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Weakly $(\pi_{\mathcal{H}}, \lambda)$-continuity in Generalized Topologies

Research Article

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Abstract: In this paper, we introduce and study the notions of weakly π-$\mathcal{H}$-open sets and weakly $(\pi_{\mathcal{H}})$-continuity in hereditary generalized topological spaces. Also we prove that a function $f : (X, \mu, \mathcal{H}) \to (Y, \lambda)$ is (μ, λ)-continuous function if and only if it is weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous function and $(\delta_{\mathcal{H}}, \lambda)$-continuous function where $\mathcal{H}$ is μ-codense.

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1. Introduction and Preliminaries

A family μ of subsets of X is called a generalized topology(GT)[1] if $\emptyset \in \mu$ and closed under arbitrary union. The generalized topology μ is said to be strong [10], if $X \in \mu$. (X, μ) is called a quasi topology [6], if μ is closed under finite intersection. A subset A of a generalized topological space (X, μ) is called μ-σ-open [3] (resp. μ-π-open [3], μ-α-open [3], μ-β-open [3]) if $A \subset c_\mu(i_\mu(A))$ (resp. $A \subset i_\mu(c_\mu(A))$ $A \subset i_\mu(c_\mu(i_\mu(A)))$, $A \subset c_\mu(i_\mu(c_\mu(A)))$. c_σ is the intersection of all μ-σ-closed containing A. A hereditary class $\mathcal{H}$ of X is a non-empty collection of subsets of X such that $A \subset B$, $B \in \mathcal{H}$ implies $A \in \mathcal{H}$ [2]. In the paper [2], for a hereditary class $\mathcal{H}$, the operator $()^*$:exp X$\to$ exp X was introduced. An operator c^*_μ : exp X $\to$ exp X was defined by using the operator $()^*$ (i.e., for $A \subset X$) $c^*_\mu(A) = A \cup A^*$, which is monotonic, enlarging and idempotent. Some properties of operators $()^*$ and c^*_μ were investigated in [2]. For every subset A of X, with respect to μ and a hereditary class $\mathcal{H}$ of subsets of X, then $\mu^* = \{A \subset X / c^*_\mu(X - A) = X - A\}$ is generalized topology[2], and $i^*_\mu(A)$ will denote the interior of A in (X, μ^*). A function $f : (X, \mu, \mathcal{H}) \to (Y, \lambda)$ is said to be (μ, λ)-continuous if for every λ-open set U in Y implies that $f^{-1}(U)$ is μ-open set in X. A subset A of a hereditary generalized topological space $(X, \mu, \mathcal{H})$ is said to be α-$\mathcal{H}$-open [2](resp. β-$\mathcal{H}$-open [2], σ-$\mathcal{H}$-open, π-$\mathcal{H}$-open [2], δ-$\mathcal{H}$-open [2], t-$\mathcal{H}$-set [7], t^*-$\mathcal{H}$-set [7]) if $A \subseteq i_\mu(c^*_\mu(i_\mu(A)))$ (resp. $A \subseteq c_\mu(i_\mu(c^*_\mu(A)))$, $A \subseteq c^*_\mu(i_\mu(A))$, $A \subseteq i_\mu(c^*_\mu(A))$, $i_\mu(c^*_\mu(A)) \subseteq c^*_\mu(i_\mu(A))$, $i_\mu(c^*_\mu) = i_\mu(A)$, $i_\mu(c^*_\mu(i_\mu(A))) = i_\mu(A)$). If A is μ^*-closed [2] if $A^* \subset A$.

Lemma 1.1 ([2]). *Let $(X, \mu, \mathcal{H})$ be a space with a hereditary generalized topological space and A, $B \subset X$, Then the following hold.*

(a) If $A \subset B$, then $A^* \subset B^*$.

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(b) $A^* = c^*_\mu(A) \subset c^*_\mu(A^*)$.

(c) *If* $A \subset A^*$, *then* $c_\mu(A) = A^* = c^*_\mu(A) = c^*_\mu(A^*)$.

(d) *If* $U \in \mu$, *then* $U \cap A^* \subset (U \cap A)^*$.

Lemma 1.2 ([7]). *If* (X, μ) *is a quasi topology with a hereditary class* $\mathcal{H}$. *Then the following are hold.*

(a) $\mathcal{H}$ *is* μ*-codense if only if* $A \subset A^*$ *for every* $A \in \mu$.

(b) *If* $A \subset A^*$, *then* $A^* = c_\mu(A^*) = c_\mu(A) = c^*_\mu(A)$.

Lemma 1.3 ([9]). *If* (X, μ) *is GTS with a hereditary class* $\mathcal{H}$. *For* $A \subset X$,

(a) $c^*_\mu(A) = X\text{-}i^*_\mu(X\text{-}A)$

(b) $i_\mu(A) \subset i^*_\mu(A) \subset A$.

Lemma 1.4 ([3]). *In a hereditary generalized topological space* $(X, \mu, \mathcal{H})$, *the following hold.*

(a) $c_\sigma(A) = A \cup i_\mu(c_\mu(A))$,

(b) $c_\sigma(A) = i_\mu(c_\mu(A))$, *if* $A \in \mu$.

2. Generalized Weakly π-$\mathcal{H}$-open Sets

Definition 2.1. *A subset* A *of a hereditary generalized topological space* $(X, \mu, \mathcal{H})$ *is said to be weakly* π*-*$\mathcal{H}$*-open, if* $A \subset c_\sigma(i_\mu(c^*_\mu(A)))$. *A subset* A *of* X *is said to be weakly* π*-*$\mathcal{H}$*-closed if its complement is weakly* π*-*$\mathcal{H}$*-open.*

Proposition 2.2. *In a hereditary generalized topological space* $(X, \mu, \mathcal{H})$, *the following hold.*

(a) Every μ*-open is weakly* π*-*$\mathcal{H}$ *open.*

(b) Every π*-*$\mathcal{H}$*-open is weakly* π*-*$\mathcal{H}$*-open.*

(c) Every weakly π*-*$\mathcal{H}$*-open is* β*-*$\mathcal{H}$*-open.*

(d) Every μ*-*α*-open is weakly* π*-*$\mathcal{H}$*-open.*

(e) Every weakly π*-*$\mathcal{H}$*-open is* μ*-*π*-open.*

Proposition 2.3. *Let* $(X, \mu, \mathcal{H})$ *be a quasi topology with a hereditary class* $\mathcal{H}$ *where* $\mathcal{H}$ *is* μ*-codense and* $A \subset X$. *Then* A *is weakly* π*-*$\mathcal{H}$*-open if and only if* A *is* π*-*$\mathcal{H}$*-open.*

Proof. By Proposition 2.2, every π-$\mathcal{H}$-open is weakly π-$\mathcal{H}$-open. Conversely if A is weakly π-$\mathcal{H}$-open implies that $A \subset c_\sigma(i_\mu(c^*_\mu(A)))$. By Lemma 1.6, it follows that $c_\sigma(i_\mu(c^*_\mu(A))) = i_\mu(c_\mu(i_\mu(c^*_\mu(A)))) = i_\mu(c^*_\mu(i_\mu(c^*_\mu(A)))) = i_\mu(c^*_\mu(A))$, therefore, $A \subset i_\mu(c^*_\mu(A))$. Hence A is π-$\mathcal{H}$-open. □

Proposition 2.4. *Let* $(X, \mu, \mathcal{H})$ *be a hereditary generalized topological space and* A, $B \subset X$. *If* A *is weakly* π*-*$\mathcal{H}$*-open and* $A \subset B \subset c^*_\mu(A)$, *then* B *is* β*-*$\mathcal{H}$*-open.*

Proof. Suppose that $A \subset B \subset c^*_\mu(A)$ and A is weakly π-$\mathcal{H}$-open implies that $A \subset c_\sigma(i_\mu(c^*_\mu(A)))$. Since $B \subset c^*_\mu(A) \subset c^*_\mu(c_\sigma(i_\mu(c^*_\mu(A)))) \subset c^*_\mu(c_\mu(i_\mu(i_\mu(c^*_\mu(A))))) \subset c^*_\mu(c_\mu(i_\mu(c^*_\mu(A)))) \subset c_\mu(i_\mu(c^*_\mu(A))) \subset c_\mu(i_\mu(c^*_\mu(B)))$. Hence B is β-$\mathcal{H}$-open. □

Proposition 2.5. *Let $(X, \mu, \mathcal{H})$ be a hereditary generealized topological space and $A \subset X$. If A is both weakly π-$\mathcal{H}$-open and μ^*-closed, then A is μ-α-open.*

Proof. If A is weakly π-$\mathcal{H}$-open implies that $A \subset c_\sigma(i_\mu(c^*_\mu(A)))$ and A is μ^*-closed implies that $A^* \subset A$. Then $A \subset c_\sigma(i_\mu(c^*_\mu(A))) \subset i_\mu(c_\mu(i_\mu(c^*_\mu(A)))) \subset i_\mu(c_\mu(i_\mu(A^* \cup A))) \subset i_\mu(c_\mu(i_\mu(A)))$. Hence A is μ-α-open. □

Corollary 2.6. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and $A \subset X$. If $A \in \mathcal{H}$ and A is weakly π-$\mathcal{H}$-open, then A is μ-α-open.*

Theorem 2.7. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space. Then arbitrary union of weakly π-$\mathcal{H}$-open sets is weakly π-$\mathcal{H}$-open.*

Proof. Since A_α is weakly π-$\mathcal{H}$-open for each $\alpha \in \Delta$, we have $A_\alpha \subset c_\sigma(i_\mu(c^*_\mu(A_\alpha))) \subset c_\sigma(i_\mu(c^*_\mu(\cup_{\alpha\in\Delta} A_\alpha)))$ and hence $\cup_{\alpha\in\Delta} A_\alpha \subset c_\sigma(i_\mu(c^*_\mu(\cup_{\alpha\in\Delta} A_\alpha)))$. Hence $\cup_{\alpha\in\Delta} A_\alpha$ is weakly π-$\mathcal{H}$-open. □

Theorem 2.8. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and A, $B \subset X$. If A is weakly π-$\mathcal{H}$-open and B is μ-open, then $A \cap B$ is weakly π-$\mathcal{H}$-open.*

Proof. If A is weakly π-$\mathcal{H}$-open implies that $A \subset c_\sigma(i_\mu(c^*_\mu(A)))$ and B is μ-α-open implies that $B = i_\mu(B)$. Then $A \cap B \subset c_\sigma(i_\mu(c^*_\mu(A))) \cap i_\mu(B) = i_\mu(c_\mu(i_\mu(c^*_\mu(A)))) \cap i_\mu(B) \subset i_\mu(c_\mu(i_\mu(c^*_\mu(A))) \cap B) = i_\mu(c_\mu(i_\mu(c^*_\mu(A) \cap i_\mu(B)))) \subset c_\sigma(i_\mu(c^*_\mu(A)) \cap i_\mu(B)) \subset c_\sigma(i_\mu(c^*_\mu(A \cap i_\mu(B)))) \subset c_\sigma(i_\mu(c^*_\mu(A \cap B)))$. Hence $A \cap B$ is weakly π-$\mathcal{H}$-open. □

Remark 2.9. *The following example shows that the intersection of weakly π-$\mathcal{H}$-open sets need not be weakly π-$\mathcal{H}$-open.*

Example 2.10. *Let $X = \{a, b, c, d\}$, $\mu = \{\emptyset, \{c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}, X\}$ and $\mathcal{H} = \{\emptyset, \{a\}, \{b\}, \{d\}, \{a, d\}\}$. Consider $A = \{a, b, c\}$ and $B = \{a, b, d\}$. Now $c_\sigma(i_\mu(c^*_\mu(A))) = c_\sigma(i_\mu(\{a, b, c\})) = c_\sigma(\{a, b, c\}) = X \supset A$ and $c_\sigma(i_\mu(c^*_\mu(B))) = c_\sigma(i_\mu(\{a, b, d\})) = c_\sigma(\{b, d\}) = \{a, b, d\} \supset B$. Hence A and B are weakly π-$\mathcal{H}$-open sets. But $c_\sigma(i_\mu(c^*_\mu(A \cap B))) = c_\sigma(i_\mu(c^*_\mu(b))) = c_\sigma(i_\mu(\{b\})) = c_\sigma(\emptyset) = \emptyset \not\supseteq A \cap B$. Therefore $A \cap B$ is not weakly π-$\mathcal{H}$-open.*

Theorem 2.11. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and $A \subset X$. Then A is weakly π-$\mathcal{H}$-closed if and only if $i_\sigma(c_\mu(i^*_\mu(A))) \subset A$.*

Proof. If A is weakly π-$\mathcal{H}$-closed, then X-A is weakly π-$\mathcal{H}$-open which implies that X-$A \subset c_\sigma(i_\mu(c^*_\mu(X\text{-}A))) = X - i_\sigma(c_\mu(i^*_\mu(A)))$. Therefore $i_\sigma(c_\mu(i^*_\mu(A))) \subset A$. Coversely, assume that $i_\sigma(c_\mu(i^*_\mu(A))) \subset A$. Then X-$A \subset c_\sigma(i_\mu(c^*_\mu(X - A)))$. Hence X-A is weakly π-$\mathcal{H}$-open. Thus A is weakly π-$\mathcal{H}$-closed. □

Proposition 2.12. *Let A be a subset of a hereditary generalized topological space $(X, \mu, \mathcal{H})$ such that $A \subset A^*$. Then the following are equivalent.*

(a) A is π-$\mathcal{H}$-open.

(b) A is weakly π-$\mathcal{H}$-open,

(c) A is μ-π-open.

Proof. $(a) \Rightarrow (b)$ and $(b) \Rightarrow (c)$. Obvious.
$(c) \Rightarrow (a)$. Since $A \subset A^*$ implies that $A \subset i_\mu(A^*) = i_\mu(A \cup A^*) = i_\mu(c^*_\mu(A))$. Also A is μ-π-open implies that $A \subset i_\mu(c_\mu(A)) = i_\mu(c^*_\mu(A))$ and hence A is π-$\mathcal{H}$-open. □

Theorem 2.13. *Let A be a subset of hereditary generalized topological space $(X, \mu, \mathcal{H})$ such that $A \subset A^*$. Then A is weakly π-$\mathcal{H}$-open if and only if $c_\sigma(A) = c_\sigma(i_\mu(c^*_\mu(A)))$.*

Proof. Since $A \subset A^*$ implies that $c_\mu(A) = c^*_\mu(A)$. If A is weakly π-$\mathcal{H}$-open implies that $A \subset c_\sigma(i_\mu(c^*_\mu(A)))$ and so $c_\sigma(i_\mu(c^*_\mu(A))) = A \cup c_\sigma(i_\mu(c^*_\mu(A))) = A \cup i_\mu(c_\mu(i_\mu(c^*_\mu(A)))) = A \cup i_\mu(c_\mu(i_\mu(c_\mu(A)))) = A \cup i_\mu(c_\mu(A)) = c_\sigma(A)$.
Conversely, assume that $c_\sigma(i_\mu(c^*_\mu(A))) = c_\sigma(A)$, then $c_\sigma(i_\mu(c_\mu(A))) = A \cup i_\mu(c_\mu(A)) = A \cup i_\mu(c_\mu(i_\mu(c_\mu(A)))) = A \cup c_\sigma(i_\mu(c_\mu(A))) = A \cup c_\sigma(i_\mu(c^*_\mu(A)))$. Thus $A \subset c_\sigma(i_\mu(c^*_\mu(A)))$ and so A is weakly π-$\mathcal{H}$-open. □

Theorem 2.14. *Let $(X, \mu, \mathcal{H})$ be a quasi topology with a hereditary class $\mathcal{H}$ and $\mathcal{H}$ be μ-codense and $A \subset X$. Then the following are equivalent.*

(a) A is α-$\mathcal{H}$-open.

(b) A is π-$\mathcal{H}$-open and δ-$\mathcal{H}$-open.

(c) A is weakly π-$\mathcal{H}$-open and δ-$\mathcal{H}$-open.

Proof. $(a) \Rightarrow (b)$. If A is α-$\mathcal{H}$-open implies that $A \subset i_\mu(c^*_\mu(i_\mu(A))) \subset i_\mu(c^*_\mu(A))$. Hence A is π-$\mathcal{H}$-open. Since A is α-$\mathcal{H}$-open implies that $A \subset i_\mu(c^*_\mu(i_\mu(A))) \subset c^*_\mu(i_\mu(A))$, therefore, $i_\mu(c^*_\mu(A)) \subset i_\mu(c^*_\mu(c^*_\mu(i_\mu(A)))) \subset i_\mu(c^*_\mu(i_\mu(A))) \subset c^*_\mu(i_\mu(A))$. Hence A is δ-$\mathcal{H}$-open.
$(b) \Rightarrow (c)$. Let A be both π-$\mathcal{H}$-open and δ-$\mathcal{H}$-open. By Proposition 2.2, every π-$\mathcal{H}$-open is weakly π-$\mathcal{H}$-open. Hence A is weakly π-$\mathcal{H}$-open and δ-$\mathcal{H}$-open.
$(c) \Rightarrow (a)$. If A is weakly π-$\mathcal{H}$-open implies that $A \subset c_\sigma(i_\mu(c^*_\mu(A)))$ and $\mathcal{H}$ is μ-codense implies that $c^*_\mu(A) = c_\mu(A)$ for $A \in \mu$. Now $A \subset c_\sigma(i_\mu(c^*_\mu(A))) = i_\mu(c_\mu(i_\mu(c^*_\mu(A)))) \subset i_\mu(c_\mu(c^*_\mu(i_\mu(A)))) = i_\mu(c_\mu(c_\mu(i_\mu(A)))) = i_\mu(c_\mu(i_\mu(A))) = i_\mu(c^*_\mu(i_\mu(A)))$. Hence A is α-$\mathcal{H}$-open. □

Definition 2.15. *A subset A of a hereditary generalized topological space $(X, \mu, \mathcal{H})$ is said to be a strong t-$\mathcal{H}$-set (resp. $\mathcal{H}_\beta$-set), if $c_\sigma(i_\mu(c^*_\mu(A))) = i_\mu(A)$ (resp. $c_\mu(i_\mu(c^*_\mu(A))) = i_\mu(A)$. A subset A of X is said to be a strong $B_{\mathcal{H}}$-set, if $A = U \cap V$, where $U \in \mu$ and V is a strong t-$\mathcal{H}$-set. A subset A of X is said to be a $S\mathcal{H}_\beta$-set, if $A = U \cap V$, where $U \in \mu$ and V is a $\mathcal{H}_\beta$-set.*

Theorem 2.16. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space, $A \subset X$ and $\mathcal{H}$ be a μ-codense. Then A is t-$\mathcal{H}$-set if and only if A is a strong t-$\mathcal{H}$-set.*

Proof. If A is a strong t-$\mathcal{H}$-set, then $i_\mu(c^*_\mu(A)) \subset c_\sigma(i_\mu(c^*_\mu(A))) = i_\mu(A) \subset i_\mu(c^*_\mu(A))$, so, $i_\mu(c^*_\mu(A)) = i_\mu(A)$. Hence A is a t-$\mathcal{H}$-set.
Conversely, If A is a t-$\mathcal{H}$-set, then $i_\mu(c^*_\mu(A)) = i_\mu(A)$. Now $i_\mu(A) \subset A \subset A \subset c_\sigma(i_\mu(c^*_\mu(A))) = i_\mu(c_\mu(i_\mu(c^*_\mu(A)))) = i_\mu(c_\mu(i_\mu(A))) = i_\mu(c^*_\mu(i_\mu(A))) \subset c^*_\mu(i_\mu(A)) = i_\mu(A)$. Hence A is a strong t-$\mathcal{H}$-set. □

Theorem 2.17. *Let $(X, \mu, \mathcal{H})$ be a strong hereditary generalized topological spaces. If $\mathcal{H}$ is μ-codense and $\subset X$, then A is a strong t-$\mathcal{H}$-set if and only if A is both t^*-$\mathcal{H}$-set and a $B_{\mathcal{H}}$-set.*

Proof. Necessity. If A is a strong t-$\mathcal{H}$-set, then $i_\mu(A) = c_\sigma(i_\mu(c^*_\mu(A)))$ and so $i_\mu(c_\mu(i_\mu(c^*_\mu(A)))) \subset A$ which implies that $i_\mu(c^*_\mu(A)) \subset A$. Therefore $i_\mu(c^*_\mu(A)) \subset i_\mu(A)$. Hence $i_\mu(A) = i_\mu(c^*_\mu(A))$ and so A is a t-$\mathcal{H}$-set. Hence $A = X \cap A$ is a $B_{\mathcal{H}}$-set. Since $i_\mu(c^*_\mu(i_\mu(A))) \subset i_\mu(c^*_\mu(A)) \subset A$, we have $i_\mu(A) \subset i_\mu(c^*_\mu(i_\mu(A))) \subset i_\mu(A)$ and so $i_\mu(A) = i_\mu(c^*_\mu(i_\mu((A))))$. Hence A is t^*-$\mathcal{H}$-set.
Sufficiency. Since $\mathcal{H}$ is μ-codense, $c_\sigma(i_\mu(c^*_\mu(A))) \subset c_\sigma(i_\mu(c^*_\mu(U \cap V))) \subset c_\sigma(i_\mu(c^*_\mu(U) \cap c^*_\mu(V))) = c_\sigma(i_\mu(c^*_\mu(U)) \cap i_\mu(c^*_\mu(V))) = c_\sigma(i_\mu(c^*_\mu(U)) \cap i_\mu(V)) = i_\mu(c_\mu(i_\mu(c^*_\mu(U)) \cap i_\mu(V))) \subset i_\mu(c_\mu(i_\mu(c^*_\mu(U))) \cap c_\mu(i_\mu(V))) = i_\mu(c_\mu(i_\mu(c^*_\mu(U)))) \cap i_\mu(c_\mu(i_\mu(V))) = i_\mu(c_\mu(U)) \cap i_\mu(V) \subset c_\mu(i_\mu(c_\mu(U)) \cap i_\mu(V)) = c_\mu(i_\mu(c_\mu(U) \cap)) = c_\mu(i_\mu(c_\mu(U \cap i_\mu(V)))) = c_\mu(i_\mu(c_\mu(i_\mu(U \cap V)))) = c_\mu(i_\mu(U \cap V)) = c_\mu(i_\mu(A)) = c^*_\mu(i_\mu(A))$. Hence $c_\sigma(i_\mu(c^*_\mu(A))) \subset c^*_\mu(i_\mu(A))$ which implies that $i_\mu(c_\sigma(i_\mu(c^*_\mu(A)))) \subset i_\mu(c^*_\mu(i_\mu(A))) = i_\mu(A)$. Thus $i_\mu(A) \subset c_\sigma(i_\mu(c^*_\mu(A))) \subset i_\mu(A)$. Hence $c_\sigma(i_\mu(c^*_\mu(A))) = i_\mu(A)$ and so A is strong t-$\mathcal{H}$-set. □

Theorem 2.18. *Let $(X, \mu, \mathcal{H})$ be a strong hereditary generalized topological space and $A \subset X$. Then the following are equivalent.*

(a) A is μ-open.

(b) A is weakly π-$\mathcal{H}$-open and strong $B_{\mathcal{H}}$-set.

(c) A is β-$\mathcal{H}$-open and an $S\mathcal{H}_\beta$-set.

Proof. The implications $(a) \Rightarrow (b)$ and $(a) \Rightarrow (c)$ are obvious, Since X is a strong t-$\mathcal{H}$-set and a $\mathcal{H}_\beta$-set.
$(b) \Rightarrow (a)$. Since A is weakly π-$\mathcal{H}$-open, we have $A \subset c_\sigma(i_\mu(c^*_\mu(A))) = c_\sigma(i_\mu(c^*_\mu(U \cap V)))$, where $A = U \cap V$, $U \in \mu$ and V is strong t-$\mathcal{H}$-set. Hence $A \subset U \cap A \subset (U \cap c_\sigma(i_\mu(c^*_\mu(V)))) \subset (U \cap c_\mu(i_\mu(c^*_\mu(V)))) = U \cap i_\mu(V) = i_\mu(A)$. Hence A is μ-open.
$(c) \Rightarrow (a)$. Let A be β-$\mathcal{H}$-open and a $S\mathcal{H}_\beta$-set. Let $A = U \cap V$, where $U \in \mu$ and V is an $\mathcal{H}_\beta$-set. Since A is β-$\mathcal{H}$-open, $A \subset c_\mu(i_\mu(c^*_\mu(A)))$ and $A = U \cap V \subset U$,we have $A \subset U \cap A \subset U \cap c_\mu(i_\mu(c^*_\mu(A))) = U \cap c_\mu(i_\mu(c^*_\mu(U \cap V))) \subset U \cap c_\mu(i_\mu(c^*_\mu(U))) \cap c_\mu(i_\mu(c^*_\mu(V))) = U \cap i_\mu(V) = i_\mu(A)$. Hence A is μ-open. □

3. Generalized Weakly π-$\mathcal{H}$-continuous Functions

Definition 3.1. *A function $f : (X, \mu, \mathcal{H}) \to (Y, \lambda)$ is said to be weakly π-$\mathcal{H}$-continuous (resp. $B_{\mathcal{H}}$-continuous) if $f^{-1}(V)$ is weakly π-$\mathcal{H}$-open (resp. $B_{\mathcal{H}}$-open) in $(X, \mu, \mathcal{H})$ for every λ-open V of (Y, λ).*

Proposition 3.2. *For a function $f : (X, \mu, \mathcal{H}) \to (Y, \lambda)$, the following hold.*

(a) Every (μ, λ)-continuous is weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous.

(b) Every $(\pi_{\mathcal{H}}, \lambda)$-continuous is weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous.

(c) Every weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous is $(\beta_{\mathcal{H}}, \lambda)$-continuous.

Theorem 3.3. *Let $f : (X, \mu, \mathcal{H}_1) \to (Y, \lambda, \mathcal{H}_2)$ and $g : (Y, \lambda, \mathcal{H}_2) \to (Z, \eta)$ be two functions, where $\mathcal{H}_1$ and $\mathcal{H}_2$ are hereditary classes on X, Y and Z respectively. Then $g \circ f$ is weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous if f is weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous and g is (λ, η)-continuous.*

Proof. Let U be any η-open in (Z, η). Then g is (λ, η)-continuous, $g^{-1}(U)$ is λ-open in $(Y, \lambda, \mathcal{H}_2)$. Since f is weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$ is weakly π-$\mathcal{H}$-open in $(X, \mu, \mathcal{H}_1)$. Hence $g \circ f$ is weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous. □

Proposition 3.4. *Let $f : (X, \mu, \mathcal{H}) \to (Y, \lambda)$ be a function and $\mathcal{H}$ be μ-codense. Then the following are equivalent.*

(a) f is (μ, λ)-continuous.

(b) f is weakly $(\pi_{\mathcal{H}}, \lambda)$-continuious and $(B_{\mathcal{H}}, \lambda)$-continuous.

(c) f is $(\beta_{\mathcal{H}}, \lambda)$-continuous and a $(S\mathcal{H}_\beta, \lambda)$-continuous.

Proposition 3.5. *Let $f : (X, \mu, \mathcal{H}) \to (Y, \lambda)$ be a function and $\mathcal{H}$ be μ-codense. Then the following are equivalent.*

(a) f is $(\alpha_{\mathcal{H}}, \lambda)$-continuous.

(b) f is weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous and $(\delta_{\mathcal{H}}, \lambda)$-continuous.

(c) f is $(\pi_{\mathcal{H}}, \lambda)$-continuous and $(\delta_{\mathcal{H}}, \lambda)$-continuous.

Definition 3.6. *Let $(X, \mu, \mathcal{H})$ be a hereditary generalized topological space and $A \subset X$ and let $x \in X$. Then A is said to be a weakly $\pi_{\mathcal{H}}$-neighbourhood of x, if there exists weakly π-$\mathcal{H}$-open set U containing x such that $U \subset A$.*

Theorem 3.7. *Let $f : (X, \mu, \mathcal{H}) \to (Y, \lambda)$ be a function between the hereditary generalized topological space $(X, \mu, \mathcal{H})$ to the generalized topological space (Y, λ). Then the following are equivalent.*

(a) f is a weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous.

(b) for each $x \in X$ and each λ-open set V in Y with $f(x) \in V$, there exists weakly π-$\mathcal{H}$-open set U containing x such that $f(U) \subset V$.

(c) for each $x \in X$ and each λ-open set V in Y with $f(x) \in V$, $f^{-1}(U)$ is a weakly $\pi_{\mathcal{H}}$-neighbourhood of x.

(d) the inverse image of each λ-closed in (Y, λ) is weakly π-$\mathcal{H}$-closed.

Proof. $(a) \Rightarrow (b)$. Let $x \in X$ and let V be any λ-open set in Y such that $f(x) \in V$. Since f is a weakly $(\pi_{\mathcal{H}}, \lambda)$-continuous, $f^{-1}(V)$ is weakly π-$\mathcal{H}$-open set. By putting $U = f^{-1}(V)$ which is containing x, we have $f(U) \in V$.
$(b) \Rightarrow (c)$. Let V be any λ-open set in Y and let $f(x) \in V$. Then by (b), there exists weakly π-$\mathcal{H}$-open set U containing x such that $f(U) \subset V$. Therefore, $x \in U \subset f^{-1}(V)$. Hence $f^{-1}(V)$ is a weakly $\pi_{\mathcal{H}}$-neighbourhood of x.
$(c) \Rightarrow (a)$. Let V be λ-open in Y and let $f(x) \in V$. Then by (c), $f^{-1}(V)$ is a weakly $\pi_{\mathcal{H}}$-neighbourhood of x. Thus for each $x \in f^{-1}(V)$, there exists weakly π-$\mathcal{H}$-open set U_x containing x such that $x \in U_x \subset f^{-1}(V)$. Hence $f^{-1}(V) \subset \cup_{x \in f^{-1}(V)} U_x$ and so $f^{-1}(V)$ is weakly π-$\mathcal{H}$-open set.
$(b) \Rightarrow (d)$. Let F be a λ-closed in Y. Take $V = Y - F$. Then V is a λ-open in Y. Let $x \in f^{-1}(V)$, by (b), there exists weakly π-$\mathcal{H}$-open set W of X containing x such that $f(W) \subset V$. Thus, we obtain $x \in W \subset c_\sigma(i_\mu(c^*_\mu(W))) \subset c_\sigma(i_\mu(c^*_\mu(f^{-1}(W))))$ and hence $f^{-1}(V) \subset c_\sigma(i_\mu(c^*_\mu(f^{-1}(W))))$. This shows that $f^{-1}(V)$ is weakly π-$\mathcal{H}$-open set in X. Hence $f^{-1}(F) = X - f^{-1}(Y - F) = X - f^{-1}(F)$ is weakly π-$\mathcal{H}$-closed in X. □

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