}) + 2\rho} \right\}$	1	ρ
6	$\bar{y}_{g6} = \bar{y} \left(\frac{\bar{X}\beta_1(x) + \beta_2(x)}{\bar{x}\beta_1(x) + \beta_2(x)} \right) \exp \left\{ \frac{\beta_1(x)(\bar{X} - \bar{x})}{\beta_1(x)(\bar{X} + \bar{x}) + 2\beta_2(x)} \right\}$	$\beta_1(x)$	$\beta_2(x)$
7	$\bar{y}_{g7} = \bar{y} \left(\frac{\bar{X}\beta_2(x) + \beta_1(x)}{\bar{x}\beta_2(x) + \beta_1(x)} \right) \exp \left\{ \frac{\beta_2(x)(\bar{X} - \bar{x})}{\beta_2(x)(\bar{X} + \bar{x}) + 2\beta_1(x)} \right\}$	$\beta_2(x)$	$\beta_1(x)$
8	$\bar{y}_{g8} = \bar{y} \left(\frac{\bar{X} + M_d}{\bar{x} + M_d} \right) \exp \left\{ \frac{(\bar{X} - \bar{x})}{(\bar{X} + \bar{x}) + 2M_d} \right\}$	1	M_d
9	$\bar{y}_{g9} = \bar{y} \left(\frac{\bar{X}C_x + \rho}{\bar{x}C_x + \rho} \right) \exp \left\{ \frac{C_x(\bar{X} - \bar{x})}{C_x(\bar{X} + \bar{x}) + 2\rho} \right\}$	C_x	ρ
10	$\bar{y}_{g10} = \bar{y} \left(\frac{\bar{X}\rho + C_x}{\bar{x}\rho + C_x} \right) \exp \left\{ \frac{\rho(\bar{X} - \bar{x})}{\rho(\bar{X} + \bar{x}) + 2C_x} \right\}$	$\beta_2(x)$	C_x

S.No.	Estimator	Values of Constants	
		a	b
11	$\bar{y}_{g11} = \bar{y} \left(\frac{\bar{X}\beta_2(x)+\rho}{\bar{x}\beta_2(x)+\rho} \right) \exp \left\{ \frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2\rho} \right\}$	$\beta_2(x)$	ρ
12	$\bar{y}_{g12} = \bar{y} \left(\frac{\bar{X}\rho+\beta_2(x)}{\bar{x}\rho+\beta_2(x)} \right) \exp \left\{ \frac{\rho(\bar{X}-\bar{x})}{\rho(\bar{X}+\bar{x})+2\beta_2(x)} \right\}$	ρ	$\beta_2(x)$
13	$\bar{y}_{g13} = \bar{y} \left(\frac{\bar{X}C_x+M_d}{\bar{x}C_x+M_d} \right) \exp \left\{ \frac{C_x(\bar{X}-\bar{x})}{C_x(\bar{X}+\bar{x})+2M_d} \right\}$	C_x	M_d
14	$\bar{y}_{g14} = \bar{y} \left(\frac{\bar{X}\beta_2(x)+M_d}{\bar{x}\beta_2(x)+M_d} \right) \exp \left\{ \frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2M_d} \right\}$	$\beta_2(x)$	M_d
15	$\bar{y}_{g15} = \bar{y} \left(\frac{\bar{X}+S_x}{\bar{x}+S_x} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2S_x} \right\}$	1	S_x
16	$\bar{y}_{g16} = \bar{y} \left(\frac{\bar{X}\beta_1(x)+S_x}{\bar{x}\beta_1(x)+S_x} \right) \exp \left\{ \frac{\beta_1(x)(\bar{X}-\bar{x})}{\beta_1(x)(\bar{X}+\bar{x})+2S_x} \right\}$	$\beta_1(x)$	S_x
17	$\bar{y}_{g17} = \bar{y} \left(\frac{\bar{X}\beta_2(x)+S_x}{\bar{x}\beta_2(x)+S_x} \right) \exp \left\{ \frac{\beta_2(x)(\bar{X}-\bar{x})}{\beta_2(x)(\bar{X}+\bar{x})+2S_x} \right\}$	$\beta_2(x)$	S_x
18	$\bar{y}_{g18} = \bar{y} \left(\frac{\bar{X}+D_1}{\bar{x}+D_1} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_1} \right\}$	1	D_1
19	$\bar{y}_{g19} = \bar{y} \left(\frac{\bar{X}+D_2}{\bar{x}+D_2} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_2} \right\}$	1	D_2
20	$\bar{y}_{g20} = \bar{y} \left(\frac{\bar{X}+D_3}{\bar{x}+D_3} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_3} \right\}$	1	D_3
21	$\bar{y}_{g21} = \bar{y} \left(\frac{\bar{X}+D_4}{\bar{x}+D_4} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_4} \right\}$	1	D_4
22	$\bar{y}_{g22} = \bar{y} \left(\frac{\bar{X}+D_5}{\bar{x}+D_5} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_5} \right\}$	1	D_5
23	$\bar{y}_{g23} = \bar{y} \left(\frac{\bar{X}+D_6}{\bar{x}+D_6} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_6} \right\}$	1	D_6
24	$\bar{y}_{g24} = \bar{y} \left(\frac{\bar{X}+D_7}{\bar{x}+D_7} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_7} \right\}$	1	D_7
25	$\bar{y}_{g25} = \bar{y} \left(\frac{\bar{X}+D_8}{\bar{x}+D_8} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_8} \right\}$	1	D_8
26	$\bar{y}_{g26} = \bar{y} \left(\frac{\bar{X}+D_9}{\bar{x}+D_9} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_9} \right\}$	1	D_9
27	$\bar{y}_{g27} = \bar{y} \left(\frac{\bar{X}+D_{10}}{\bar{x}+D_{10}} \right) \exp \left\{ \frac{(\bar{X}-\bar{x})}{(\bar{X}+\bar{x})+2D_{10}} \right\}$	1	D_{10}

To obtain the bias and MSE of the general chain ratio-ratio-type exponential estimator \bar{y}_{gCRR_e} , we write

$$\bar{y} = \bar{Y}(1 + e_0)$$

$$\bar{x} = \bar{X}(1 + e_1)$$

$$E(e_0) = E(e_1) = 0$$

and

$$E(e_0^2) = \frac{(1-f)}{n} C_y^2,$$

$$E(e_1^2) = \frac{(1-f)}{n} C_x^2,$$

$$E(e_0 e_1) = \frac{(1-f)}{n} \rho C_y C_x = \frac{(1-f)}{n} C C_x^2.$$

Expressing \bar{y}_{gCRR_e} at (32) in term of e's we have

$$\bar{y}_{gCRR_e} = \bar{Y}(1 + e_0)(1 + \tau e_1)^{-1} \exp \left\{ -\frac{\tau e_1}{2} \left(1 + \frac{\tau e_1}{2} \right)^{-1} \right\}, \tag{33}$$

where $\tau = \frac{a\bar{X}}{a\bar{X}+b}$. We assume that $|\tau e_1| < 1$ so that $(1 + \tau e_1)^{-1}$ is expandable. Expanding the right hand side of (33), multiplying out and neglecting terms of e's having power greater than two we have

$$\bar{y}_{gCRR e} \cong \bar{Y} \left(1 + e_0 - \frac{3}{2}\tau e_1 - \frac{3}{2}\tau e_0 e_1 + \frac{15}{8}e_1^2 \right)$$

or

$$(\bar{y}_{gCRR e} - \bar{Y}) \cong \bar{Y} \left(e_0 - \frac{3}{2}\tau e_1 - \frac{3}{2}\tau e_0 e_1 + \frac{15}{8}e_1^2 \right) \tag{34}$$

Taking expectation of both sides of (34) we get the bias of $\bar{y}_{gCRR e}$ to the first degree of

$$B(\bar{y}_{gCRR e}) = \frac{3(1-f)}{8n} \bar{Y} C_x^2 (5 - 4\tau C) \tag{35}$$

Squaring both sides of (34) and neglecting terms of e's having power greater than two we have

$$(\bar{y}_{gCRR e} - \bar{Y})^2 \cong \bar{Y}^2 \left(e_0^2 + \frac{9}{4}\tau^2 e_1^2 - 3\tau e_0 e_1 \right) \tag{36}$$

Taking expectation of both sides of (36) we get the mean squared error of $\bar{y}_{gCRR e}$ to the first degree of approximation as

$$MSE(\bar{y}_{gCRR e}) = \frac{(1-f)}{n} \bar{Y}^2 \left[C_y^2 + \frac{3}{4}\tau C_x^2 (3\tau - 4C) \right] \tag{37}$$

We note that the biases and mean squared errors of the estimators listed in Table 1 can be easily obtained from (35) and (37) just by putting suitable value of constants (a, b). We, now, consider the following estimators for population mean \bar{Y} :

$$\bar{y}_R^* = \bar{y} \left(\frac{a\bar{X} + b}{a\bar{x} + b} \right) \tag{38}$$

and its chained version is given by

$$\bar{y}_{CR}^* = \bar{y} \left(\frac{a\bar{X} + b}{a\bar{x} + b} \right)^2 \tag{39}$$

which is due Singh and Rathour [7]. Bahl and Tuteja [1] ratio-type exponential estimator for population \bar{y} as

$$\bar{y}_{Re}^* = \bar{y} \exp \left\{ \frac{a(\bar{X} - \bar{x})}{a(\bar{X} + \bar{x}) + 2b} \right\} \tag{40}$$

To the first degree of approximation, the bias and mean squared error of the estimators \bar{y}_R^* , \bar{y}_{CR}^* and \bar{y}_{Re}^* to first degree of approximation are respectively given by

$$B(\bar{y}_R^*) = \frac{(1-f)}{n} \bar{Y} C_x^2 \tau (\tau - C) \tag{41}$$

$$B(\bar{y}_{CR}^*) = \frac{(1-f)}{n} \bar{Y} C_x^2 \tau (3\tau - 2C) \tag{42}$$

$$B(\bar{y}_{Re}^*) = \frac{(1-f)}{8n} \bar{Y} C_x^2 \tau (3\tau - 4C) \tag{43}$$

$$MSE(\bar{y}_R^*) = \frac{(1-f)}{n} \bar{Y}^2 [C_y^2 + \tau C_x^2 (\tau - 2C)] \tag{44}$$

$$MSE(\bar{y}_{CR}^*) = \frac{(1-f)}{n} \bar{Y}^2 [C_y^2 + 4\tau C_x^2 (1 - C)] \tag{45}$$

$$MSE(\bar{y}_{Re}^*) = \frac{(1-f)}{n} \bar{Y}^2 \left[C_y^2 + \frac{1}{4}\tau C_x^2 (\tau - 4C) \right] \tag{46}$$

From (1), (44), (45) and (46) can be shown that the proposed general chain ratio-ratio type exponential estimator $\bar{y}_{gCRR e}$ is more efficient than:

(a) the usual unbiased estimator \bar{y} if

$$\text{either } C > \frac{3\tau}{4}, \tau > 0 \tag{47}$$

$$\text{or } C < \frac{3\tau}{4}, \tau < 0 \tag{48}$$

(b) ratio-type estimator \bar{y}_R^* if

$$\text{either } C > \frac{5\tau}{4}, \tau > 0 \tag{49}$$

$$\text{or } C < \frac{5\tau}{4}, \tau < 0 \tag{50}$$

(c) the chain ratio-type estimator \bar{y}_{CR}^* due to Singh and Rathour [7] if

$$\text{either } C > \frac{7\tau}{4}, \tau > 0 \tag{51}$$

$$\text{or } C < \frac{7\tau}{4}, \tau < 0 \tag{52}$$

(d) ratio-type exponential estimator \bar{y}_{Re}^* if

$$\text{either } C > \tau, \tau > 0 \tag{53}$$

$$\text{or } C < \tau, \tau < 0 \tag{54}$$

Combined the inequalities (47), (49), (51) and (53) we get that the proposed class of general chain-ratio-ratio-type estimator \bar{y}_{gCRRRe} is more efficient than $\bar{y}, \bar{y}_R^*, \bar{y}_{Re}^*, \bar{y}_{CR}^*$. If

$$\frac{5\tau}{4} < C < \frac{7\tau}{4}, \tau > 0 \tag{55}$$

Further combining the inequalities (48), (50), (52) and (54) we obtained that the proposed general chain ratio-ratio-type estimator is more efficient than $\bar{y}, \bar{y}_R^*, \bar{y}_{Re}^*, \bar{y}_{CR}^*$. If

$$\frac{7\tau}{4} < C < \frac{3\tau}{4}, \tau > 0 \tag{56}$$

Remark 3.1. When the correlation between the study variate y and the auxiliary variate x is negative (high) we define a chain product-product-type exponential estimator for the population mean \bar{y} as

$$\bar{y}_{gCPPe} = \bar{y} \left(\frac{a\bar{x} + b}{a\bar{X} + b} \right) \exp \left\{ \frac{a(\bar{x} - \bar{X})}{a(\bar{x} + \bar{X}) + 2b} \right\} \tag{57}$$

where (a, b) are same as defined for general chain ratio-ratio- type exponential estimator \bar{y}_{gCRRRe} defined by (32). A large number of product-product-type exponential estimators can be generated from the proposed general chain product-product-type exponential estimator given by (55) for suitable values of (a, b) . Using the slandered technique the bias and MSE of \bar{y}_{gCPPe} can be easily defined.

4. Empirical Study

To judge the performance of the proposed chain-ratio-ratio-type exponential estimator \bar{y}_{CRRe} over \bar{y} , \bar{y}_R , \bar{y}_{CR} , \bar{y}_{Re} and \bar{y}_{CRe} , we consider some data which is earlier considered by Kadilar and Cingi [3]. Description of the data is given below:

\bar{y} : the level of apple production

\bar{x} : the member of apple trees

and the required values are given in following scheme.

Table 2. Data Statistics

$N = 100$	$S_x = 49189.08$
$n = 20$	$S_y = 6425.00$
$\rho = 0.82$	$S_{yx} = 25777869226$
$\bar{X} = 24375.50$	$C_y = 4.18$
$\bar{Y} = 1536.77$	$C_x = 2.02$

We have computed the percent relative efficiencies (PREs) of the estimators \bar{y}_R , \bar{y}_{CR} , \bar{y}_{Re} and \bar{y}_{CRe} with respect to usual unbiased estimator \bar{y} for data statistics given in Table 2 and the findings are shown in Table 3.

Table 3. PREs of different estimators of the population mean \bar{y} with respect to \bar{y} .

Estimator	\bar{y}	\bar{y}_R	\bar{y}_{CRe}	\bar{y}_{CR}	\bar{y}_{Re}	\bar{y}_{CRRe}
PRE(\bullet, \bar{y})	100.00	226.76	226.76	286.48	151.03	297.05

Table 3 clearly indicates that the proposed chain ratio-ratio-type exponential estimator \bar{y}_{CRRe} is more efficient than usual unbiased estimator \bar{y} , usual ratio estimator \bar{y}_R , and proposed chain exponential estimator \bar{y}_{CRe} , Kadilar and Cingi [3] chain ratio-type estimator \bar{y}_{CR} and Bahl and Tuteja [1] ratio type exponential estimator \bar{y}_{Re} . Thus the procedure outlined in the paper is justified.

Acknowledgements

The authors are thankful to the editor and learned referees for their valuable suggestions on the research paper.

References

- [1] S.Bahl and R.K.Tuteja, *Ratio and product type exponential estimators*, Jour. Inform. Opt. Sci., 12(1)(1991), 159-164.
- [2] W.G.Cochran, *The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce*, Jour. Agri. Sci., 30(1940), 262-275.
- [3] C.Kadilar and H.Cingi, *Ratio estimators in stratified random sampling*, Biometrical Jour., 45(2)(2003), 218-225.
- [4] C.Kadilar and H.Cingi, *An improvement in estimating the population mean by using the correlation coefficient*, Hacett. Jour. Math. Statist, 35(1)(2006), 103-109.
- [5] M.N.Murthy, *Sampling theory and Method*, Statistical Publishing Society, Calcutta, (1967).
- [6] D.S.Robson, *Applications of multivariate polykays to the theory of unbiased ratio-type estimation*, Jour. Amer. Statist. Assoc., 52(280)(1957), 511-522.
- [7] H.P.Singh and A.Rathour, *A study on the class of chain ratio-type estimators*. Jour. Statist. Res. Iron, 7(2010), 187-199.

- [8] H.P.Singh and R.Tailor, *Use of known correlation coefficient in estimating the finite population mean*, Statist. Trans., 6(4)(2003), 555-560.
- [9] S.Singh, *Advanced Sampling Theory with Applications*, How Michael "selected" Amy. Kluwer Academic Publishers. The Netherlands, (2003).
- [10] J.Subramani and G.Kumarapandiyan, *Variance estimation using median of the auxiliary variable*, Inter. Jour. Prob. Statist. Applica., 1(3)(2012), 62-66.
- [11] J.Subramani and G.Kumarapandiyan, *Variance estimation using quartiles and their functions of an auxiliary variable*, Inter. Jour. Statist. Applica., 2(5)(2012), 67-72.
- [12] J.Subramani and G.Kumarapandiyan, *Estimation of finite population mean using deciles of an auxiliary variable*, Statistics in Transition-new series 14(1)(2013), 75-88.
- [13] J.Subramani and G.Kumarapandiyan, *New modified ratio estimator for estimation of population mean when median of the auxiliary variable is known*, Pak. Jour. Stat. Opera. Res., 9(2)(2013), 137-145.
- [14] A.K.P.C.Swain, *On an improved ratio type estimator of finite population mean in sample surveys*, Revista Invest. Opera., 35(1)(2014), 49-57.
- [15] L.N.Upadhyaya and H.P.Singh, *On a dual to ratio estimator for estimating finite population variance*, Nepal Math. Sci. Rep., 11(1)(1986), 37-42.
- [16] L.N.Upadhyaya and H.P.Singh, *Use of transformed auxiliary variable in estimating the finite population mean*, Biometrical Journal, 41(5)(1999), 627-636.
- [17] Z.Yan and B.Tain, *Ratio method to the mean estimation using coefficient of skewness of auxiliary variable*, ICICA, Part II, CCIS106(2010), 103-110.
- [18] B.V.S.Sisodia and V.K.Dwivedi, *A modified ratio estimator using coefficient of variation of auxiliary variable*, Jour. Indi. Soci. Agri. Statist, 33(1981), 13-18.
- [19] H.P.Singh, *A general class of estimators of ratio, product and mean using supplementary information on an auxiliary character in PPSWR sampling scheme*, Gujarat Statist. Rev., 13(2)(1986), 1-30.
- [20] R.S.Solanki, H.P.Singh and S.K.Pal, *Improved estimation of finite population mean in sample surveys*, Jour. Adva. Comp., 2(2)(2013), 70-78.