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Soft β-Hausdorff and Soft β-Regular Spaces via Soft Ideals

Research Article

S.Brindha Devi[1]*, V.Inthumathi[1] and V.Chitra[1]

1 Department of Mathematics, N.G.M. College, Pollachi, Coimbatore, Tamilnadu, India.

Abstract: In this paper, we introduce the notions of soft β-I-Hausdorff spaces which is weaker than soft semi-I-Hausdorff spaces. Also, we establish the relationships between the existing spaces. Further, we define soft β-I-regular spaces and investigate some of their properties.

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1. Introduction

In many complicated problems arising in the fields of engineering, social science, economics, medical science, etc involving uncertainities, classical methods are found to be inadequate in recent times. Molodtsov [17] pointed out that the important existing theories such as Probability theory, Fuzzy set theory, Intuitionistic fuzzy set theory, Rough set theory etc which can be considered as a mathematical tools for dealing with uncertainities, have their own difficulties. So, in 1999 he initiated the concept of Soft set as a new mathematical tool for dealing with uncertainities. Soft set theory, initiated by Molodtsov [17], is free of the difficulties present in these theories. Later Maji et al [16] presented some new definitions on soft sets such as soft subset, the complement of a soft set etc. Recently, Shabir and Naz [18] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. Kandil et al [14] introduced the notion of soft semi separation axioms. Later, Arokia Lancy and Arockiarani [4] gave the definition of soft β-separation axioms.

The notion of soft ideal was initiated for the first time by Kandil et al. They also introduced the concept of soft local function. In 2014, Kandil et al [12] introduced the notion of soft quasi-I-open sets, soft quasi-I-closed sets and defined soft quasi-I-Hausdorff spaces. Also, he introduced soft semi Hausdorff spaces via soft ideals. Then, A.C.Guler and G.Kale [5] introduced the concept of soft I-regularity and soft I-normality. Kandil et al [13] later gave the definition of semi regular and semi normal spaces via soft ideals.

In this paper, we introduce the notions of soft β-I-Hausdorff spaces which is weaker than soft semi-I-Hausdorff spaces. Also, we establish the relationships between the existing spaces. Further, we define soft β-I-regular spaces and investigate some of their properties.

* *E-mail: brindhadevi195@gmail.com*

2. Preliminaries

Definition 2.1 ([17]). *Let X be an initial universal set and E be a set of parameters. Let $P(X)$ denote the power set of X and A be a non-empty subset of E. A pair (F, A) denoted by F_A is called a soft set over X, where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular $e \in A$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, A) and if $e \notin A$, then $F(e)=\phi$. That is $F_A=\{F(e){:}e\in A\subseteq E, F{:}A\to P(X)\}$. The family of all these soft sets are denoted by $SS(X)_A$.*

Definition 2.2 ([16]). *Let $F_A, G_B \in SS(X)_E$. Then F_A is a soft subset of G_B denoted by $F_A \tilde{\subseteq} G_B$, if*

(1) $A \subseteq B$, and

(2) $F(e)\subseteq G(e), \forall\, e \in A$. In this case, F_A is said to be a soft subset of G_B and G_B is said to be a soft superset of F_A.

Definition 2.3 ([3]). *The complement of a soft set (F, A), denoted by $(F, A)^{'}$, is defined by $(F, A)^{'} = (F^{'}, A)$, $F^{'} : A \to P(X)$ is a mapping given by $F^{'}(e) = X - F(e)$, $\forall e \in A$. Clearly $(F^{'})^{'}$ is the same as F and $((F, A)^{'})^{'} = (F, A)$.*

Definition 2.4 ([18]). *The difference of two soft sets (F, E) and (G, E) over the common universe X, denoted by $(F, E) - (G, E)$ is the soft set (H, E) where for all $e \in E$, $H(e) = F(e) - G(e)$.*

Definition 2.5 ([18]). *Let $x \in X$. Then x_E denote the soft set over X for which $x_E(e) = \{x\}$, $\forall e \in E$ and is called the singleton soft point.*

Definition 2.6 ([16]). *The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C=A\cup B$ and for all $e \in C$,*

$$H(e) = \begin{cases} F(e) & , \quad e \in A - B \\ G(e) & , \quad e \in B - A \\ F(e) \cup G(e) & , \quad e \in A \cap B \end{cases}$$

Definition 2.7 ([16]). *The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C=A\cap B$ and for all $e \in C$, $H(e)=F(e)\cap G(e)$.*

Definition 2.8 ([18]). *Let τ be a collection of soft sets over a universe X with a fixed set of parameters E, then $\tau \subseteq SS(X)_E$ is called a soft topology on X if*

(1) $\tilde{X}, \tilde{\phi} \in \tau$, where $\tilde{\phi}(e) = \phi$ and $\tilde{X}(e) = X$, $\forall e \in E$.

(2) The union of any number of soft sets in τ belongs to τ.

(3) The intersection of any two soft sets in τ belongs to τ. The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.9 ([18]). *Let (X, τ, E) be a soft topological space and $(F, A) \in SS(X)_E$. The soft closure of (F, A), denoted by $cl(F, A)$ is the intersection of all closed soft super sets of (F, A). Clearly $cl(F, A)$ is the smallest closed soft set over X which contains (F, A). i.e., $cl(F, A) = \tilde{\bigcap}\{(H,C){:}(H,C)$ is closed soft set and $(F,A) \tilde{\subseteq}(H, C)\}$.*

Definition 2.10 ([2]). *Let (X, τ, E) be a soft topological space and $(F, A) \in SS(X)_E$. The soft interior of (G,B), denoted by $int(G, B)$ is the union of all open soft subsets of (G,B). Clearly $int(G, B)$ is the largest open soft set over X which is contained in (G,B). i.e., $int(G, B) = \tilde{\bigcup}\{((H,C){:}$ (H,C) is an open soft set and $(H,C) \tilde{\subseteq}(G, B)\}$.*

Definition 2.11 ([2]). *The soft set $(F, E) \in SS(X)_E$ is called a soft point in X_E if there exist $x \in X$ and $e \in E$ such that $F(e)=\{x\}$ and $F(e^{'}) = \phi$ for each $e^{'} \in E - \{e\}$, and the soft point (F, E) is denoted by x_e.*

Definition 2.12 ([2]). *A soft set (G,B) in a soft topological space (X,τ,E) is called a soft neighborhood(briefly:nbd) of the soft point $x_e\tilde{\in}X_E$, if there exists an open soft set (H,C) such that $x_e\tilde{\in}(H,C)\tilde{\subseteq}(G,B)$. A soft set (G,B) in a soft topological space (X,τ,E) is called a soft neighborhood of the set (F,A), if there exists an open soft set (H,C) such that $(F,A)\tilde{\in}(H,C)\tilde{\subseteq}(G,B)$. The neighborhood system of a soft point x_e, denoted by $N\tau(x_e)$, is the family of all its neighborhoods.*

Definition 2.13 ([18]). *Let (X,τ,E) be a soft topological space and Y be a non-null subset of X. Then $\tau_Y=\{(F_Y,E):(F,E)\in\tau\}$ is said to be the relative soft topology on Y and (Y,τ_Y,E) is called a soft subspace of (X,τ,E).*

Theorem 2.14 ([18]). *Let (Y,τ,E) be a soft subspace of a soft topological space (X,τ,E) and $(F,E)\in SS(X)_E$. Then*

(1) If (F,E) is open soft set in Y and $\tilde{Y}\in\tau$, then $(F,E)\in\tau$.

(2) (F,E) is open soft set in Y if and only if $(F,E)=\tilde{Y}\tilde{\cap}\ (G,E)$ for some $(G,E)\in\tau$.

(3) (F,E) is closed soft set in Y if and only if $(F,E)=\tilde{Y}\tilde{\cap}(H,E)$ for some (H,E) is τ-closed soft set.

Definition 2.15 ([1]). *Let $SS(X)_A$ and $SS(Y)B$ be families of soft sets,$u:X\to Y$ and $p:A\to B$ be mappings. Let $f_{pu}:SS(X)_A\to SS(Y)B$ be a mapping. Then*

(1) If $(F,A)\in SS(X)_A$. Then the image of (F,A) under f_{pu}, written as $f_{pu}(F,A)=(f_{pu}(F),p(A))$, is a soft set in $SS(Y)_B$ such that

$$f_{pu}(F)(b)=\begin{cases}\bigcup_{x\in p^{-1}(b)\cap A}u(F(a)) & , \quad p^{-1}(b)\cap A\neq 0\\ \phi & , \quad otherwise\end{cases}$$

for all $b\in B$.

(2) If $(G,B)\in SS(Y)_B$. Then the inverse image of (G,B) under f_{pu}, written as $f_{pu}^{-1}(G,B)=(f_{pu}^{-1}(G),p^{-1}(B))$, is a soft set in $SS(X)_A$ such that

$$f_{pu}^{-1}(G)(a)=\begin{cases}u^{-1}(G(p(a))) & , \quad p(a)\in B\\ \phi & , \quad otherwise\end{cases}$$

for all $a\in A$. The soft function f_{pu} is called surjective if p and u are surjective, it is said to be injecive if p and u are injective.

Definition 2.16 ([2, 9, 15]). *Let (X,τ_1,A) and (Y,τ_2,B) be soft topological spaces and $f_{pu}:SS(X)_A\to SS(Y)B$ be a function. Then the function f_{pu} is called*

(1) Continuous soft if $f_{pu}^{-1}(G,B)\in\tau_1,\forall(G,B)\in\tau_2$.

(2) open soft if $f_{pu}(G,A)\in\tau_2$, $(G,A)\in\tau_1$.

(3) Homeomorphism soft if it is bijective, continuous soft and f_{pu}^{-1} is continuous soft.

Definition 2.17 ([8]). *A non-empty collection I of subsets of a set X is called an ideal on X, if it satisfies the following conditions:*

(1) $A\in I$ and $B\in I\Rightarrow A\cup B\in I$.

(2) $A\in I$ and $B\subseteq A\Rightarrow B\in I$. i.e., I is closed under finite unions and subsets.

Definition 2.18 ([11]). *Let $\tilde{I}$ be a non-null collection of soft sets over a universe X with a fixed set of parameters E, then $\tilde{I}\subseteq SS(X)_E$ is called a soft ideal on X with a fixed set E if*

(1) $(F,E)\in\tilde{I}$ *and* $(G,E)\in\tilde{I}\Rightarrow(F,E)\tilde{\cup}(G,E)\in\tilde{I}$,

(2) $(F,E)\in\tilde{I}$ *and* $(G,E)\tilde{\subseteq}(F,E)\Rightarrow(G,E)\in\tilde{I}$. *i.e.,* $\tilde{I}$ *is closed under finite soft unions and soft subsets.*

Definition 2.19 ([18]). *Let* (X,τ,E) *be a soft topological space and* $x,y\in X$ *such that* $x\neq y$. *Then* (X,τ,E) *is called a soft Hausdorff space or soft* T_2*-space if there exist open soft sets (F,E) and (G,E) such that* $x\in(F,E)$,$y\in(G,E)$ *and* $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$.

Definition 2.20 ([14]). *Let* (X,τ,E) *be a soft topological space and* $x,y\in X$ *such that* $x\neq y$. *Then* (X,τ,E) *is called a soft semi Hausdorff space or soft semi* T_2*-space if there exist semi open soft sets (F,E) and (G,E) such that* $x\in(F,E)$,$y\in(G,E)$ *and* $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$.

Definition 2.21 ([12]). *Let* $(X,\tau,E,\tilde{I})$ *be a soft topological space with soft ideal and* $x,y\in X$ *such that* $x\neq y$. *Then* $(X,\tau,E,\tilde{I})$ *is called a soft* β*-Hausdorff space or soft* β*-*T_2*-space if there exist* β*-open soft sets (F,E) and (G,E) such that* $x\in(F,E),y\in(G,E)$ *and* $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$.

Definition 2.22 ([12]). *Let* $(X,\tau,E,\tilde{I})$ *be a soft topological space with soft ideal and* $x,y\in X$ *such that* $x\neq y$. *Then* $(X,\tau,E,\tilde{I})$ *is called a soft* $\tilde{I}$*-Hausdorff space or soft* $\tilde{I}$*-*T_2*-space if there exist* $\tilde{I}$*-open soft sets (F,E) and (G,E) such that* $x\in(F,E),y\in(G,E)$ *and* $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$.

Definition 2.23 ([12]). *Let* $(X,\tau,E,\tilde{I})$ *be a soft topological space with soft ideal and* $x,y\in X$ *such that* $x\neq y$. *Then* $(X,\tau,E,\tilde{I})$ *is called a soft semi-*$\tilde{I}$*-Hausdorff space or soft-semi-*$\tilde{I}$*-*T_2*-space if there exist semi-*$\tilde{I}$*-open soft sets (F,E) and (G,E) such that* $x\in(F,E),y\in(G,E)$ *and* $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$.

Definition 2.24 ([12]). *Let* $(X,\tau,E,\tilde{I})$ *be a soft topological space with soft ideal and* $x,y\in X$ *such that* $x\neq y$. *Then* $(X,\tau,E,\tilde{I})$ *is called a soft quasi-*$\tilde{I}$*-Hausdorff space or soft quasi-*$\tilde{I}$*-*T_2*-space if there exist quasi-*$\tilde{I}$*-open soft sets (F,E) and (G,E) such that* $x\in(F,E),y\in(G,E)$ *and* $(F,E)\tilde{\cap}(G,E)=\tilde{\phi}$.

Definition 2.25 ([4]). *Let* (X,τ,E) *be a soft topological space over* X, *and let* (G,E) *be a soft closed set in* X *and* $x\in X$ *such that* $x\tilde{\notin}(G,E)$. *If there exist two soft* β*-open sets* (F_1,E) *and* (F_2,E) *such that* $x\tilde{\in}(F_1,E)$, $(G,E)\tilde{\subseteq}(F_2,E)$ *and* $(F_1,E)\tilde{\cap}(F_2,E)=\tilde{\phi}$, *then* (X,τ,E) *is called a soft* β*-regular space.*

Definition 2.26 ([5]). *Let* (X,τ,A) *be a soft topological space on* X *with* I *be soft ideal and let* (G,A) *be a soft closed set over* X *such that* $E^x_\alpha\tilde{\notin}(G,A)$ *for* $x\in X$. *If there exist disjoint soft open sets* (U,A) *and* (V,A) *such that* $E^x_\alpha\tilde{\in}(U,A)$, $(G,A)-(V,A)\in I$ *and then* (X,τ,A) *is called a soft* I*-regular space.*

Definition 2.27 ([13]). *Let* $(X,\tau,E,\tilde{I})$ *be a soft topological space with soft ideal and* (H,E) *be a semi closed soft set over* X *such that* $x\notin(H,E)$ *for* $x\in X$. *Then* $(X,\tau,E,\tilde{I})$ *is called a soft semi-*$\tilde{I}$*-regular space if there exist disjoint semi open soft sets* (F,E) *and* (G,E) *such that* $x\in(F,E)$ *and* $(H,E)-(G,E)\in\tilde{I}$.

Definition 2.28 ([11]). *Let* (X,τ,E) *be a soft topological space,*$\tilde{I}$ *be a soft ideal over* X *with the same set of parameters* E. *Then*

$(F,E)^*(\tilde{I},\tau)(F^*_E)=\tilde{\cup}\{x_e\in\epsilon:O_{x_e}\tilde{\cap}(F,E)\tilde{\notin}\tilde{I}\quad\forall O_{x_e}\in\tau\}$

Definition 2.29 ([11]). *Let* (X,τ,E) *be a soft topological space,* $\tilde{I}$ *be a soft ideal over* X *with the same set of parameters* E. *Then the operator* $cl^*:SS(X)_E\to SS(X)_E$ *defined by* $cl^*(F,E)=(F,E)\tilde{\cup}(F,E))^*$ *is a soft closure operator.*

Theorem 2.30 ([10]). *Let* $(X_1,\tau_1,A,\tilde{I})$ *be a soft topological space with soft ideal,* (X_2,τ_2,B) *be a soft topological space and* $f_{pu}:(X_1,\tau_1,A,\tilde{I})\to(X_2,\tau_2,B)$ *be a soft function. Then* $f_{pu}(\tilde{I})=\{f_{pu}((F,A)):(F,A)\in\tilde{I}\}$ *is a soft ideal on* X_2.

3. Soft β-I-Hausdorff Spaces

Definition 3.1. *Let $(X,\tau,E,\tilde{I})$ be a soft topological space with soft ideal and $x,y \in X$ such that $x \neq y$. Then $(X,\tau,E,\tilde{I})$ is called a soft β-$\tilde{I}$-Hausdorff space or soft β-$\tilde{I}$-T_2-space if there exist β-$\tilde{I}$-open soft sets (F,E) and (G,E) such that $x \in (F,E), y \in (G,E)$ and $(F,E)\tilde{\cap}(G,E) = \tilde{\phi}$.*

Proposition 3.2. *Every soft β-$\tilde{I}$-Hausdorff space is soft β-Hausdorff.*

Proof. Let $(X,\tau,E,\tilde{I})$ be a soft β-$\tilde{I}$-Hausdorff space. Since every soft β-$\tilde{I}$-open set is soft β-open, it follows that $(X,\tau,E,\tilde{I})$ is a soft β-Hausdorff space. □

The converse of the above proposition is not true as seen from the following example.

Example 3.3. *Let $X = \{h_1,h_2,h_3\}, E = \{e\}$ and $\tau = \{\tilde{X},\tilde{\phi},(F_1,E)\}$ where (F_1,E) is a soft set over X defined as $F_1(e) = \{h_1,h_2\}$.Let $\tilde{I} = \{\tilde{\phi},\{e,\{h_1\}\}\}$. Then $(X,\tau,E,\tilde{I})$ is a soft β-Hausdorff space, but not soft β-$\tilde{I}$-Hausdorff space. Because for every $h_1,h_2 \in X$ such that $h_1 \neq h_2$, there are no disjoint β-$\tilde{I}$-open soft sets (F,E) and (G,E) such that $h_1 \in (F,E), h_2 \in (G,E)$.*

The following examples shows that the notion of soft quasi-I-Hausdorff spaces and soft semi-I-Hausdorff spaces are independent of each other.

Example 3.4.

(1) Let $X = \{h_1,h_2\}, E = \{e\}$ and $\tau = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E)\}$ where $(F_1,E),(F_2,E)$ are soft sets over X defined as $F_1(e) = \{h_1\}$, $F_2(e) = \{h_2\}$. Let $\tilde{I} = SS(X)_E$. Then $(X,\tau,E,\tilde{I})$ is a soft semi-$\tilde{I}$-Hausdorff space, but not soft quasi-$\tilde{I}$-Hausdorff space.

(2) Let $X = \{h_1,h_2,h_3\}, E = \{e\}$ and $\tau = \{\tilde{X},\tilde{\phi},(F_1,E)\}$ where (F_1,E) is a soft set over X defined as $F_1(e) = \{h_1,h_3\}$. Let $\tilde{I} = \{\phi\}$. Then $(X,\tau,E,\tilde{I})$ is a soft quasi-$\tilde{I}$-Hausdorff space, but not soft semi-$\tilde{I}$-Hausdorff space.

Proposition 3.5. *Every soft quasi-$\tilde{I}$-Hausdorff space is soft β-$\tilde{I}$-Hausdorff.*

Proof. Let $(X,\tau,E,\tilde{I})$ be a soft quasi-$\tilde{I}$-Hausdorff space. Then $(F,E)\tilde{\subseteq}cl(int[(F,E)^*])\tilde{\subseteq}cl(int(cl^*(F,E)).$ Thus $(F,E)\tilde{\subseteq}cl(int(cl^*(F,E)))$. Hence $(X,\tau,E,\tilde{I})$ is a soft β-$\tilde{I}$-Hausdorff space. □

The converse of the above proposition is not true as seen from the following example.

Example 3.6. *Let $X = \{h_1,h_2,h_3\}, E = \{e\}$ and $\tau = \{\tilde{X},\tilde{\phi},(F_1,E),(F_2,E),(F_3,E)\}$ where $(F_1,E),(F_2,E),(F_3,E)$ are soft sets over X defined as $F_1(e) = \{h_1\}$, $F_2(e) = \{h_2\}$, $F_3(e) = \{h_1,h_2\}$. Let $\tilde{I} = SS(X)_E$. Then $(X,\tau,E,\tilde{I})$ is a soft β-$\tilde{I}$-Hausdorff space, but not soft quasi-$\tilde{I}$-Hausdorff space.*

Proposition 3.7. *Every soft semi-$\tilde{I}$-Hausdorff space is soft β-$\tilde{I}$-Hausdorff.*

Proof. It is obvious from their definitions. □

The converse of the above proposition is not true as seen from the following example.

Example 3.8. *In Example 3.4(2), it is seen that the soft ideal topological space $(X,\tau,E,\tilde{I})$ is soft β-$\tilde{I}$-Hausdorff but it is not soft semi-$\tilde{I}$-Hausdorff.*

Remark 3.9. *The following implications shows the relationships among the existing spaces.*

$$\begin{array}{ccccc} \text{Soft } I\text{-}T_2 & \Rightarrow & \text{Soft semi-}I\text{-}T_2 & \Rightarrow & \text{Soft semi-}T_2 \\ \Downarrow & & \Downarrow & & \Downarrow \\ \text{Soft quasi-}I\text{-}T_2 & \Rightarrow & \text{Soft } \beta\text{-}I\text{-}T_2 & \Rightarrow & \text{Soft } \beta\text{-}T_2 \end{array}$$

4. Soft β-regular Space via Soft Ideals

Definition 4.1. *Let $(X, \tau, E, \tilde{I})$ be a soft topological space with soft ideal and (H, E) be a β-closed soft set over X such that $x \notin (H, E)$ for $x \in X$. Then $(X, \tau, E, \tilde{I})$ is called a soft β-$\tilde{I}$-regular space if there exist disjoint β-open soft sets (F, E) and (G, E) such that $x \in (F, E)$ and $(H, E) - (G, E) \in \tilde{I}$.*

Example 4.2. *Let $X = \{h_1, h_2, h_3\}, E = \{e\}$ and $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E)\}$ where $(F_1, E), (F_2, E), (F_3, E)$ are soft sets over X defined as follows:*

$$F_1(e) = \{h_1\},$$

$$F_2(e) = \{h_2\},$$

$$F_3(e_) = \{h_1, h_2\}.$$

Then τ defines a soft topology on X. Let $\tilde{I} = SS(X)_E$. Then $(X, \tau, E, \tilde{I})$ is a soft β-$\tilde{I}$-regular space.

Proposition 4.3.

(1) Every soft β-regular space is soft β-$\tilde{I}$-regular.

(2) If $\tilde{I} = \tilde{\phi}$, then $(X, \tau, E, \tilde{I})$ is a soft β-regular space if and only if it is a soft β-$\tilde{I}$-regular space.

Theorem 4.4. *A soft subspace $(Y, \tau_Y, E, \tilde{I_Y})$ of a soft β-$\tilde{I}$-regular space $(X, \tau, E, \tilde{I})$ is soft β-$\tilde{I_Y}$-regular space.*

Let $y \in Y$ and (G,E) be a β-closed soft set in Y such that $y \notin (G, E)$. Then $(G, E) = (Y, E)\tilde{\cap}(H, E)$ for some β-closed soft set (H,E) in X from Theorem 2.14 and $y \notin (H, E)$. Since $(X, \tau, E, \tilde{I})$ is soft β-$\tilde{I}$-regular space, there exist disjoint β-open soft sets (F_1, E) and (F_2, E) in X such that $y \in (F_1, E)$ and $(H, E) - (F_2, E) \in \tilde{I}$. Then, we have $((H, E) - (F_2, E))\tilde{\cap}(Y, E) \in \tilde{I_Y}$. Since $(Y, E)\tilde{\cap}(F_1, E)$ and $(Y, E)\tilde{\cap}(F_2, E)$ are disjoint soft β-open sets, $(Y, \tau_Y, E, \tilde{I_Y})$ is a soft β-$\tilde{I_Y}$-regular space.

Proposition 4.5.

(1) Every soft I-regular space is soft semi-I-regular.

(2) Every soft semi-I-regular space is soft β-I-regular.

Proof. The proof is obvious from their definitions. □

The converse of the above proposition is not true as seen from the following example.

Example 4.6.

(1) Let $X = \{h_1, h_2, h_3\}, E = \{e\}$ and $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E)\}$ where (F_1, E) is a soft set over X defined as $F_1(e) = \{h_1, h_2\}$. Then τ defines a soft topology on X. Let $\tilde{I} = SS(X)_E$ be a soft ideal over X. Then $(X, \tau, E, \tilde{I})$ is a soft β-$\tilde{I}$-regular space but it is neither soft semi-I-regular nor soft I-regular.

(2) Let $X = \{h_1, h_2, h_3, h_4, h_5\}, E = \{e\}$ *and* $\tau = \{\tilde{X}, \tilde{\phi}, (F_1, E), (F_2, E), (F_3, E), (F_4, E)\}$ *where* $(F_1, E), (F_2, E), (F_3, E), (F_4, E)$ *are soft sets over* X *defined as follows:*

$$F_1(e) = \{h_1\},$$

$$F_2(e) = \{h_2\},$$

$$F_3(e) = \{h_1, h_2\},$$

$$F_4(e) = \{h_1, h_2, h_3\}.$$

Then τ *defines a soft topology on* X. *Let* $\tilde{I}$ *be the soft discrete topology over* X. *Then* $(X, \tau, E, \tilde{I})$ *is a soft semi-*$\tilde{I}$*-regular space but not a soft* I*-regular space.*

Theorem 4.7. *Let* $(X, \tau, E, \tilde{I})$ *be a soft ideal topological space and* $x \in X$. *Then the following are equivalent:*

(1) $(X, \tau, E, \tilde{I})$ *is a soft* β*-*I*-regular space.*

(2) For every β*-closed soft set (F,E) such that* $x_E \tilde{\cap} (F, E) = \tilde{\phi}$, *there exist disjoint* β*-open soft sets* (G_1, E) *and* (G_2, E) *such that* $x_E - (G_1, E) \in \tilde{I}$ *and* $(F, E) - (G_2, E) \in \tilde{I}$.

Proof. (1) $\Rightarrow$ (2) : Let (F,E) be a β-closed soft set such that $x_E \tilde{\cap} (F, E) = \tilde{\phi}$. Then $x \notin (F, E)$. By hypothesis, there exist disjoint β-open soft sets (G_1, E) and (G_2, E) such that $x_E - (G_1, E) \in \tilde{I}$ and $(F, E) - (G_2, E) \in \tilde{I}$.
(2) $\Rightarrow$ (1) : Let (F,E) be a β-closed soft set such that $x \notin (F, E)$. Then $x_E \tilde{\cap} (F, E) = \tilde{\phi}$. From (2), there exist β-open soft sets (G_1, E) and (G_2, E) such that $x_E - (G_1, E) \in \tilde{I}$ and $(F, E) - (G_2, E) \in \tilde{I}$. Therefore, $(X, \tau, E, \tilde{I})$ is a soft β-I-regular space. ☐

Theorem 4.8. *Let* (X, τ, E) *be a soft topological space and* $x \in X$. *Then the following are equivalent:*

(1) $(X, \tau, E, \tilde{I})$ *is soft* β*-*$\tilde{I}$*-regular space.*

(2) For every β*-open soft set* (A, E) *such that* $x \in (A, E)$, *there exists a* β*-open soft set* (F, E) *such that* $x \in (F, E)$ *and* $\beta cl(F, E) - (A, E) \in \tilde{I}$.

(3) For every β*-closed soft set* (B, E) *such that* $x \notin (B, E)$, *there exists a* β*-open soft set* (F, E) *such that* $x \notin (F, E)$ *and* $\beta cl(F, E) \tilde{\cap} (B, E) \in \tilde{I}$.

Proof. (1) $\Rightarrow$ (2): Let (A,E) be a β-open soft set such that $x \in (A, E)$. Then, $(A, E)'$ is a β-closed soft set such that $x \notin (A, E)'$. It follows by (1), there exists disjoint β-open soft sets (G,E) and (H,E) such that $x \in (G, E)$ and $(A, E)' - (H, E) = (H, E)' - (A, E) \in \tilde{I}$. Since $(G, E) \tilde{\cap} (H, E) = \tilde{\phi}$. Then, $(G, E) \tilde{\subseteq} (H, E)'$. So, $\beta cl(G, E) \tilde{\subseteq} (H, E)'$. Hence, $\beta cl(G, E) - (A, E) \tilde{\subseteq} (H, E)' - (A, E) \in \tilde{I}$.
(2) $\Rightarrow$ (3): Let (B,E) be a β-closed soft set such that $x \notin (B, E)$. Then, $(B, E)'$ is a β-open soft set such that $x \in (B, E)'$. From (2), there exists a β-open soft set (F,E) such that $x \in (F, E)$ and $\beta cl(F, E) - (B, E)' = \beta cl(F, E) \tilde{\cap} (B, E) \in \tilde{I}$.
(3) $\Rightarrow$ (1): Let (H,E) be a β-closed soft set such that $x \notin (H, E)$. From (3), there exists a β-open soft set (F,E) such that $x \in (F, E)$ and $\beta cl(F, E) \tilde{\cap} (H, E) = (H, E) - [\beta cl(F, E)]' \in \tilde{I}$, where (F,E) and $\beta cl(F, E)]'$ are disjoint β-open soft sets. Therefore, $(X, \tau, E, \tilde{I})$ is a soft β-$\tilde{I}$-regular space. ☐

Lemma 4.9 ([13]). *Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be an injective soft function. Then,* $f_{pu}[F_A - G_A] = f_{pu}(F_A) - f_{pu}(G_A)$.

Theorem 4.10. *Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a soft function which is bijective, irresolute soft and irresolute open soft. If* (X, τ_1, A) *is a soft* β*-*$\tilde{I}$*-regular space, then* (Y, τ_2, B) *is a soft* β*-*$f_{pu}(\tilde{I})$*-regular space.*

Proof. Let (G,B) be a β-closed soft set in Y and $y \in Y$ such that $y \notin (G,B)$. Since f_{pu} is a homeomorphism soft function, then $\exists x \in X$ such that $u(x) = y$ and $(F,A) = f_{pu}^{-1}(G,B)$ is a β-closed soft set in X such that $x \notin (F,A)$. By hypothesis, there exist disjoint β-open soft sets (H,A) and (K,A) such that $x \in (H,A)$ and $(K,A) - (F,A) \in \tilde{I}$. It follows that, $x \in H_A(e)$ for all $e \in A$ and $f_{pu}[(K,A) - (F,A)] \in f_{pu}(\tilde{I})$ from Theorem 2.30. Hence, $u(x) = y \in u[H_A(e)]$ for all $e \in A$ and $f_{pu}(K,A) - f_{pu}(F,A) \in f_{pu}(\tilde{I})$ from Lemma 4.9. Therefore, $y \in f_{pu}(H,A) = (W,B)$ and $(U,B) - (V,B) \in f_{pu}(\tilde{I})$, where (W,B) is β-open soft set in Y. Therefore, (Y, τ_2, B) is a soft β-$f_{pu}(\tilde{I})$-regular space. □

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