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$\breve{g}$-closed and $\breve{g}$-open Maps in Topological Spaces

Research Article

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Abstract: A set A in a topological space (X, τ) is said to be $\breve{g}$-closed set if cl(A)$\subseteq$U whenever A$\subseteq$U and U is B-open in X. In this paper, we introduce $\breve{g}$-closed map from a topological space X to a topological space Y as the image of every closed set is $\breve{g}$-closed, and also we prove that the composition of two $\breve{g}$-closed maps need not be a $\breve{g}$-closed map. We also obtain some properties of $\breve{g}$-closed maps.

MSC: 54C10, 54C08, 54C05.

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1. Introduction

Malghan [8] introduced the concept of generalized closed maps in topological spaces. Devi [5] introduced and studied sg-closed maps and gs-closed maps. Recently, Sheik John [16] defined ω-closed maps and studied some of their properties. In this paper, we introduce $\breve{g}$-closed maps, $\breve{g}$-open maps, $\breve{g}^\star$-closed maps and $\breve{g}^\star$-open maps in topological spaces and obtain certain characterizations of these classes of maps.

2. Preliminaries

Throughout this paper, (X, τ), (Y, σ) and (Z, η) (or X, Y and Z) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For any subset A of a space (X, τ), the closure of A, the interior of A and the complement of A are denoted by cl(A), int(A) and A^c respectively.

We recall the following definitions which are useful in the sequel.

Definition 2.1. *A subset A of a space (X, τ) is called semi-open set [7] if A$\subseteq$cl(int(A)). The complement of semi-open set is semi-closed.*

The semi-closure [4] of a subset A of X, denoted by scl(A), is defined to be the intersection of all semi-closed sets of (X, τ) containing A. It is known that scl(A) is a semi-closed set. For any subset A of an arbitrarily chosen topological space, the semi-interior [4] of A, denoted by sint(A), is defined to be the union of all semi-open sets of (X, τ) contained in A.

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Definition 2.2. *A subset A of a space (X, τ) is called:*

(1). a generalized closed (briefly, g-closed) set [6] if cl(A)⊆U whenever A⊆U and U is open in X. The complement of g-closed set is called g-open set;

(2). a $\hat{g}$-closed set [18](= ω-closed set [16]) if cl(A)⊆U whenever A⊆U and U is semi-open in X. The complement of $\hat{g}$-closed set is called $\hat{g}$-open set;

(3). a semi-generalized closed (briefly, sg-closed) set [2] if scl(A)⊆U whenever A⊆U and U is semi-open in X. The complement of sg-closed set is called sg-open set;

(4). a generalized semi-closed (briefly, gs-closed) set [1] if scl(A)⊆U whenever A⊆U and U is open in X. The complement of gs-closed set is called gs-open set;

(5). a $\ddot{g}$-closed set [10] if cl(A)⊆U whenever A⊆U and U is sg-open in X. The complement of $\ddot{g}$-closed set is called $\ddot{g}$-open set;

(6). ψ-closed set [9, 19] if scl(A)⊆U whenever A⊆U and U is sg-open in X. The complement of Ψ-closed set is called Ψ-open set;

(7). a A-closed set [11] if cl(A)⊆U whenever A⊆U and U is $\ddot{g}$-open in X. The complement of A-closed set is called A-open set;

(8). a B-closed set [11] if scl(A)⊆U whenever A⊆U and U is A-open in X. The complement of B-closed set is called B-open set;

(9). a $\breve{g}$-closed set [11] if cl(A)⊆U whenever A⊆U and U is B-open in X. The complement of $\breve{g}$-closed set is called $\breve{g}$-open set.

The collection of all $\breve{g}$-closed sets of X is denoted by $\breve{g}C(X)$.

Definition 2.3 ([12]).

(1). For any A⊆X, $\breve{g}$-int(A) is defined as the union of all $\breve{g}$-open sets contained in A. In symbols, $\breve{g}$-int(A) = ∪{G : G⊆A and G is $\breve{g}$-open}.

(2). For every set A⊆X, we define the $\breve{g}$-closure of A to be the intersection of all $\breve{g}$-closed sets containing A. In symbols, $\breve{g}$-cl(A)=∩{F : A⊆F ∈ $\breve{g}$ C(X)}.

Definition 2.4 ([12]). *Let $\tau_{\breve{g}}$ be the topology on X generated by $\breve{g}$-closure in the usual manner. That is, $\tau_{\breve{g}} = \{U \subseteq X : \breve{g}\text{-}cl(U^c) = U^c\}$.*

Definition 2.5 ([13]). *Let (X, τ) be a topological space. Let x be a point of X and G be a subset of X. Then G is called an $\breve{g}$-neighborhood of x (briefly, $\breve{g}$-nbhd of x) in X if there exists an $\breve{g}$-open set U of X such that x∈U⊆G.*

Definition 2.6. *A space X is called*

(1). T_ω-space if every ω-closed set in it is closed [16].

(2). $T_{\breve{g}}$-space if every $\breve{g}$-closed set in it is closed [15].

Definition 2.7. *A map f : (X, τ)→(Y, σ) is called*

(1). ω-continuous [16] if the inverse image of every closed set in (Y, σ) is ω-closed in (X, τ).

(2). ğ-continuous [14] if the inverse image of every closed set in (Y, σ) is ğ-closed in (X, τ).

(3). ğ-irresolute [14] if the inverse image of every ğ-closed set in (Y, σ) is ğ-closed in (X, τ).

(4). strongly ğ-continuous [14] if the inverse image of every ğ-open set in (Y, σ) is open in (X, τ).

(5). B-irresolute [14] if $f^{-1}(V)$ is B-open in (X, τ) for every B-open subset V in (Y, σ).

Proposition 2.8 ([12]). *For any A⊆X, the following hold:*

(1). ğ-int(A) is the largest ğ-open set contained in A.

(2). A is ğ-open if and only if ğ-int(A) = A.

Proposition 2.9 ([12]). *For any A⊆X, the following hold:*

(1). ğ-cl(A) is the smallest ğ-closed set containing A.

(2). A is ğ-closed if and only if ğ-cl(A) = A.

Proposition 2.10 ([12]). *For any two subsets A and B of (X, τ), the following hold:*

(1). If A⊆B, then ğ-cl(A)⊆ğ-cl(B).

(2). ğ-cl(A∩B)⊆ğ-cl(A)∩ğ-cl(B).

Theorem 2.11 ([14]). *A subset A of X is ğ-open if and only if F⊆int(A) whenever F is B-closed and F⊆A.*

Definition 2.12. *A map f : (X, τ)→(Y, σ) is called*

(1). g-closed [8] if f(V) is g-closed in (Y, σ) for every closed set V of (X, τ).

(2). sg-closed [5] if f(V) is sg-closed in (Y, σ) for every closed set V of (X, τ).

(3). gs-closed [5] if f(V) is gs-closed in (Y, σ) for every closed set V of (X, τ).

(4). ψ-closed [9] if f(V) is ψ-closed in (Y, σ) for every closed set V of (X, τ).

3. ğ-closed Maps

We introduce the following definition:

Definition 3.1. *A map f : (X, τ)→(Y, σ) is said to be ğ-closed if the image of every closed set in (X, τ) is ğ-closed in (Y, σ).*

Example 3.2. *Let X = Y = {a, b, c} with τ = {ϕ, {a}, {b}, {a, b}, X} and σ = {ϕ , {a, b}, Y}. Let f : (X, τ)→(Y, σ) be the identity map. Then f is an ğ-closed map.*

Proposition 3.3. *A map f : (X, τ)→(Y, σ) is ğ-closed if and only if ğ-cl(f(A)) ⊆ f(cl(A)) for every subset A of (X, τ).*

Proof. Suppose that f is ğ-closed and A⊆X. Then cl(A) is closed in X and so f(cl(A)) is ğ-closed in (Y, σ). We have f(A)⊆ f(cl(A)) and by Propositions 2.9 and 2.10, ğ-cl(f(A))⊆ ğ-cl(f(cl(A))) = f(cl(A)).

Conversely, let A be any closed set in (X, τ). Then A = cl(A) and so f(A) = f(cl(A))⊇ğ-cl(f(A)), by hypothesis. We have f(A)⊆ ğ-cl(f(A)). Therefore f(A) = ğ-cl(f(A)). i.e., f(A) is ğ-closed by Proposition 2.9 and hence f is ğ-closed. □

Proposition 3.4. *Let f : (X, τ)→(Y, σ) be a map such that $\breve{g}$-cl(f(A))⊆f(cl(A)) for every subset A⊆X. Then the image f(A) of a closed set A in (X, τ) is $\breve{g}$-closed in (Y, σ).*

Proof. Let A be a closed set in (X, τ). Then by hypothesis $\breve{g}$-cl(f(A))⊆f(cl(A)) = f(A) and so $\breve{g}$-cl(f(A)) = f(A). Therefore f(A) is $\breve{g}$-closed in (Y, σ). □

Theorem 3.5. *A map f : (X, τ)→(Y, σ) is $\breve{g}$-closed if and only if for each subset S of (Y, σ) and each open set U containing f^{-1}(S) there is an $\breve{g}$-open set V of (Y, σ) such that S⊆V and f^{-1}(V)⊆U.*

Proof. Suppose f is $\breve{g}$-closed. Let S⊆Y and U be an open set of (X, τ) such that f^{-1}(S)⊆U. Then V = $(f(U^c))^c$ is an $\breve{g}$-open set containing S such that f^{-1}(V)⊆ U.
For the converse, let F be a closed set of (X, τ). Then $f^{-1}((f(F))^c)$⊆ F^c and F^c is open. By assumption, there exists an $\breve{g}$-open set V in (Y, σ) such that $(f(F))^c$⊆ V and f^{-1}(V)⊆F^c and so F⊆$(f^{-1}(V))^c$. Hence V^c⊆f(F)⊆ f($(f^{-1}(V))^c$)⊆V^c which implies f(F) = V^c. Since V^c is $\breve{g}$-closed, f(F) is $\breve{g}$-closed and therefore f is $\breve{g}$-closed. □

Proposition 3.6. *If f : (X, τ)→(Y, σ) is B-irresolute $\breve{g}$-closed and A is an $\breve{g}$-closed subset of (X, τ), then f(A) is $\breve{g}$-closed in (Y, σ).*

Proof. Let U be an B-open set in (Y, σ) such that f(A)⊆U. Since f is B-irresolute, f^{-1}(U) is an B-open set containing A. Hence cl(A)⊆f^{-1}(U) as A is $\breve{g}$-closed in (X, τ). Since f is $\breve{g}$-closed, f(cl(A)) is an $\breve{g}$-closed set contained in the B-open set U, which implies that cl(f(cl(A)))⊆U and hence cl(f(A))⊆ U.Therefore, f(A) is an $\breve{g}$-closed set in (Y, σ). □

The following example shows that the composition of two $\breve{g}$-closed maps need not be a $\breve{g}$-closed.

Example 3.7. *Let (X, τ), (Y, σ) and f be as in Example 3.2. Let Z = {a, b, c} and η = { ϕ, {a}, {a, b}, Z}. Let g : (Y, σ)→(Z, η) be the identity map. Then both f and g are $\breve{g}$-closed maps but their composition g o f : (X, τ)→(Z, η) is not an $\breve{g}$-closed map, since for the closed set {a, c} in (X, τ), (g o f)({a, c}) = {a, c}, which is not an $\breve{g}$-closed set in (Z, η).*

Corollary 3.8. *Let f : (X, τ)→(Y, σ) be $\breve{g}$-closed and g : (Y, σ)→(Z, η) be $\breve{g}$-closed and B-irresolute, then their composition g o f : (X, τ)→(Z, η) is $\breve{g}$-closed.*

Proof. Let A be a closed set of (X, τ). Then by hypothesis f(A) is an $\breve{g}$-closed set in (Y, σ). Since g is both $\breve{g}$-closed and B-irresolute by Proposition 3.6, g(f(A)) = (g o f)(A) is $\breve{g}$-closed in (Z, η) and therefore g o f is $\breve{g}$-closed. □

Proposition 3.9. *Let f : (X, τ)→(Y, σ) and g : (Y, σ)→(Z, η) be $\breve{g}$-closed maps where (Y, σ) is a $T_{\breve{g}}$-space. Then their composition g o f : (X, τ)→(Z, η) is $\breve{g}$-closed.*

Proof. Let A be a closed set of (X, τ). Then by assumption f(A) is $\breve{g}$-closed in (Y, σ). Since (Y, σ) is a $T_{\breve{g}}$-space, f(A) is closed in (Y, σ) and again by assumption g(f(A)) is $\breve{g}$-closed in (Z, η). i.e., (g o f)(A) is $\breve{g}$-closed in (Z,η) and so g o f is $\breve{g}$-closed. □

Proposition 3.10. *If f : (X, τ)→(Y, σ) is $\breve{g}$-closed, g : (Y, σ)→(Z, η) is $\breve{g}$-closed, (resp. ψ-closed, sg-closed and gs-closed) and (Y, σ) is a $T_{\breve{g}}$-space, then their composition g o f : (X, τ)→(Z, η) is g-closed(resp. ψ-closed, sg-closed and gs-closed).*

Proof. Let A be a closed set of (X, τ). Then by assumption f(A) is $\breve{g}$-closed in (Y, σ). Since (Y, σ) is a $T_{\breve{g}}$-space, f(A) is closed in (Y, σ) and again by assumption g(f(A)) is g-closed (resp. ψ-closed, sg-closed and gs-closed) in (Z, η). i.e., (g o f)(A) is g-closed (resp. ψ-closed, sg-closed and gs-closed) in (Z, η) and so g o f is g-closed (resp. ψ-closed, sg-closed and gs-closed). □

Proposition 3.11. *Let f : (X, τ)→(Y, σ) be a closed map and g : (Y, σ)→(Z, η) be an $\breve{g}$-closed map, then their composition g o f : (X, τ)→(Z, η) is $\breve{g}$-closed.*

Proof. Let A be a closed set of (X, τ). Then by assumption f(A) is closed in (Y, σ) and again by assumption g(f(A)) is $\breve{g}$-closed in (Z, η). i.e., (g o f)(A) is $\breve{g}$-closed in (Z, η) and so g o f is $\breve{g}$-closed. □

Remark 3.12. *If f : (X, τ)→(Y, σ) is an $\breve{g}$-closed and g : (Y, σ)→(Z, η) is closed, then their composition need not be an $\breve{g}$-closed map as seen from the following example.*

Example 3.13. *Let (X, τ), (Y, σ) and f be as in Example 3.2. Let Z = {a, b, c} and η = {ϕ, {a}, {a, b}, Z}. Let g : (Y, σ)→(Z, η) be the identity map. Then f is an $\breve{g}$-closed map and g is a closed map. But their composition g o f : (X, τ)→(Z, η) is not an $\breve{g}$-closed map, since for the closed set {a, c} in (X, τ), (g o f)({a, c}) = {a, c}, which is not an $\breve{g}$-closed set in (Z, η).*

Theorem 3.14. *Let f : (X, τ)→(Y, σ) and g : (Y, σ)→(Z, η) be two maps such that their composition g o f : (X, τ)→(Z, η) is an $\breve{g}$-closed map. Then the following statements are true.*

(1). If f is continuous and surjective, then g is $\breve{g}$-closed.

(2). If g is $\breve{g}$-irresolute and injective, then f is $\breve{g}$-closed.

(3). If f is ω-continuous, surjective and (X, τ) is a T_ω-space, then g is $\breve{g}$-closed.

(4). If g is strongly $\breve{g}$-continuous and injective, then f is closed.

Proof. (1). Let A be a closed set of (Y, σ). Since f is continuous, f^{-1}(A) is closed in (X, τ) and since g o f is $\breve{g}$-closed, (g o f)(f^{-1}(A)) is $\breve{g}$-closed in (Z, η). That is g(A) is $\breve{g}$-closed in (Z, η), since f is surjective. Therefore g is an $\breve{g}$-closed map.

(2). Let B be a closed set of (X, τ). Since g o f is $\breve{g}$-closed, (g o f)(B) is $\breve{g}$-closed in (Z, η). Since g is $\breve{g}$-irresolute, g^{-1}((g o f)(B)) is $\breve{g}$-closed set in (Y, σ). That is f(B) is $\breve{g}$-closed in (Y, σ), since g is injective. Thus f is an $\breve{g}$-closed map.

(3). Let C be a closed set of (Y, σ). Since f is ω-continuous, f^{-1}(C) is ω-closed in (X, τ). Since (X, τ) is a T_ω-space, f^{-1}(C) is closed in (X, τ) and so as in (1), g is an $\breve{g}$-closed map.

(4). Let D be a closed set of (X, τ). Since g o f is $\breve{g}$-closed, (g o f)(D) is $\breve{g}$-closed in (Z, η). Since g is strongly $\breve{g}$-continuous, g^{-1}((g o f)(D)) is closed in (Y, σ). That is f(D) is closed set in (Y, σ), since g is injective. Therefore f is a closed map. □

In the next theorem we show that normality is preserved under continuous $\breve{g}$-closed maps.

Theorem 3.15. *If f : (X, τ)→(Y, σ) is a continuous, $\breve{g}$-closed map from a normal space (X, τ) onto a space (Y, σ), then (Y, σ) is normal.*

Proof. Let A and B be two disjoint closed subsets of (Y, σ). Since f is continuous, f^{-1}(A) and f^{-1}(B) are disjoint closed sets of (X, τ). Since (X, τ) is normal, there exist disjoint open sets U and V of (X, τ) such that $f^{-1}(A)\subseteq U$ and $f^{-1}(B)\subseteq V$. Since f is $\breve{g}$-closed, by Theorem 3.5, there exist disjoint $\breve{g}$-open sets G and H in (Y, σ) such that $A\subseteq G$, $B\subseteq H$, $f^{-1}(G)\subseteq U$ and $f^{-1}(H)\subseteq V$. Since U and V are disjoint, int(G) and int(H) are disjoint open sets in (Y, σ). Since A is closed, A is B-closed and therefore we have by Theorem 2.11, $A\subseteq$int(G). Similarly $B\subseteq$int(H) and hence (Y, σ) is normal. □

Analogous to an $\breve{g}$-closed map, we define an $\breve{g}$-open map as follows:

Definition 3.16. *A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be an $\breve{g}$-open map if the image f(A) is $\breve{g}$-open in (Y, σ) for each open set A in (X, τ).*

Proposition 3.17. *For any bijection $f : (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent:*

(1). $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is $\breve{g}$-continuous.

(2). f is $\breve{g}$-open map.

(3). f is $\breve{g}$-closed map.

Proof. (1) ⇒ (2) Let U be an open set of (X, τ). By assumption, $(f^{-1})^{-1}(U) = f(U)$ is $\breve{g}$-open in (Y, σ) and so f is $\breve{g}$-open.
(2) ⇒ (3) Let F be a closed set of (X, τ). Then F^c is open set in (X, τ). By assumption, $f(F^c)$ is $\breve{g}$-open in (Y, σ). That is $f(F^c) = (f(F))^c$ is $\breve{g}$-open in (Y, σ) and therefore f(F) is $\breve{g}$-closed in (Y, σ). Hence f is $\breve{g}$-closed.
(3) ⇒ (1) Let F be a closed set of (X, τ). By assumption, f(F) is $\breve{g}$-closed in (Y, σ). But $f(F) = (f^{-1})^{-1}(F)$ and therefore f^{-1} is $\breve{g}$-continuous. □

In the next two theorems, we obtain various characterizations of $\breve{g}$-open maps.

Theorem 3.18. *Assume that the collection of all $\breve{g}$-open sets of Y is closed under arbitrary union. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map. Then the following statements are equivalent:*

(1). f is an $\breve{g}$-open map.

(2). For a subset A of (X, τ), $f(int(A)) \subseteq \breve{g}\text{-}int(f(A))$.

(3). For each $x \in X$ and for each neighborhood U of x in (X, τ), there exists an $\breve{g}$-neighborhood W of f(x) in (Y, σ) such that $W \subseteq f(U)$.

Proof. (1) ⇒ (2). Suppose f is $\breve{g}$-open. Let $A \subseteq X$. Then int(A) is open in (X, τ) and so f(int(A)) is $\breve{g}$-open in (Y, σ). We have $f(int(A)) \subseteq f(A)$. Therefore by Proposition 2.8, $f(int(A)) \subseteq \breve{g}\text{-}int(f(A))$.
(2) ⇒ (3). Suppose (2) holds. Let $x \in X$ and U be an arbitrary neighborhood of x in (X, τ). Then there exists an open set G such that $x \in G \subseteq U$. By assumption, $f(G) = f(int(G)) \subseteq \breve{g}\text{-}int(f(G))$. This implies $f(G) = \breve{g}\text{-}int(f(G))$. By Proposition 2.8, we have f(G) is $\breve{g}$-open in (Y, σ). Further, $f(x) \in f(G) \subseteq f(U)$ and so (3) holds, by taking $W = f(G)$.
(3) ⇒ (1). Suppose (3) holds. Let U be any open set in (X, τ), $x \in U$ and $f(x) = y$. Then $y \in f(U)$ and for each $y \in f(U)$, by assumption there exists an $\breve{g}$-neighborhood W_y of y in (Y, σ) such that $W_y \subseteq f(U)$. Since W_y is an $\breve{g}$-neighborhood of y, there exists an $\breve{g}$-open set V_y in (Y, σ) such that $y \in V_y \subseteq W_y$. Therefore, $f(U) = \cup\{V_y : y \in f(U)\}$ is an $\breve{g}$-open set in (Y, σ) by the given condition. Thus f is an $\breve{g}$-open map. □

Theorem 3.19. *A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\breve{g}$-open if and only if for any subset S of (Y, σ) and for any closed set F containing $f^{-1}(S)$, there exists an $\breve{g}$-closed set K of (Y, σ) containing S such that $f^{-1}(K) \subseteq F$.*

Proof. Similar to Theorem 3.5. □

Corollary 3.20. *A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\breve{g}$-open if and only if $f^{-1}(\breve{g}\text{-}cl(B)) \subseteq cl(f^{-1}(B))$ for each subset B of (Y, σ).*

Proof. Suppose that f is $\breve{g}$-open. Then for any $B \subseteq Y$, $f^{-1}(B) \subseteq cl(f^{-1}(B))$. By Theorem 3.19, there exists an $\breve{g}$-closed set K of (Y, σ) such that $B \subseteq K$ and $f^{-1}(K) \subseteq cl(f^{-1}(B))$. Therefore, $f^{-1}(\breve{g}\text{-}cl(B)) \subseteq (f^{-1}(K)) \subseteq cl(f^{-1}(B))$, since K is an $\breve{g}$-closed set in (Y, σ).
Conversely, let S be any subset of (Y, σ) and F be any closed set containing $f^{-1}(S)$. Put $K = \breve{g}\text{-}cl(S)$. Then K is an $\breve{g}$-closed set and $S \subseteq K$. By assumption, $f^{-1}(K) = f^{-1}(\breve{g}\text{-}cl(S)) \subseteq cl(f^{-1}(S)) \subseteq F$ and therefore by Theorem 3.19, f is $\breve{g}$-open. □

Finally in this section, we define another new class of maps called $\breve{g}^{\star}$-closed maps which are stronger than $\breve{g}$-closed maps.

Definition 3.21. *A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be $\breve{g}^{\star}$-closed if the image f(A) is $\breve{g}$-closed in (Y, σ) for every $\breve{g}$-closed set A in (X, τ).*

For example the map f in Example 3.2 is an $\breve{g}^{\star}$-closed map. Analogous to $\breve{g}^{\star}$-closed map we can also define $\breve{g}^{\star}$-open map.

Remark 3.22. *Since every closed set is an $\breve{g}$-closed set we have $\breve{g}^{\star}$-closed map is an $\breve{g}$-closed map. The converse is not true in general as seen from the following example.*

Example 3.23. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be the identity map. Then f is an $\breve{g}$-closed but not $\breve{g}^{\star}$-closed map. Since $\{a, c\}$ is $\breve{g}$-closed set in (X, τ), but its image under f is $\{a, c\}$ which is not $\breve{g}$-closed set in (Y, σ).*

Proposition 3.24. *A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is $\breve{g}^{\star}$-closed if and only if $\breve{g}$-cl(f(A))$\subseteq$ f($\breve{g}$-cl(A)) for every subset A of (X, τ).*

Proof. Similar to Proposition 3.3. □

Proposition 3.25. *For any bijection $f : (X, \tau) \rightarrow (Y, \sigma)$, the following statements are equivalent:*

(1). $f^{-1} : (Y, \sigma) \rightarrow (X, \tau)$ is $\breve{g}$-irresolute.

(2). f is $\breve{g}^{\star}$-open map.

(3). f is $\breve{g}^{\star}$-closed map.

Proof. Similar to Proposition 3.17. □

Proposition 3.26. *If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is B-irresolute and $\breve{g}$-closed, then it is an $\breve{g}^{\star}$-closed map.*

Proof. The proof follows from Proposition 3.6. □

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