



Nano Generalized Locally Closed Sets and NGLC-Continuous Functions in Nano Topological Spaces

Research Article

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Abstract: The purpose of this paper is to introduce a new class of sets called Nano generalized locally closed set in a Nano Topological space, also introduce Nano Generalized Locally closed continuous function, Nano Generalized Locally closed irresolute function and studied some of its properties.

Keywords: Nano Topology, Ng-closed set, NGLC-closed set, NGLC*-closed set, NGLC**-closed set

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1. Introduction

Balachandran [1] et.al, introduced the concept of Generalized locally closed set in Topology. They also investigated the classes of GLC-continuous maps and GLC-irresolute maps. The notion of Nano topology was introduced by Lellis Thivagar[4] which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it and also defined Nano closed sets ,Nano-interior and Nano-closure.He has also defined Nano continuous functions , Nano open mapping , Nano closed mapping and Nano Homeomorphism. In [2] Bhuvaneshwari et.al, introduced and studied some properties of Nano generalized closed sets in Nano topological spaces. In this paper we introduce a new class of sets called Nano generalized locally closed sets and discuss some of its properties. We also introduce Nano Generalized Locally closed continuous function and Nano Generalized Locally closed irresolute function and studied some of its properties.

2. Preliminaries

Definition 2.1 ([7]). A subset A of (X, τ) is called a generalized closed set (briefly g -closed) if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

Definition 2.2 ([5]). Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$. Then,

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- (1). The lower approximation of X with respect to R is the set of all objects which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X) = \cup\{R(x) : R(x) \subseteq X, x \in U\}$ where $R(x)$ denotes the equivalence class determined by $x \in U$.
- (2). The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = \cup\{R(x) : R(x) \cap X \neq \Phi, x \in U\}$
- (3). The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not- X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

Property 2.3 ([5]). If (U, R) is an approximation space and $X, Y \subseteq U$, then

- (1). $L_R(X) \subseteq X \subseteq U_R(X)$
- (2). $L_R(\Phi) = U_R(\Phi) = \Phi$
- (3). $L_R(U) = U_R(U) = U$
- (4). $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
- (5). $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
- (6). $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
- (7). $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
- (8). $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
- (9). $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
- (10). $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
- (11). $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.4 ([5]). Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \Phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

- (1). U and $\Phi \in \tau_R(X)$
- (2). The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
- (3). The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X . $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U . Elements of $[\tau_R(X)]^c$ are called Nano closed sets with $[\tau_R(X)]^c$ being called dual Nano topology of $\tau_R(X)$.

Definition 2.5 ([5]). If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then

- (1). The Nano interior of the set A is defined as the union of all Nano open subsets contained in A and is denoted by $NInt(A)$. $NInt(A)$ is the largest Nano open subset of A .
- (2). The Nano closure of the set A is defined as the intersection of all Nano closed sets containing A and is denoted by $NCl(A)$. $NCl(A)$ is the smallest Nano closed set containing A .

Definition 2.6 ([2]). A subset A of $(U, \tau_R(X))$ is called Nano generalized closed set (briefly Ng-closed) if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is Nano open in $(U, \tau_R(X))$.

Definition 2.7 ([3]). Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a mapping $f : (U, \tau_R(X)) \rightarrow (V, \tau_{R'}(Y))$ is Nano g -continuous on U if the inverse image of every Nano open set in V is Nano g -open in U .

Definition 2.8 ([1]). A subset S of (X, τ) is called Generalized locally closed set (briefly, glc) if $S = G \cap F$ where G is g -open in (X, τ) and F is g -closed in (X, τ) .

Definition 2.9 ([1]). For a subset S of (X, τ) , $S \in GLC^*(X, \tau)$ if there exist a g -open set G and a closed set F of (X, τ) , respectively, such that $S = G \cap F$.

Definition 2.10 ([1]). For a subset S of (X, τ) , $S \in GLC^{**}(X, \tau)$ if there exist an open set G and a g -closed set F of (X, τ) , respectively, such that $S = G \cap F$.

Definition 2.11 ([1]). A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called GLC -continuous (resp. GLC^* -continuous, resp. GLC^{**} -continuous) if $f^{-1}(V) \in GLC(X, \tau)$ (resp. $f^{-1}(V) \in GLC^*(X, \tau)$, resp. $f^{-1}(V) \in GLC^{**}(X, \tau)$) for each $V \in \sigma$.

3. Nano Generalized Locally Closed Sets

Definition 3.1. A subset A of $(U, \tau_{R(X)})$, is called Nano locally closed if $A = G \cap F$, where $G \in \tau_{R(X)}$ and F is Nano Closed in $(U, \tau_{R(X)})$.

Remark 3.2. The following are well known.

- (1). A subset A of $(U, \tau_{R(X)})$, is Nano locally closed if and only if its complement $U - A$ is the union of a Nano open set and a Nano closed set.
- (2). Every Nano open (resp- Nano closed) subset of U is Nano locally closed.
- (3). The complement of a Nano locally closed set need not be Nano locally closed.

Now we introduce the following:

Definition 3.3. A subset A of $(U, \tau_{R(X)})$, is called Nano Generalized locally closed set (briefly Ngcl) if $A = G \cap F$, where G is Ng-open in $(U, \tau_{R(X)})$ and F is Ng-closed in $(U, \tau_{R(X)})$. The collection of all Nano Generalized locally closed sets of $(U, \tau_{R(X)})$ will be denoted by $NGLC(U, \tau_{R(X)})$.

The following two collections of subsets of $(U, \tau_{R(X)})$ (i.e) $NGLC^*(U, \tau_{R(X)})$ and $NGLC^{**}(U, \tau_{R(X)})$ are defined as follows.

Definition 3.4. For a subset A of $(U, \tau_{R(X)})$, $A \in NGLC^*(U, \tau_{R(X)})$, if there exist a Ng- open set G and a Nano closed set F of $(U, \tau_{R(X)})$, respectively, such that $A = G \cap F$.

Definition 3.5. For a subset A of $(U, \tau_{R(X)})$, $A \in NGLC^{**}(U, \tau_{R(X)})$, if there exist a Nano open set G and a Ng-closed set F of $(U, \tau_{R(X)})$, respectively, such that $A = G \cap F$.

Example 3.6. Let $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{b, d\}\}$ and $X = \{a, b\}$. Then $\tau_{R(X)} = \{\emptyset, \{a\}, \{a, b, d\}, \{b, d\}, U\}$. Then the $NGLC$ sets are $P(U)$ and NLC sets are $\{\emptyset, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, U\}$.

Remark 3.7. Every Ng-closed set (resp- Ng-open set) is Ngcl

Theorem 3.8. Let A be a subset of $(U, \tau_R(X))$.

- (1). If A is Nano locally closed, then $A \in NGLC^*(U, \tau_R(X))$ but not conversely
- (2). If $A \in NGLC^*(U, \tau_R(X))$ or $A \in NGLC^{**}(U, \tau_R(X))$, then A is Nglc.

Proof. The proof is obvious from the Example 3.6. In the example, $NGLC^*$ are $\{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, U\}$ and $NGLC^{**}$ is $P(U)$. \square

The following result is a characterization of $NGLC^*(U, \tau_R(X))$.

Theorem 3.9. For a subset A of $(U, \tau_R(X))$, the following are equivalent.

- (1). $A \in NGLC^*(U, \tau_R(X))$
- (2). $A = P \cap NCl(A)$ for some Ng-open set P
- (3). $Ncl(A) - A$ is Ng-closed
- (4). $A \cup (U - NCl(A))$ is Ng-open.

Proof.

- (1) \Rightarrow (2) Let $A \in NGLC^*(U, \tau_R(X))$. Then $A = P \cap F$ where P is Ng-open and F is Nano closed. Since $A \subseteq P$ and $A \subseteq NCl(A)$, $A \subseteq P \cap NCl(A)$. Conversely, since $A \subseteq F$, $NCl(A) \subseteq F$, we have $A = P \cap F$ contains $P \cap NCl(A)$. That is $P \cap NCl(A) \subseteq A$. Therefore we have $A = P \cap NCl(A)$.
- (2) \Rightarrow (1) Since P is Ng-open and $NCl(A)$ is Nano closed, $P \cap NCl(A) \in NGLC^*(U, \tau_R(X))$ by Definition 3.4 of $NGLC^*(U, \tau_R(X))$.
- (2) \Rightarrow (3) $A = P \cap NCl(A)$ implies that $NCl(A) - A = NCl(A) \cap P^c$ which is Ng-closed, Since P^c is Ng-closed.
- (3) \Rightarrow (2) Let $P = [NCl(A) - A]^c$. Then by assumption, P is Ng-open in $(U, \tau_R(X))$ and $A = P \cap NCl(A)$.
- (3) \Rightarrow (4) $A \cup (U - NCl(A)) = A \cup (NCl(A))^c = [NCl(A) - A]^c$ and by assumption $[NCl(A) - A]^c$ is Ng-open and $A \cup (U - NCl(A))$ is Ng-open.
- (4) \Rightarrow (3) Let $P = A \cup (NCl(A))^c$. Then P^c is Ng-closed and $P^c = NCl(A) - A$ and therefore $NCl(A) - A$ is Ng-closed. \square

Theorem 3.10. For a subset A of $(U, \tau_R(X))$, the following statements are equivalent.

- (1). $A \in NGLC(U, \tau_R(X))$
- (2). $A = P \cap Ng - Cl(A)$ for some Ng-open set P
- (3). $NgCl(A) - A$ is Ng-closed
- (4). $A \cup (Ng - Cl(A))^c$ is Ng-open
- (5). $A \subseteq Ng - int(A \cup (Ng - Cl(A))^c)$

Proof.

- (1) \Rightarrow (2) Let $A \in NGLC(U, \tau_R(X))$. Then $A = P \cap F$, where P is Ng-open and F is Ng-closed. Since $A \subseteq F$, $Ng - Cl(A) \subseteq F$ and therefore $P \cap Ng - Cl(A) \subseteq A$. Also $A \subseteq P$ and $A \subseteq Ng - Cl(A)$ implies $A \subseteq P \cap Ng - Cl(A)$ and therefore $A = P \cap Ng - Cl(A)$.
- (2) \Rightarrow (3) $A = P \cap Ng - Cl(A)$ implies $Ng - Cl(A) - A = Ng - Cl(A) \cap P^c$ which is Ng-closed since P^c is Ng-closed.
- (3) \Rightarrow (4) $A \cup (Ng - Cl(A))^c = (Ng - Cl(A) - A)^c$ and by assumption $(Ng - Cl(A) - A)^c$ is Ng-open and so is $A \cup (Ng - Cl(A))^c$.

(4) \Rightarrow (5) By assumption, $A \cup (Ng - Cl(A))^c = Ng - int(A \cup (Ng - Cl(A))^c)$ and hence $A \subseteq Ng - int(A \cup (Ng - Cl(A))^c)$.

(5) \Rightarrow (1) By assumption and since $A \subseteq Ng - Cl(A)$, $A = Ng - int(A \cup (Ng - Cl(A))^c) \cap Ng - Cl(A) \in NGLC(U, \tau_{R(X)})$. \square

Theorem 3.11. *Let A be a subset of $(U, \tau_{R(X)})$. Then $A \in NGLC^{**}(U, \tau_{R(X)})$ if and only if $A = P \cap Ng - Cl(A)$ for some Nano open set P .*

Proof. Let $A \in NGLC^{**}(U, \tau_{R(X)})$. Then $S = P \cap F$ where P is Nano open and F is Ng-closed. Since $S \subseteq F$, $Ng - Cl(A) \subseteq F$. Now $A = A \cap Ng - Cl(A) = P \cap F \cap Ng - Cl(A) = P \cap Ng - Cl(A)$. Here the converse part is trivial. \square

Corollary 3.12. *Let A be a subset of $(U, \tau_{R(X)})$. If $A \in NGLC^{**}(U, \tau_{R(X)})$, then $Ng - Cl(A) - A$ is Ng-closed and $A \cup (Ng - Cl(A))^c$ is Ng-open.*

Proof. Let $A \in NGLC^{**}(U, \tau_{R(X)})$. Then by Theorem 3.11, $A = P \cap Ng - Cl(A)$ for some Nano open set P and $Ng - Cl(A) - A = Ng - Cl(A) \cap P^c$ is Ng-closed in $(U, \tau_{R(X)})$. If $F = Ng - Cl(A) - A$, then $F^c = A \cup (Ng - Cl(A))^c$ and F^c is Ng-open and therefore $A \cup (Ng - Cl(A))^c$ is Ng-open. \square

4. NGLC-Continuous and NGLC-Irresolute Functions

Definition 4.1. *A function $f : (U, \tau_{R(X)}) \rightarrow (V, \tau_{R'(Y)})$ is called Nano locally closed continuous function [shortly, NLC-continuous] if $f^{-1}(B)$ is a Nano locally closed set of $(U, \tau_{R(X)})$ for each Nano open set B of $(V, \tau_{R'(Y)})$.*

Definition 4.2. *A function $f : (U, \tau_{R(X)}) \rightarrow (V, \tau_{R'(Y)})$ is called Nano Generalized locally closed continuous function [shortly, NGLC continuous], (resp. $NGLC^*$ -continuous, resp. $NGLC^{**}$ -continuous) if $f^{-1}(B) \in NGLC((U, \tau_{R(X)})$ (resp. $f^{-1}(B) \in NGLC^*((U, \tau_{R(X)})$, $f^{-1}(B) \in NGLC^{**}((U, \tau_{R(X)})$ for each $B \in \tau_{R'(Y)}$.*

Example 4.3. *Let $U = V = \{a, b, c, d\}$ and $\tau_{R(X)} = \{\emptyset, \{a\}, \{a, b, d\}, \{b, d\}, U\}$ and $\tau_{R'(Y)} = \{\emptyset, \{a, c, d\}, \{a, d\}, \{c\}, V\}$. Then $NGLC(U, \tau_{R(X)}) = P(U)$ and the identity map $f : (U, \tau_{R(X)}) \rightarrow (V, \tau_{R'(Y)})$ is NGLC-continuous.*

Definition 4.4. *A function $f : (U, \tau_{R(X)}) \rightarrow (V, \tau_{R'(Y)})$ is called NGLC-irresolute (resp- $NGLC^*$ -irresolute, resp. $NGLC^{**}$ -irresolute) if $f^{-1}(B) \in NGLC((U, \tau_{R(X)})$ (resp. $f^{-1}(B) \in NGLC^*((U, \tau_{R(X)})$, $f^{-1}(B) \in NGLC^{**}((U, \tau_{R(X)})$ for each $B \in NGLC(V, \tau_{R'(Y)})$ (resp. $B \in NGLC^*(V, \tau_{R'(Y)})$, $B \in NGLC^{**}(V, \tau_{R'(Y)})$.*

Example 4.5. *The map in the Example 4.3 is NGLC-irresolute.*

Theorem 4.6. *Let $f : (U, \tau_{R(X)}) \rightarrow (V, \tau_{R'(Y)})$ be a function.*

- (1). *If f is NLC-continuous, then it is $NGLC^*$ -continuous.*
- (2). *If f is $NGLC^*$ -continuous or $NGLC^{**}$ -continuous, then it is NGLC-continuous.*
- (3). *If f is NGLC-irresolute (resp. $NGLC^*$ -irresolute, resp. $NGLC^{**}$ -irresolute), then it is NGLC-continuous (resp. $NGLC^*$ -continuous, resp. $NGLC^{**}$ -continuous)*

Proof. Suppose that f is NLC-continuous. Let B be a Nano open set of $(V, \tau_{R'(Y)})$. Then

- (1). $f^{-1}(B)$ is Nano locally closed in $(U, \tau_{R(X)})$ by definition. By Theorem 3.8, $f^{-1}(B) \in NGLC^*((U, \tau_{R(X)})$ for each $B \in \tau_{R'(Y)}$. Therefore f is $NGLC^*$ -continuous.
- (2). Since every $NGLC^*$ set is NGLC set and every $NGLC^{**}$ set is NGLC set, the proof follows.
- (3). Follows from the fact that every Nano open set in $\tau_{R'(Y)}$ is NGLC, $NGLC^*$ and $NGLC^{**}$.

□

Remark 4.7. *The converse of the above theorem is not true as seen from the following examples.*

Example 4.8. *Let $U = V = \{a, b, c, d\}$ and $\tau_{R(X)} = \{\emptyset, \{a\}, \{a, b, d\}, \{b, d\}, U\}$ and $\tau_{R'(Y)} = \{\emptyset, \{a, c, d\}, \{a, d\}, \{c\}, V\}$. Define $f : (U, \tau_{R(X)}) \rightarrow (V, \tau_{R'(Y)})$ as $f(a) = a$, $f(b) = c$, $f(c) = b$ and $f(d) = d$. Then f is NGLC-continuous, NGLC*-continuous and NGLC**-continuous but not NLC-continuous because $NLC(U, \tau_{R(X)}) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}, U\}$. $NGLC(U, \tau_{R(X)}) = NGLC^{**}(U, \tau_{R(X)}) = P(U)$ and $NGLC^*(U, \tau_{R(X)}) = P(U) - \{\{b, c\}, \{c, d\}, \{a, b, c\}, \{a, c, d\}\}$.*

Example 4.9. *Let $U, V, \tau_{R(X)}$ and $\tau_{R'(Y)}$ be as in Example 4.3. Then the identity map $f : (U, \tau_{R(X)}) \rightarrow (V, \tau_{R'(Y)})$ is NGLC-continuous but not NGLC* continuous.*

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