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On $**b$-$t**$-sets and $**b$-$**t$-sets in Topological Spaces

Research Article

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Abstract: The aim of this paper is to introduce the new concepts of $**t$-sets and $t**$-sets, $**b$-$t**$- sets,$**b$-$**t$-sets in the topological space and discuss some properties and characterization of these new notions.

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Keywords: $**b$-open sets, B-sets,$B*$-sets,t-sets,$t*$-sets.

1. Introduction

Levine [10], [1963] introduced the notion of semi-open sets and semi-continuity in topological spaces and studied their properties. Andrijevic [3] [1996] introduced a class of generalised open sets in a topological space called b-open sets. The class of b-open sets is contained in the class of semi-open sets and pre-open sets.Mashhour [11] [1982] introduced pre-open sets in topological spaces. From this he introduced some theorems on pre-continuous and weak pre-continuous mappings in topological space. The concept of semi-pre open sets was introduced by D.Andrijevic [4] [1986] and studied the properties of semi-pre-open sets,which is equivalent to the β-open sets. The notion of β-open sets was introduced by M.E.Abd El.Monsef [1]. Tong [15] [1989] introduced the concepts of t-set and B-set in topological spaces. Indira and Rekha [8] [2012] introduced the concepts of $*b$-open set,$**b$-open set,$t*$-set $B*$-set, locally $*b$-closed set, locally $**b$-closed set, $*b$-continuous, $**b$-continuous, $t*$-continuous, $B*$-continuous,locally $**b$-closed continuous, $D(c,*b)$-continuous, $D(c,**b)$-continuous functions in topological space it is also proved in [8], the class of $*b$-open set is both semi-open and pre-open and discussed the properties of the above sets. Rekha and Indira [13] [2013] introduced the notion of $**b$-t-set, $**b$-$t*$-set, $**b$-B-set, $**b$-$B*$-set, $**b$-semi open and $**b$-pre open sets in topological space and discuss the properties of the above sets.

1.1. Preliminaries

Definition 1.1 ([9]). *Given a subset A of a topological space (X,τ), the interior of A is defined as the union of all open sets contained in A,it is denoted by $int(A)$.And the closure of A is defined as the intersection of all closed sets containing A,it is denoted by $cl(A)$*

Definition 1.2. *A subset A of a space (X,τ) is said to be,*

(1). semi-open [10] if $A \subseteq cl(int(A))$.

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(2). pre-open [11] if $A \subseteq int\,(cl\,(A))$.

(3). an α*-open [12] if* $A \subseteq int\,(cl\,(int\,(A)))$.

(4). semi-pre-open [1, 4] if $A \subseteq cl\,(int\,(cl\,(A)))$.

(5). regular-open [3] if $A = int\,(cl\,(A))$.

Definition 1.3 ([3])**.** *A subset A of a space* (X, τ) *the semi-interior(resp. pre-interior,semi-pre-interior) of A, denoted by* $Sint\,(A)\,(resp.Pint\,(A)\,, Spint\,(A))$*,is the union of all semi-open(resp.pre-open,semi-pre-open) subsets of X contained in A.*

Definition 1.4 ([3])**.** *A subset A of a space* (X, τ) *the semi-closure(resp. pre-closure,semi-pre-closure) of A,denoted by* $Scl\,(A)\,(resp.Pcl\,(A)\,, Spcl\,(A))$ *is the intersection of all semi-closed(resp.pre-closed,semi-pre-closed) subsets of X containing A.*

Definition 1.5. *A subset A of a space* (X, τ) *is said to be,*

(1). b-open [3] if $A \subseteq cl\,(int\,(A)) \cup int\,(cl\,(A))$.

*(2). *b-open [6] if* $A \subseteq cl\,(int\,(A)) \cap int\,(cl\,(A))$.

*(3). b**-open [5] if* $A \subseteq int\,(cl\,(int\,(A))) \cup cl\,(int\,(cl\,(A)))$.

*(4). **b-open [6] if* $A \subseteq int\,(cl\,(int\,(A))) \cap cl\,(int\,(cl\,(A)))$.

Definition 1.6. *A subset A of a space* (X, τ) *is called,*

(1). t-set [15] if $int\,(A)$=$int\,(cl\,(A))$.

(2). B-set [15] if A =$U \cap V$*, where* $U \in \tau$ *and V is a t-set.*

(3). t-set [6] if* $cl\,(A)$=$cl\,(int\,(A))$.

(4). B-set [6] if A=*$U \cap V$*, where* $U \in \tau$ *and V is a t*-set.*

Definition 1.7 ([7])**.** *A subset A of a space* (X, τ) *is said to be,*

*(1). *b-t-set if* $int\,(A)$=$int\,({}^*bcl\,(A))$.

*(2). *b-t*-set if* $cl\,(A)$=$cl\,({}^*bint\,(A))$.

*(3). *b-B-set if* $A = U \cap V$*, where* $U \in \tau$ *and V is a *b-t-set.*

*(4). *b-B*-set if* $A = U \cap V$*, where* $U \in \tau$ *and V is a *b-t*-set.*

*(5). *b-semi-open if* $A \subseteq cl\,({}^*bint\,(A))$.

*(6). *b-pre-open if* $A \subseteq int\,({}^*bcl\,(A))$.

Definition 1.8 ([14])**.** *A subset A of a space* (X, τ) *is said to be,*

*(1). **b-t-set if* $int\,(A) = int\,({}^{**}bcl\,(A))$.

*(2). **b-t*-set if* $cl\,(A) = cl\,({}^{**}bint\,(A))$.

*(3). **b-B-set if* $A = U \cap V$*, where* $U \in \tau$ *and V is a **b-t-set.*

*(4). **b-B*-set if $A = U \cap V$, where $U \in \tau$ and V is a **b-t*-set.*

*(5). **b-semi-open if $A \subseteq cl\left({}^{**}bint\left(A\right)\right)$.*

*(6). **b-pre-open if $A \subseteq int\left({}^{**}bcl\left(A\right)\right)$.*

Lemma 1.9 ([2]). *Let A be a subset of a space (X, τ). Then*

(1). $bcl\left(int\left(A\right)\right) = int\left(bcl\left(A\right)\right) = int\left(cl\left(int\left(A\right)\right)\right)$.

(2). $bint\left(cl\left(A\right)\right) = cl\left(bint\left(A\right)\right) = cl\left(int\left(cl\left(A\right)\right)\right)$.

Result 1.10 ([2]). *In a extremally disconnected space (X, τ),*

(1). $int\left(cl\left(A\right)\right) = cl\left(int\left(A\right)\right)$.

(2). closure of every open set is open.

Result 1.11 ([8]).

(1). Every regular open set is a t-set.

(2). Every regular closed set is a t-set.*

(3). Every locally closed set is a B-set.

Result 1.12 ([14]).

*(1). Every **b-closed set is a **b-t-set.*

*(2). Every **b-open set is a **b-t*-set.*

*(3). Every open set is a **b-pre-open set.*

2. Properties of t**-sets and **t-Sets

Definition 2.1. *A subset A of a space (X, τ) is called*

*(1). **t**-set** if $int\left(A\right) = int\left(cl\left(int\left(A\right)\right)\right)$*

*(2). ****t-set** if $cl\left(A\right) = cl\left(int\left(cl\left(A\right)\right)\right)$*

Definition 2.2. *A subset A of a space (X, τ) is called*

*(1). **B**-set** if $A = U \cap V$, where $U \in \tau$ and V is a t**-set.*

*(2). ****B-set** if $A = U \cap V$, where $U \in \tau$ and V is a **t-set.*

Example 2.3. *Let $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $\tau' = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$*

*(1). The collection of t**-sets $= \{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$*

*(2). The collection of **t-sets $= \{X, \phi, \{c\}, \{d\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}\}$*

Result 2.4.

*(1). Every regular open set is a **t-set.*

*(2). Every regular closed set is a t**-set.*

Theorem 2.5. *Let A be a subset of (X, τ). Then to prove the following,*

*(1). A is a t**-set iff it is semi-pre-closed.*

*(2). A is a **t-set iff it is semi-pre-open.*

Proof.

(1). Let A be a t**-set.Then we have $int\,(A) = int\,(cl\,(int\,(A)))$. $\Rightarrow int\,(cl\,(int\,(A))) \subseteq A$. Therefore A is semi-pre-closed. Conversely, assume that A is semi-pre-closed.we have $int\,(cl\,(int\,(A))) \subseteq A$. $\Rightarrow int\,(cl\,(int\,(A))) \subseteq int\,(A)$. Since we also have $int\,(A) \subseteq int\,(cl\,(int\,(A)))$.We have,$int\,(A) = int\,(cl\,(int\,(A)))$. $\Rightarrow A$ is t**-set.

(2). Let A be a **t-set.Then we have $cl\,(A) = cl\,(int\,(cl\,(A)))$. $\Rightarrow A \subseteq cl\,(int\,(cl\,(A)))$. Therefore A is semi-pre-open. Conversely, assume that A is semi-pre-open.We have $A \subseteq cl\,(int\,(cl\,(A)))$.$\Rightarrow cl\,(A) \subseteq cl\,(int\,(cl\,(A)))$. Since we also have $cl\,(int\,(cl\,(A))) \subseteq A$. We have,$cl\,(A) = cl\,(int\,(cl\,(A)))$. $\Rightarrow A$ is **t-set.

□

Theorem 2.6.

*(1). If A and B are t**-sets, then $A \cap B$ is a t**-set.*

*(2). If A and B are **t-sets, then $A \cup B$ is a **t-set.*

Proof.

(1). Let A and B be a t**-set. Since

$$\begin{aligned}
int(A \cap B) &\subseteq int[cl(int(A \cap B))] \\
&\subseteq int[cl(int(A)) \cap cl(int(B))] \\
&= int(cl(int(A))) \cap int(cl(int(B))) = int(A \cap B) \\
int(A \cap B) &\subseteq int(cl(int(A \cap B))) \subseteq int(A \cap B) \\
int(A \cap B) &= int(cl(int(A \cap B))) \Rightarrow A \cap B \text{ is a } t^{**} - \text{set.}
\end{aligned}$$

(2). Let A and B be a **t-set. Since

$$\begin{aligned}
cl(A \cup B) &= cl(int(cl(A) \cup cl(B))) \\
&\subseteq cl[int(cl(A)) \cup int(cl(B))] \\
cl(A \cup B) &\subseteq cl(int(cl(A \cup B))) \\
Since,\ \ int(cl(A \cup B)) &\subseteq (A \cup B) \\
\Rightarrow cl(int(cl(A \cup B))) &\subseteq cl(A \cup B) \\
cl(A \cup B) &= cl(int(cl(A \cup B))) \Rightarrow A \cup B \text{ is a } {}^{**}t - \text{set.}
\end{aligned}$$

□

Theorem 2.7. *A set A is a t^{**}-set iff its complement is a $^{**}t$-set.*

Proof. Let A be a t^{**}-set.

$$\begin{aligned} Then \;\; int(A) &= int(cl(int(A))) \\ \Leftrightarrow X - int(A) &= X - int(cl(int(A))) \\ \Leftrightarrow cl(X - A) &= cl(int(cl(X - A))) \\ \Leftrightarrow cl(A^c) &= cl(int(cl(A^c))) \end{aligned}$$

$\Leftrightarrow A^c$ is a $^{**}t$-set. ☐

Theorem 2.8. *For a subset A of a space (X, τ),the following are equivalent:*

(1). A is open.

(2). A is α-open and B^{**}-set.

Proof. To prove: (1) $\Rightarrow$ (2):Let A be open,

$$\begin{aligned} A &= int(A) \\ cl(A) &= cl(int(A)). \\ \Rightarrow A \subseteq cl(int(A)) & \\ int(A) &\subseteq int(cl(int(A))). \\ \Rightarrow A &\subseteq int(cl(int(A))) \end{aligned}$$

$\Rightarrow A$ is α-open.let $U = A \in \tau$ and $V = X$ be a t^{**}-set containing A.$\Rightarrow A = U \cap V$

$\Rightarrow A$ is B^{**}-set.

To prove: (2) $\Rightarrow$ (1): Let A be a α-open and a B^{**}-set.$\Rightarrow$ $A = U \cap V$, where $U \in \tau$ and V is a t^{**}-set. Since V is a t^{**}-set.We have $int\,(V) = int\,(cl\,(int\,(V)))$. Since A is α-open.

$$\begin{aligned} A &\subseteq int(cl(int(A))) \\ &= int(cl(int(U \cap V))) \\ &= int(cl(int(U))) \cap int(cl(int(V))) \\ \Rightarrow U \cap V &= int(cl(U)) \cap int(V) \\ consider, \;\; U \cap V &= (U \cap V) \cap U \\ U \cap V &\subseteq U \cap int(V) \\ \Rightarrow V &\subseteq int(V) \\ Also \;\; int(V) &\subseteq V \\ int(V) &= V \\ A &= U \cap int(V) \\ A &= int(A) \end{aligned}$$

$\Rightarrow A$ is open. ☐

Theorem 2.9. *If A is regular open. Then it is semi-pre-open and **t-set.*

Proof. Given A is regular open. $A = int(cl(A)) .\Rightarrow cl(A) = cl(int(cl(A)))$.We have $A \subseteq cl(int(cl(A))) .\Rightarrow A$ is semi-pre-open. Also $cl(A) \subseteq cl(int(cl(A)))$.And $int(cl(A)) \subseteq A .\Rightarrow cl(int(cl(A))) \subseteq cl(A)$. We get $cl(A) = cl(int(cl(A))) .\Rightarrow A$ is semi-pre-open and **t-set. ☐

Example 2.10. *The converse of the above theorem is not true is verified by the following example. Let $X = \{a, b, c, d\}$*
$\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$
$\tau' = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. *The set $\{a, c\}$ is a semi-pre-open and **t-set but it is not a regular open set.*

Theorem 2.11. *If A is regular closed. Then it is semi-pre-closed and t**-set.*

Proof. Given A is regular closed. $A = cl(int(A)) .\Rightarrow int(A) = int(cl(int(A)))$.We have $(int(cl(int(A))) \subseteq A .\Rightarrow A$ is semi-pre-closed. Also $int(cl(int(A))) \subseteq int(A)$.And $A \subseteq cl(int(A)) .\Rightarrow int(A) \subseteq int(cl(int(A)))$. We get $int(A) = int(cl(int(A))) .\Rightarrow A$ is semi-pre-closed and t**-set. ☐

Example 2.12. *The converse of the above theorem is not true is verified by the following example. Let $X = \{a, b, c, d\}$*
$\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$
$\tau' = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. *Here $\{a\}$ is a semi-pre-closed and t**-set but it is not a regular closed set.*

Result 2.13.

(1). The intersection of t-set and t-set is a t-set.*

*(2). The intersection of t**-set and **t-set is a t**-set.*

Result 2.14.

(1). The union of t-set and t-set is a t*-set.*

*(2). The union of t**-set and **t-set is a **t-set.*

Theorem 2.15. *Let (X, τ) be a extremally disconnected space.Let A be a t-set iff it is a t**-set.*

Proof. Given that (X, τ) be a extremally disconnected space.Let A be a t-set. int(A)=int(cl(A))=cl(int(A)).
int(A)=int(cl(int(A))) $\Rightarrow A$ is a t**-set.Conversely, Assume that A be a t**-set.
int(A)=int(cl(int(A)))=int(int(cl(A)))=int(cl(A)) $\Rightarrow A$ is a t-set. ☐

Theorem 2.16. *Let A be a t**-set and open set. Then it is a b**-open.*

Proof. Given that A is a t**-set and open set.$int(A) = int(cl(int(A)))$.Since A is open.$\Rightarrow int(A) = A$. From this we get,$A = int(cl(int(A))) .\Rightarrow A \subseteq int(cl(int(A)))$.Therefore A is b**-open. ☐

Example 2.17. *The converse of the above theorem is not true is verified from the following example.*
Let $X = \{a, b, c, d\}$
$\tau = \{X, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{a, c, d\}\}$
$\tau' = \{X, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c\}\}$

*(1). The collection of t**-set and open $= \{x, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{b, d\}, \{a, c, d\}\}$*

*(2). The collection of b**-sets $= \{X, \phi, \{a\}, \{b\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}, \{b, c, d\}\}$*

*From the above $\{a, d\}$ be a b**-open but it is not t**-set and open.*

3. $^{**}b$-$^{**}t$-Sets and $^{**}b$-t^{**}-Sets in the Topological Space

Definition 3.1. *A subset A of a space (X,τ) is called $^{**}b$-t^{**}-set if $int\,(A) = int\,(cl\,(^{**}bint\,(A)))$.*

Definition 3.2. *A subset A of a space (X,τ) is called $^{**}b$-$^{**}t$-set if $cl\,(A) = cl\,(int\,(^{**}bcl\,(A)))$.*

Definition 3.3. *A subset A of a space (X,τ) is called $^{**}b$-B^{**}-set if $A = U \cap V$, where $U \in \tau$ and V is a $^{**}b$-t^{**}-set.*

Definition 3.4. *A subset A of a space (X,τ) is called $^{**}b$-$^{**}B$-set if $A = U \cap V$, where $U \in \tau$ and V is a $^{**}b$-$^{**}t$-set.*

Definition 3.5. *A subset A of a space (X,τ) is called $^{**}b$-α-open if $A \subseteq int\,(cl\,(^{**}bint\,(A)))$.*

Definition 3.6. *A subset A of a space (X,τ) is called $^{**}b$-semi-pre-open if $A \subseteq cl\,(int\,(^{**}bcl\,(A)))$.*

Result 3.7.

(1). Every $^{**}b$-closed set is a $^{**}b$-t^{**}-set.

(2). Every $^{**}b$-open set is a $^{**}b$-$^{**}t$-set.

Theorem 3.8. *Let A be a subset of (X,τ). Then to prove the following,*

(1). A is a $^{**}b$-t^{**}-set iff it is $^{**}b$-semi-pre-closed.

(2). A is a $^{**}b$-$^{**}t$-set iff it is $^{**}b$-semi-pre-open.

Proof.

(1). Let A be a $^{**}b$-t^{**}-set. Then we have $int\,(A) = int\,(cl\,(^{**}bint\,(A)))$. $\Rightarrow int\,(cl\,(^{**}bint\,(A))) \subseteq A$. Therefore A is $^{**}b$-semi-pre-closed. Conversely, assume that A is $^{**}b$-semi-pre-closed.We have $int\,(cl\,(^{**}bint\,(A))) \subseteq A$. $\Rightarrow$ $int\,(cl\,(^{**}bint\,(A))) \subseteq int\,(A)$. Since we also have $int\,(A) \subseteq int\,(cl\,(^{**}bint\,(A)))$. We have,$int\,(A) = int\,(cl\,(^{**}bint\,(A)))$. $\Rightarrow A$ is $^{**}b$-t^{**}-set.

(2). Let A be a $^{**}b$-$^{**}t$-set.Then we have $cl\,(A) = cl\,(int\,(^{**}bcl\,(A)))$. $\Rightarrow A \subseteq cl\,(int\,(^{**}bcl\,(A)))$. Therefore A is $^{**}b$-semi-pre-open. Conversely, assume that A is $^{**}b$semi-pre-open.We have $A \subseteq cl\,(int\,(^{**}bcl\,(A)))$. $\Rightarrow cl\,(A) \subseteq cl\,(int\,(^{**}bcl\,(A)))$. Since we also have $cl\,(int\,(^{**}bcl\,(A))) \subseteq A$. We have,$cl\,(A) = cl\,(int\,(^{**}cl\,(A)))$. $\Rightarrow A$ is a $^{**}b$-$^{**}t$-set.

□

Theorem 3.9.

(1). If A and B are $^{**}b$-t^{**}-sets, then $A \cap B$ is a $^{**}b$-t^{**}-set.

(2). If A and B are $^{**}b$-$^{**}t$-sets, then $A \cup B$ is a $^{**}b$-$^{**}t$-set.

Proof.

(1). Let A and B be a $^{**}b$-t^{**}-set.

$$\begin{aligned} Since\ \ int(A\cap B) &\subseteq int[cl(^{**}bint(A\cap B))] \\ &= int(A\cap B) \\ int(A\cap B) &\subseteq int(cl(^{**}bint(A\cap B))) \subseteq int(A\cap B) \\ int(A\cap B) &= int(cl(^{**}bint(A\cap B))) \end{aligned}$$

$\Rightarrow A \cap B$ is a $^{**}b$-t^{**}-set.

(2). Let A and B be a **b-**t-set.

$$\begin{aligned} Since\ \ cl(A\cup B) &= cl(A)\cup cl(B)\\ &\subseteq cl[int(^{**}bcl(A))\cup int(^{**}bcl(B))]\\ cl(A\cup B) &\subseteq cl(int(^{**}bcl(A\cup B)))\\ Since\ \ int(^{**}bcl(A\cup B)) &\subseteq (A\cup B)\\ \Rightarrow cl(int(^{**}bcl(A\cup B))) &\subseteq cl(A\cup B)\\ cl(A\cup B) &= cl(int(^{**}cl(A\cup B))) \end{aligned}$$

$\Rightarrow A\cup B$ is a **b-**t-set.

□

Theorem 3.10. *For a subset A of a space (X,τ), the following properties hold:*

*(1). If A is a t**-set, then it is **b-t**-set.*

*(2). If A is a **b-t**-set,then it is **b-B**-set.*

*(3). If A is a B**-set, then it is **b-B**-set.*

Proof.

(1). Let A be a t**-set.$int\,(A) = int\,(cl\,(int\,(A)))$. $\Rightarrow int\,(cl\,(int\,(A))) \subseteq A$. $\Rightarrow A$ is semi-pre-closed. Since every semi-pre-closed set is a **b-semi-pre-closed set. $int\,(cl\,(^{**}bint\,(A))) \subseteq A$. $\Rightarrow A$ is a **b-t**-set.[by using theorem 3.8]

(2). Let A be a **b-t**-set. $int\,(A) = int\,(cl\,(^{**}bint\,(A)))$. Let $U = X \in \tau$ be sn open set containing A and $V = A$ be a **b-t**-set.$\Rightarrow A = U \cap V$. Therefore A is a **b-B**-set.

(3). Let A be a B**-set. $A = U \cap V$, where $U \in \tau$ be an open set and V is a t**-set.since every t**-set is a **b-t**-set. $\Rightarrow A$ is a **b-t**-set.

□

Theorem 3.11. *For a subset A of a space (X,τ), the following properties hold:*

*(1). If A is a **t-set, then it is **b-**t-set.*

*(2). If A is a **b-**t-set,then it is **b-**B-set.*

*(3). If A is a **B-set, then it is **b-**B-set.*

Theorem 3.12. *For a subset A of a space (X,τ),the following are equivalent:*

*(1). A is **b-open.*

*(2). A is **b-α-open and **b-B**-set.*

Proof. To prove: (1) $\Rightarrow$ (2):Let A be **b-open,

$$\begin{aligned} A &= {}^{**}bint(A). \\ cl(A) &= cl({}^{**}bint(A)). \\ \Rightarrow A &\subseteq cl({}^{**}bint(A)) \\ int(A) &\subseteq int(cl({}^{**}bint(A))) \end{aligned}$$

since every **b-open set is an open set.

$$\Rightarrow A \subseteq int(cl({}^{**}bint(A)))$$

$\Rightarrow A$ is **b-α-open.let $U = A \in \tau$ be an open set containing A and $V = X$ be a **b-t**-set containing A.$\Rightarrow A = U \cap V$ $\Rightarrow A$ is **b-B**-set.Hence A is **b-α-open and **b-B**-set.

To prove: (2) $\Rightarrow$ (1): Let A be a **b-α-open and a **b-B**-set.$\Rightarrow A = U \cap V$, where $U \in \tau$ and V is a **b-t**-set.since V is a **b-t**-set.we have $int\left(V\right) = int\left(cl\left({}^{**}bint\left(V\right)\right)\right)$. since A is **b-α-open.

$$\begin{aligned} A &\subseteq int(cl({}^{**}bint(A))) \\ &= int(cl({}^{**}bint(U \cap V))) \\ &= int(cl({}^{**}bint(U))) \cap int(cl({}^{**}bint(V))) \\ \Rightarrow U \cap V &= int({}^{**}bcl(U)) \cap int(V) \\ \text{consider} U \cap V &= (U \cap V) \cap U \\ \Rightarrow V &\subseteq int(V) \\ int(V) &= V \\ A &= U \cap int(V) \\ A &= int(A) \end{aligned}$$

$\Rightarrow A$ is open. We know that every open set is a **b-open set $\Rightarrow A$ is a **b-open set. ☐

Theorem 3.13. *If A is **b-regular open.Then it is **b-semi-pre-open and **b-**t-set.*

Proof. Given A is **b-regular open. $A = int\left({}^{**}bcl\left(A\right)\right) \Rightarrow cl\left(A\right) = cl\left(int\left({}^{**}bcl\left(A\right)\right)\right)$. We have $A \subseteq cl\left(int\left({}^{**}bcl\left(A\right)\right)\right) \Rightarrow$ A is **b-semi-pre-open. Also $cl\left(A\right) \subseteq cl\left(int\left({}^{**}bcl\left(A\right)\right)\right)$. And $int\left({}^{**}bcl\left(A\right)\right) \subseteq A$.$\Rightarrow cl\left(int\left({}^{**}bcl\left(A\right)\right)\right) \subseteq cl\left(A\right)$. We get $cl\left(A\right) = cl\left(int\left({}^{**}bcl\left(A\right)\right)\right) \Rightarrow A$ is **b-semi-pre-open and **b-**t-set. ☐

Example 3.14. *The converse of the above theorem is not true. Let* $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ *and* $\tau' = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. *The set* $\{b, c\}$ *is a **b-semi-pre-open and **t-**t-set but it is not a **b-regular open set.*

Theorem 3.15. *If A is **b-regular closed.Then it is **b-semi-pre-closed and **b-t**-set.*

Proof. Given A is **b-regular closed. $A = cl\left({}^{**}bint\left(A\right)\right) \Rightarrow int\left(A\right) = int\left(cl\left({}^{**}bint\left(A\right)\right)\right)$. We have $\left(int\left(cl\left({}^{**}bint\left(A\right)\right)\right) \subseteq\right.$ A.$\Rightarrow A$ is **b-semi-pre-closed. Also $int\left(cl\left({}^{**}bint\left(A\right)\right)\right) \subseteq int\left(A\right)$. And $A \subseteq cl\left({}^{**}bint\left(A\right)\right) \Rightarrow int\left(A\right) \subseteq int\left(cl\left({}^{**}bint\left(A\right)\right)\right)$. We get $int\left(A\right) = int\left(cl\left({}^{**}bint\left(A\right)\right)\right) \Rightarrow A$ is **b-semi-pre-closed and **b-t**-set. ☐

Example 3.16. *The converse of the above theorem is not true. Let* $X = \{a, b, c, d\}$, $\tau = \{X, \phi, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$ *and* $\tau' = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. *Here* $\{a\}$ *is a **b-semi-pre-closed and **b-**t-set but it is not a **b-regular closed set.*

Theorem 3.17. *Let* (X, τ) *be a extremally disconnected space.Let A beb a **b-t-set iff it is a **b-t**-set.*

Proof. Given that (X, τ) be a extremally disconnected space. $int\,(cl\,(A)) = cl\,(int\,(A))$
Let A be a **b-t-set.

$$\begin{aligned} int(A) &= int(^{**}bcl(A)) \\ &= int(cl(A)) \\ &= cl(int(A)) \\ int(A) &= int(cl(int(A))) \\ int(A) &= int(cl(^{**}int(A))) \end{aligned}$$

$\Rightarrow A$ is a **b-t**-set.Conversely, Assume that A be a **b-t**-set.

$$\begin{aligned} int(A) &= int(cl(^{**}bint(A))) \\ \Rightarrow int(A) &= int(cl(int(A))) \\ &= int(int(cl(A))) \\ &= int(cl(A)) \\ int(A) &= int(^{**}bcl(A)) \end{aligned}$$

$\Rightarrow A$ is a **b-t-set. □

Theorem 3.18. *Let* (X, τ) *be a extremally disconnected space.Let A be a **b-t*-set iff it is a **b-**t-set*

Theorem 3.19. *A set A is a **b-t**-set iff its complement is a **b-**t-set.*

Proof. Let A be a **b-t**-set.

$$\begin{aligned} \text{Then } int(A) &= int(cl(^{**}bint(A))) \\ \Leftrightarrow X - int(A) &= X - int(cl(^{**}bint(A))) \\ \Leftrightarrow cl(X - A) &= cl(int(^{**}bcl(X - A))) \\ \Leftrightarrow cl(A^c) &= cl(int(^{**}bcl(A^c))) \end{aligned}$$

$\Leftrightarrow A^c$ is a **b-**t-set. □

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