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Semicircle : A Case Study on Concept of Trigonometrical Ratios of Acute Angles

Research Article

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Abstract: Trigonometric ratios of an acute angle in a right angle triangle express the relationship between the angle and length of its sides. Angles in a semicircle is a right angle. The angle inscribed in a semicircle is always 90° In the present paper, author tried to present the properties of semicircle in which it is possible to explain the concept regarding trigonometric ratios of acute angles.

Keywords: Semicircle,trigonometric ratios, acute angle.

1. Introduction

The concept of trigonometrical ratio of acute angle arises from the right angle triangles. Generally, Trigonometrical ratio of acute angle are considered on the basis of the sides of a triangle. The right angle triangle concept can be analyze with the help of a semi-circle. The geometrical properties on semicircle is also useful to understand the trigonometrical ratio of acute angle. The beauty of whole trigonometry lies on the trigonometrical ratio of acute angle. Therefore, whole conception is based on the sides of a triangle. The sides of a triangle are straight lines. Here, trigonometrical ratio of acute angle will be reduced from the geometrical properties of semicircle (i.e a curve) to a straight line.

2. Mathematical Relation Between Semicircle and Right Angle Triangle

Definition 2.1. *In the Latin semi means "half"and Semi circle is Half a circle. A closed shape consisting of half a circle and a diameter of that circle. An alternative definition is that it is an open arc. A semicircle is a half circle, formed by cutting a whole circle along a diameter line. Any diameter of a circle cuts it into two equal semicircles.*

Definition 2.2. *A triangle is said to be right angle triangle if one of its angle is equal to* 90°.

Theorem 2.3. *The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.*

Theorem 2.4. *Angles in a semicircle is a right angle. The angle inscribed in a semicircle is always* 90°.

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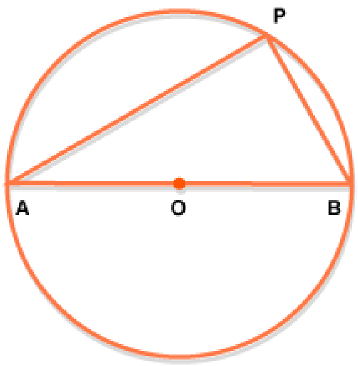

Figure 1. **Diagram to show a right angle triangle inside a circle [6]**

The angle inscribed by an arc at the center is double the angle subtended by it at any point on the remaining part of the circle. Hence, Angle in a semicircle is a right angle.

Theorem 2.5 (Thales Theorem [8, 9])**.** *The diameter of a circle always subtends a right angle to any point on the circle.*

Theorem 2.6 (Basic Proportionality Theorem (Thales theorem) [8, 9])**.** *If a line is drawn parallel to one side of a triangle intersecting other two sides, then it divides the two sides in the same ratio.*

For example, In $\triangle ABC$, if $DE \| BC$ and intersects AB in D and AC in E then $\frac{AD}{DB} = \frac{AE}{EC}$.

3. Application Of Properties On Semicircle/Circle In Trigonometric Ratio

The word 'trigonometry'is derived from the Greek words 'tri'(meaning three), 'gon'(meaning sides)and 'metron'(meaning measure).In fact, trigonometry is the study of relationships between sides and angle of a triangle. The study of ratio of sides of a right triangle with respect to its acute angles, called trigonometric ratios of the angles. The trigonometric ratios of an acute angle in a right triangles express the relationship between the angles and length of its sides. The concept regarding trigonometric ratio can be viewed inside a semicircle. Inside a semicircle a right angle triangle can be drawn.

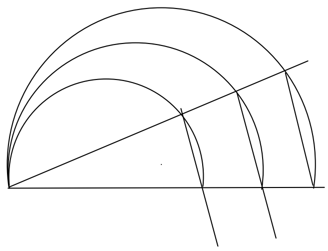

Figure 2. **Combination of semicircle and right angle triangle inside it**

So, there exist a method to explain the conception of trigonometric ratio of acute angle from uses/properties of semicircle.

3.1. Steps/Algorithm to Understand the Concept of Trigonometric Ratios of Acute Angles in Semicircle

Step I : Draw a semi circle. Angles in a semicircle is a right angle. The angle inscribed in a semicircle is always 90°.

Step II : If one of the angles of a triangle is 90° (a right angle), the triangle is called a right angled triangle. We indicate the 90° (right) angle by placing a box in its corner.) Because the three (internal) angles of a triangle add up to 180°, the other two angles are each less than 90°; that is they are acute.

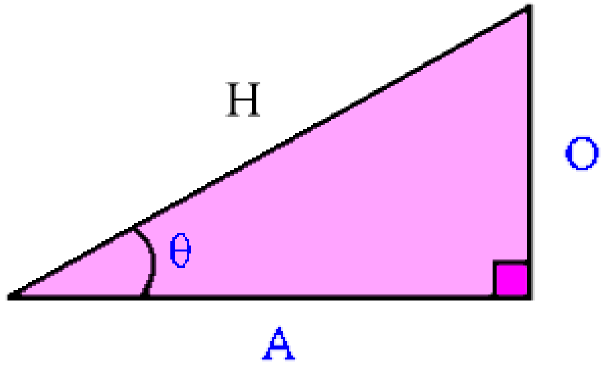

Figure 3. **A right angle triangle and Opposite sides and adjacent sides is relative to θ [10]**

Step III : Use of Pythagoras Theorem [11]: In any right angled triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides. This means that given any two sides of a right angled triangle, the third side is completely determined.

Step IV : Values of the trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle, if the angle remains the same [4].

From figure2, it is seen that the following types of triangle can be extracted/drawn with the combination of different types of semicircle.

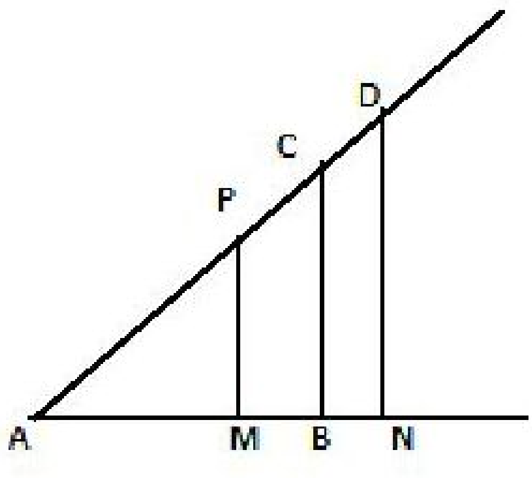

Figure 4. **Combination of right angle triangle**

For example, if we take a point P on the hypotenuse AC or a point D on AC extended, of the right triangle $\triangle ABC$. Here, PM is perpendicular to AB and DN perpendicular to AB. So, triangle $\triangle PAM$ are similar to triangle $\triangle CAB$. Therefore, by the property of similar triangles, the corresponding sides of the triangles are proportional. So,

$$\frac{AM}{AB} = \frac{AP}{AC} = \frac{MP}{BC}$$

Similarly, triangle $\triangle PAM$ and triangle $\triangle DAN$ are similar and

$$\frac{AM}{AN} = \frac{AP}{AD} = \frac{PM}{DN}$$

Angle	Right triangle $\triangle APM$	Right triangle $\triangle ABC$	Right triangle $\triangle AND$	Remarks
sinA	$\frac{PM}{AP}$	$\frac{BC}{AC}$	$\frac{DN}{AD}$	$\frac{Oppositeside}{hypotenuse}$
cosA	$\frac{AM}{AP}$	$\frac{AB}{AC}$	$\frac{AN}{AD}$	$\frac{adjacentside}{hypotenuse}$
tanA	$\frac{PM}{AM}$	$\frac{CB}{AB}$	$\frac{DN}{AN}$	$\frac{oppositesides}{adjacentside}$
cotA	$\frac{AM}{PM}$	$\frac{AB}{CB}$	$\frac{AN}{DN}$	$\frac{adjacentside}{oppositeside}$
secA	$\frac{AP}{AM}$	$\frac{AC}{AB}$	$\frac{AD}{AN}$	$\frac{hypotenuse}{adjacentside}$
cosecA	$\frac{AP}{PM}$	$\frac{AC}{BC}$	$\frac{AD}{DN}$	$\frac{adjacentside}{oppositeside}$

Hence , it is possible to see the trigonometrical ratios of acute angle by using the properties of semicircle.

4. Conclusion and Future Scope

The concept of trigonometrical ratios of acute angle can be used by any diameter of a circle. A curve like geometrical figure can be used to develop the trigonometry. Also there is a scope to understand mathematical relation between angles of a triangles inside a circle using trigonometry.

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