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Fuzzy Quasi-maximal Spaces

Research Article

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Abstract: In this paper the concept of Fuzzy Quasi-maximal spaces are introduced. Several properties and examples of fuzzy Quasi-maximal spaces are also studied. Relations between fuzzy Quasi-maximal space and fuzzy Baire space, fuzzy D-Baire space are studied.

Keywords: Fuzzy dense, fuzzy nowhere dense, fuzzy first category, fuzzy Quasi-maximal space, fuzzy Baire space and fuzzy D-Baire space.

1. Introduction

The theory of fuzzy sets was initiated by L.A.Zadeh in his classical paper [15] in the year 1965 as an attempt to develop a mathematically precise framework in which to treat systems or phenomena which cannot themselves be characterized precisely. The potential of fuzzy notion was realized by the researchers and has successfully been applied for investigations in all the branches of Science and Technology. The paper of C.L.Chang [9] in 1968 paved the way for the subsequent tremendous growth of the numerous fuzzy topological concepts.

The concept of Quasi-maximal spaces have been studied in classical topology in 'Chandan chattopadhyay'and 'Uttam kumar roy'in [8]. In this paper, we introduce the concept of Quasi-maximal spaces in fuzzy setting and investigate several characterizations of fuzzy Quasi-maximal spaces. In section 4, the relations between fuzzy Quasi-maximal spaces and fuzzy Baire space, fuzzy D-Baire space are studied.

2. Preliminaries

Now we introduce some basic notions and results used in the sequel. In this work by (X, T) or simply by X, we will denote a fuzzy topological space due to Chang [9].

Definition 2.1 ([9]). *Let λ and μ be any two fuzzy sets in a fuzzy topological space (X, T). Then we define:*

(i). $\lambda \vee \mu : X \to [0,1]$ as follows: $(\lambda \vee \mu)(x) = max\ \{\lambda(x),\ \mu(x)\}$;

(ii). $\lambda \wedge \mu : X \to [0,1]$ as follows: $(\lambda \wedge \mu)(x) = min\ \{\lambda(x),\ \mu(x)\}$;

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(iii). $\mu = \lambda^c \Leftrightarrow \mu(x) = 1 - \lambda(x)$.

For a family $\{\lambda_i / i \in I\}$ *of fuzzy sets in* (X, T)*, the union* $\psi = \vee_i \lambda_i$ *and intersection* $\delta = \wedge_i \lambda_i$ *are defined respectively as* $\psi(x) = sup_i\{\lambda_i(x), x \in X\}$ *and* $\delta(x) = inf_i\{\lambda_i(x), x \in X\}$.

Definition 2.2 ([1])**.** *Let* (X, T) *be a fuzzy topological space. For a fuzzy set* λ *of* X*, the interior and the closure of* λ *are defined respectively as* $int(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu \in T\}$ *and* $cl(\lambda) = \wedge\{\mu/\lambda \leq \mu, 1 - \mu \in T\}$.

Definition 2.3 ([14])**.** *A fuzzy set* λ *in a fuzzy topological space* (X, T) *is called fuzzy dense if there exists no fuzzy closed set* μ *in* (X, T) *such that* $\lambda < \mu < 1$ *That is* $cl(\lambda) = 1$.

Definition 2.4 ([12])**.** *A fuzzy set* λ *in a fuzzy topological space* (X, T) *is called fuzzy nowhere dense if there exists no non-zero fuzzy open set* μ *in* (X, T) *such that* $\mu < cl(\lambda)$ *That is,* $int\ cl(\lambda) = 0$.

Definition 2.5 ([14])**.** *Let* (X, T) *be a fuzzy topological space. A fuzzy set* λ *in* (X, T) *is called fuzzy first category set if* $\lambda = \bigvee_{i=1}^{\infty} \lambda_i$*, where* λ_i*'s are fuzzy nowhere dense sets in* (X, T)*. A fuzzy set which is not fuzzy first category set is called a fuzzy second category set in* (X, T).

Theorem 2.6 ([4])**.** *A fuzzy set* λ *in a fuzzy topological space* (X, T) *is called Fuzzy pre-open set if* $\lambda = pint(\lambda)$ *and fuzzy pre-closed set if* $\lambda = pcl(\lambda)$.

Definition 2.7 ([3])**.** *A fuzzy set* λ *in a fuzzy topological space* (X, T) *is called a fuzzy* G_δ*-set in* (X, T) *if* $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$*, where* $\lambda_i \in T$ *for* $i \in I$.

Definition 2.8. *Let* (X, T) *be any fuzzy topological space and* λ *be any fuzzy set in* (X, T)*. We define the fuzzy pre-closure, fuzzy semi-closure, fuzzy* β*-closure and the fuzzy pre-interior, fuzzy semi-interior, fuzzy* β*-interior of* λ *as follows:*

(i). $pcl(\lambda) = \wedge\{\mu/\lambda \leq \mu, \mu$ *is fuzzy pre-closed set of* $X\}$ *[10].*

(ii). $pint(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu$ *is fuzzy pre-open set of* $X\}$ *[10].*

(iii). $scl(\lambda) = \wedge\{\mu/\lambda \leq \mu, \mu$ *is fuzzy semi-closed set of* $X\}$ *[6].*

(iv). $sint(\lambda) = \vee\{\mu/\mu \leq \lambda, \mu$ *is fuzzy semi-open set of* $X\}$ *[6].*

(v). $\beta\text{-}cl(\lambda) = \wedge\{\mu/\lambda \leq \mu,\ \ \mu$ *is fuzzy* β*-closed set of* $X\}$ *[2].*

(vi). $\beta\text{-}int(\lambda) = \vee\ \{\ \mu\ \leq\ \lambda,\ \mu$ *is fuzzy* β*-open set of* $X\}$ *[2].*

Definition 2.9. *A fuzzy set* λ *in a fuzzy topological space* X *is called*

(i). Fuzzy pre-open if $\lambda \leq int\ cl(\lambda)$ *and fuzzy pre-closed if* $cl\ int(\lambda) \leq \lambda$ *[6].*

(ii). Fuzzy semi-open if $\lambda \leq cl\ int(\lambda)$ *and fuzzy semi-closed if* $int\ cl(\lambda) \leq \lambda$ *[6].*

(iii). Fuzzy β*-open if* $\lambda \leq cl\ int\ cl(\lambda)$ *and fuzzy* β*-closed if* $int\ cl\ int(\lambda) \leq \lambda$ *[5].*

(iv). $cl(\lambda) = \lambda$ *and fuzzy regular closed if* $cl\ int(\lambda) = \lambda$ *[1].*

Definition 2.10 ([12])**.** *Let (X, T) be a fuzzy topological space. Then* (X, T) *is called a fuzzy Baire space if* $int(\vee_{i=1}^{\infty}(\lambda_i)) = 0$*, where*$(\lambda_i)$*'s are fuzzy nowhere dense sets in* (X, T).

Definition 2.11 ([13])**.** *A fuzzy topological space* (X, T) *is called a fuzzy D-Baire space if every fuzzy first category set in* (X, T) *is a fuzzy nowhere dense set in* (X, T).

3. Fuzzy Quasi-maximal Space

Motivated by the classical concept introduced in [8] we shall now define:

Definition 3.1. *A fuzzy topological space (X,T) is said to be a Fuzzy Quasi-maximal space if for every fuzzy dense set λ in (X,T) with $int(\lambda) \neq 0$ (the null set), $int(\lambda)$ is also fuzzy dense in (X,T) .*

Example 3.2. *Let $X = \{a,b,c\}$. The fuzzy sets λ, μ and γ are defined on X as follows:*

$\lambda : X \to [0,1]$ defined as $\lambda(a) = 0.9; \lambda(b) = 0.7; \lambda(c) = 0.6$.

$\mu : X \to [0,1]$ defined as $\mu(a) = 0.5; \mu(b) = 0.6; \mu(c) = 0.8$.

$\gamma : X \to [0,1]$ defined as $\gamma(a) = 0.7; \gamma(b) = 0.8; \gamma(c) = 0.9$.

Then $T = \{0, \lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \wedge \mu), \lambda \vee \gamma, (\lambda \wedge \gamma), (\mu \vee \lambda \wedge \gamma), 1\}$ is a fuzzy topology on X.

Now consider the following fuzzy sets defined on X as follows:

$\alpha : X \to [0,1]$ defined as $\alpha(a) = 0.8; \alpha(b) = 0.6; \alpha(c) = 0.6$.

$\beta : X \to [0,1]$ defined as $\beta(a) = 0.7; \beta(b) = 0.6; \beta(c) = 0.7$.

$\delta : X \to [0,1]$ defined as $\delta(a) = 0.5; \delta(b) = 0.8; \delta(c) = 0.8$. The fuzzy dense sets in (X,T) are $\lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \wedge \mu), (\lambda \vee \gamma), (\lambda \wedge \gamma), (\mu \vee \lambda \wedge \gamma), \alpha, \beta, \delta$. Now $int(\lambda) \neq 0, int(\mu) \neq 0, int(\gamma) \neq 0, int(\lambda \vee \mu) \neq 0, int(\lambda \wedge \mu) \neq 0, int(\lambda \vee \gamma) \neq 0, int(\lambda \wedge \gamma) \neq 0, int(\mu \vee (\lambda \wedge \gamma)) \neq 0, int(\alpha) \neq 0, int(\beta) \neq 0, int(\delta) \neq 0$ and $int(\lambda), int(\mu), int(\gamma), int(\lambda \vee \mu), int(\lambda \wedge \mu), int(\lambda \vee \gamma), int(\lambda \wedge \gamma), int(\mu \vee (\lambda \wedge \gamma)), int(\alpha), int(\beta), int(\delta)$ is fuzzy dense in (X,T). Therefore (X,T) is fuzzy Quasi-maximal space.

Example 3.3. *Let $X = \{a,b,c\}$. The fuzzy sets λ and μ are defined on X as follows:*

$\lambda : X \to [0,1]$ defined as $\lambda(a) = 0.8; \lambda(b) = 0.1; \lambda(c) = 0.6$.

$\mu : X \to [0,1]$ defined as $\mu(a) = 0.2; \mu(b) = 0.9; \mu(c) = 0.5$.

Then $T = \{0, \lambda, \mu, (\lambda \vee \mu), (\lambda \wedge \mu), 1\}$ is a fuzzy topology on X.

Now consider the following fuzzy sets defined on X as follows:

$\alpha : X \to [0,1]$ defined as $\alpha(a) = 0.9; \alpha(b) = 0.9; \alpha(c) = 0.5$.

$\beta : X \to [0,1]$ defined as $\beta(a) = 0.9; \beta(b) = 0.6; \beta(c) = 0.5$.

$\delta : X \to [0,1]$ defined as $\delta(a) = 0.9; \delta(b) = 0.7; \delta(c) = 0.5$. The fuzzy sets $\lambda, (\lambda \vee \mu), \alpha, \beta, \delta$ are fuzzy dense in (X,T) and $int(\lambda) \neq 0, int(\lambda \vee \mu) \neq 0, int(\alpha) \neq 0, int(\beta) \neq 0, int(\delta) \neq 0$. Now the fuzzy sets α, β and δ are fuzzy dense in (X,T) and $int(\alpha) \neq 0, int(\beta) \neq 0, int(\delta) \neq 0$ but $int(\alpha), int(\beta)$ and $int(\delta)$ are not fuzzy dense sets in (X,T). Therefore (X,T) is not of fuzzy Quasi-maximal space.

Proposition 3.4. *Let (X,T) be a fuzzy Quasi-maximal space. If λ is a fuzzy dense set in (X,T), then $cl\ int(\lambda) = 1$.*

Proof. Let (X,T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense set in (X,T). Therefore $cl(\lambda) = 1$ with $int(\lambda) \neq 0$ and $int(\lambda)$ is fuzzy dense in (X,T). Since, λ be a fuzzy dense set in a fuzzy Quasi-maximal space (X,T). Suppose $int(\lambda) = \mu$ implies that $cl\ int(\lambda) = cl(\mu)$ implies that $cl\ int(\lambda) = 1$. Since $cl(\mu) = 1$ in (X,T). □

Proposition 3.5. *Let (X,T) be a fuzzy Quasi-maximal space, then there exists no non-zero fuzzy dense set λ with emty interior in (X,T).*

Proof. Let (X,T) be a fuzzy Quasi-maximal space. Suppose a fuzzy dense set λ is empty interior in (X,T). Therefore $int(\lambda) = 0$ implies that $cl\ int(\lambda) = 0$, by Proposition 3.4, $1 = 0$ which is a contradiction. Hence no non-zero fuzzy dense λ set with empty interior in (X,T). □

Proposition 3.6. *Let (X,T) be a fuzzy Quasi-maximal space. If any two fuzzy dense sets λ and μ is non-empty interior in (X,T), then $cl\ int(\lambda \vee \mu) = 1$ in (X,T).*

Proof. Let (X, T) be a fuzzy Quasi-maximal space. If the fuzzy dense sets λ and μ is non-empty interior in (X, T). Therefore by Proposition 3.4, $cl\ int(\lambda) = 1$ and $cl\ int(\mu) = 1$. Now $cl\ int(\lambda \vee \mu) \leq cl\ int(\lambda) \vee cl\ int(\mu)$ implies that $cl\ int(\lambda \vee \mu) \leq 1 \vee 1 = 1$. Hence $cl\ int(\lambda \vee \mu) = 1$ in (X, T). □

Remark 3.7. *If λ and μ are fuzzy dense sets and non-empty interior in a fuzzy Quasi-maximal space (X, T), then $cl\ int(\lambda \wedge \mu) \neq 1$ in (X, T). Consider the following example.*

Example 3.8. *Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ are defined on X as follows:*
$\lambda : X \to [0, 1]$ defined as $\lambda(a) = 1; \lambda(b) = 0.2; \lambda(c) = 0.7$.
$\mu : X \to [0, 1]$ defined as $\mu(a) = 0.3; \mu(b) = 0.1; \mu(c) = 0.2$.
$\gamma : X \to [0, 1]$ defined as $\gamma(a) = 0.7; \gamma(b) = 0.4; \gamma(c) = 1$.
Then $T = \{0, \lambda, \mu, \gamma, (\lambda \vee \gamma), (\lambda \wedge \gamma), 1\}$ is a fuzzy topology on X. Now the fuzzy dense sets in (X, T) are $\lambda, \gamma, \lambda \vee \gamma$ and $cl\ int(\lambda) = 1$, $cl\ int(\gamma) = 1$, $cl\ int(\lambda \vee \gamma) = 1$. Therefore (X, T) is a fuzzy Quasi-maximal space. Now the fuzzy sets λ and γ are fuzzy dense and non-empty interior in (X, T) then $cl\ int(\lambda \wedge \gamma) = 1 - \mu \neq 1$.

Proposition 3.9. *If a fuzzy dense set λ is in a fuzzy Quasi-maximal space (X, T), then $1 - \lambda$ is fuzzy nowhere dense set (X, T).*

Proof. Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T), then, by Proposition 3.4, $cl\ int(\lambda) = 1$ implies that $1 - cl\ int(\lambda) = 0$ imples that $int\ cl(1 - \lambda) = 0$. Hence $1 - \lambda$ is fuzzy nowhere dense set in (X, T). □

Proposition 3.10. *If a fuzzy dense set λ is in a fuzzy Quasi-maximal space (X, T), then,*

(i). $int\ cl\ int(\lambda) = 1$.

(ii). $cl\ int\ cl(1 - \lambda) = 0$.

(iii). $1 \leq cl\ int\ cl(\lambda)$.

(iv). $0 \geq int\ cl\ int(1 - \lambda)$.

Proof.

(i). Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T), then, by Proposition 3.9, $cl\ int(\lambda) = 1$ implies that $int\ cl\ int(\lambda) = int(1) = 1$. Hence $int\ cl\ int(\lambda) = 1$.

(ii). Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T), then by Proposition 3.4, $int\ cl(1 - \lambda) = 0$ implies that $cl\ int\ cl(1 - \lambda) = 0$.

(iii). Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T). Now $\lambda \leq cl(\lambda)$ implies that $cl\ int(\lambda) \leq cl\ int\ cl(\lambda)$ implies that $1 \leq cl\ int\ cl(\lambda)$. Since by Proposition 3.4 $cl\ int(\lambda) = 1$.

(iv). Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T), then, by above result (iii) of Proposition 3.10, $1 \leq cl\ int\ cl(\lambda)$ implies that $1 - 1 \geq int\ cl\ int(1 - \lambda)$. Hence $0 \geq int\ cl\ int(1 - \lambda)$.

□

Theorem 3.11 ([7]). *Let λ be a fuzzy set of a fuzzy topological space (X, T) then, $pcl(\lambda) \geq \lambda[\vee cl\ int(\lambda)]$.*

Proposition 3.12. *If a fuzzy set λ be a fuzzy dense in a fuzzy Quasi-maximal space (X, T) then, $pcl(\lambda) \geq 1$.*

Proof. Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T), then, by Theorem 3.11, $pcl(\lambda) \geq \lambda \vee [cl\ int(\lambda)] \geq \lambda \vee 1$ (Since by Proposition 3.4), implies that $pcl(\lambda) \geq 1$. □

Proposition 3.13. *If a fuzzy set λ be a fuzzy dense in a fuzzy Quasi-maximal space (X, T) then,*

(i). If λ is a fuzzy pre-closed, then $1 \leq \lambda$.

(ii). If λ is a fuzzy β-closed, then $1 \leq \lambda$.

(iii). If λ is a fuzzy semi-open, then $\lambda \leq 1$.

(iv). If λ is a fuzzy β-open, then $\lambda \leq 1$.

(v). If λ is a fuzzy regular-closed, then $1 = \lambda$.

Proof.

(i). Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T). If λ be a fuzzy pre-closed set in (X, T), $cl\ int(\lambda) \leq \lambda$ implies that $1 \leq \lambda$ (Since by Proposition 3.4). Hence $1 \leq \lambda$.

(ii). Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T). If λ be a fuzzy β-closed set in (X, T), $int\ cl\ int(\lambda) \leq \lambda$ implies that $1 \leq \lambda$ (Since by Proposition 3.4). Hence $1 \leq \lambda$.

(iii). Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T). If λ be a fuzzy semi-open set in (X, T), $\lambda \leq cl\ int(\lambda)$ implies that $\lambda \leq 1$ (Since by Proposition 3.4). Hence $\lambda \leq 1$.

(iv). Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T), If λ be a fuzzy β-open set in (X, T), $\lambda \leq cl\ int\ cl(\lambda)$ implies that $\lambda \leq 1$ (Since by Proposition 3.4). Hence $\lambda \leq 1$.

(v). Let (X, T) be a fuzzy Quasi-maximal space. The fuzzy set λ be a fuzzy dense in (X, T), If λ be a fuzzy regular-closed set in (X, T), $cl\ int(\lambda) = \lambda$ implies that $1 = \lambda$ (Since by Proposition 3.4). Hence $1 = \lambda$.

4. Fuzzy Quasi-maximal Space, Fuzzy Baire Space and Fuzzy D-Baire Space.

A fuzzy Quasi-maximal space need not be fuzzy Baire space. Consider the following example.

Example 4.1. *Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and γ are defined on X as follows:*

$\lambda : X \to [0, 1]$ *defined as* $\lambda(a) = 1; \lambda(b) = 0.2; \lambda(c) = 0.7$.

$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0.3; \mu(b) = 1; \mu(c) = 0.2$.

$\gamma : X \to [0, 1]$ *defined as* $\gamma(a) = 0.7; \gamma(b) = 0.4; \gamma(c) = 1$.

Then $T = \{0, \lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \wedge \mu), (\lambda \vee \gamma), (\lambda \wedge \gamma), (\mu \vee \gamma), (\mu \wedge \gamma), \lambda \vee (\mu \wedge \gamma), \mu \vee (\lambda \wedge \gamma), \gamma \wedge (\lambda \vee \mu), 1\}$ *is a fuzzy topology on X. Now* $\lambda, \mu, \gamma, (\lambda \vee \mu), (\lambda \wedge \mu), (\mu \vee \gamma), \lambda \vee (\mu \wedge \gamma), \mu \vee (\lambda \wedge \gamma)$ *are fuzzy dense sets in (X, T) and* $cl\ int(\lambda) = 1, cl\ int(\mu) = 1, cl\ int(\gamma) = 1, cl\ int(\lambda \vee \mu) = 1, cl\ int(\lambda \wedge \mu) = 1, cl\ int(\mu \vee \gamma) = 1, cl\ int(\lambda \vee (\mu \wedge \gamma)) = 1, cl\ int(\mu \vee (\lambda \wedge \gamma)) = 1$. *Hence (X, T) be a fuzzy Quasi-maximal space, but not a fuzzy Baire space. Since, the fuzzy nowhere dense sets in (X, T) are* $1 - \lambda, 1 - \mu, 1 - \gamma, 1 - (\lambda \vee \mu), 1 - (\lambda \vee \gamma), 1 - (\mu \vee \gamma), 1 - [\lambda \vee (\mu \wedge \gamma)], 1 - [\mu \vee (\lambda \wedge \gamma)]$. *Now* $int[(1 - \lambda) \vee (1 - \mu) \vee (1 - \gamma) \vee (1 - (\lambda \vee \mu)) \vee (1 - (\lambda \vee \gamma)) \vee (1 - (\mu \vee \gamma)) \vee (1 - (\lambda \vee \mu \wedge \gamma)) \vee (1 - (\mu \vee \lambda \wedge \gamma))] = \gamma \wedge (\lambda \vee \mu) \neq 0$.

Remark 4.2. *A fuzzy Baire space need not be fuzzy Quasi-maximal space. Consider the following example.*

Example 4.3. *Let $X = \{a, b, c\}$. The fuzzy sets λ, μ and $\alpha, \beta, \gamma, \delta$ are defined on X as follows:*

$\lambda : X \to [0, 1]$ *defined as* $\lambda(a) = 0.3; \lambda(b) = 0.6; \lambda(c) = 0.5$.

$\mu : X \to [0, 1]$ *defined as* $\mu(a) = 0.5; \mu(b) = 0.4; \mu(c) = 0.7$.

$\alpha : X \to [0, 1]$ *defined as* $\alpha(a) = 0.4; \alpha(b) = 0.5; \alpha(c) = 0.2$.

$\beta : X \to [0, 1]$ *defined as* $\beta(a) = 0.3; \beta(b) = 0.4; \beta(c) = 0.2$.

$\gamma : X \to [0, 1]$ *defined as* $\gamma(a) = 0.5; \gamma(b) = 0.7; \gamma(c) = 0.5$.

$\delta : X \to [0, 1]$ *defined as* $\delta(a) = 0.5; \delta(b) = 0.5; \delta(c) = 0.3$.

Then $T = \{0, \lambda, \mu, (\lambda \vee \mu), (\lambda \wedge \mu), 1\}$ is a fuzzy topology on X. Now $1 - \mu, 1 - (\lambda \vee \mu), \alpha, \beta, \delta$ are fuzzy nowhere dense sets in (X, T) implies that $int[(1 - \mu) \vee (1 - (\lambda \vee \mu)) \vee \alpha \vee \beta \vee \delta] = 0$. Hence (X, T) is a fuzzy Baire space, but not a fuzzy Quasi-maximal space. Since the fuzzy set γ is a fuzzy dense and $int(\gamma) \neq 0$ in (X, T) but $cl\ int(\gamma) = 1 - (\lambda \wedge \mu) \neq 1$.

Clearly, the following implications are true.

Fuzzy Quasi-maximal space $\not\Leftrightarrow$ Fuzzy Baire space

Proposition 4.4. *If $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$, where (λ_i)'s are fuzzy dense sets in a fuzzy Quasi-maximal space (X, T). Then (X, T) is a fuzzy Baire space.*

Proof. Let (λ_i) is are fuzzy dense sets in a fuzzy Quasi-maximal space (X, T). Then, by Proposition 3.9, $(1 - \lambda_i)$'s are fuzzy nowhere dense sets in (X, T). Now $cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 1$ implies that $1 - cl(\wedge_{i=1}^{\infty}(\lambda_i)) = 0$. Then $int(\vee_{i=1}^{\infty}(1 - \lambda_i)) = 0$, where $1 - (\lambda_i)$'s are fuzzy nowhere dense sets in (X, T). Hence (X, T) is a fuzzy Baire space. □

A fuzzy Quasi-maximal space need not be fuzzy D-Baire space.

Proof. By Example 4.1, (X, T) be a fuzzy Quasi-maximal space and $1 - (\lambda \wedge \mu)$ is fuzzy first category set in (X, T), then $int\ cl[1 - (\lambda \wedge \mu)] = int[1 - (\lambda \wedge \mu)] = \gamma \wedge (\lambda \vee \mu) \neq 0$. Hence (X, T) is not of fuzzy D-Baire space. Hence (X, T) is fuzzy Quasi-maximal space but not of fuzzy D-Baire space. □

Remark 4.5. *A fuzzy D-Baire space need not be fuzzy Quasi-maximal space.*

Proof. By Example 4.3, $1 - \mu$ is a fuzzy first category set in (X, T), then $int\ cl(1 - \mu) = 0$, therefore (X, T) is a fuzzy D-Baire space but (X, T) is not of fuzzy Quasi-maximal space (By Example 4.3). □

Clearly, the following implications are true.

Fuzzy Quasi-maximal space $\not\Leftrightarrow$ Fuzzy D-Baire space

Theorem 4.6 ([11]). *If λ is a fuzzy dense and fuzzy G_δ-set in a fuzzy topological space (X, T), then $(1 - \lambda)$ is a fuzzy first category set in (X, T).*

Proposition 4.7. *If every fuzzy dense set λ is fuzzy G_δ-set in a fuzzy Quasi-maximal space (X, T), then (X, T) is a fuzzy D-Baire space.*

Proof. Let λ be a fuzzy dense and fuzzy G_δ-set in a fuzzy Quasi-maximal space (X, T), then by Proposition 3.10, $(1 - \lambda)$ is a fuzzy nowhere dense set in (X, T). Therefore $int\ cl(1 - \lambda) = 0$. Now by Theorem 4.6, $(1 - \lambda)$ is a fuzzy first category set. Hence (X, T) is a fuzzy D-Baire space. □

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