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Inverse Accurate Total Domination in Fuzzy Graphs

Research Article

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Abstract: Let $G = (V, E)$ be a graph. Let D be a minimum accurate dominating set of G. If $V - D$ contains an accurate dominating set D' of G, then D' is called an fuzzy inverse accurate dominating set with respect to D. The fuzzy inverse accurate domination number $\gamma_{fa}^{-'}(G)$ of G is the minimum cardinality of an fuzzy inverse accurate dominating set of G. Let D be a minimum accurate total dominating set of G, if $V - D$ contains an accurate total dominating set D' of G, then D' is called an fuzzy inverse accurate total dominating set with respect to D. The fuzzy inverse accurate total dominating number $\gamma_{fat}^{-'}(G)$ of G is the minimum cardinality of an fuzzy inverse accurate total dominating set of G. In this paper we study a fuzzy inverse accurate total domination in fuzzy graphs and investigate the relationship of with other known parameters.

Keywords: Fuzzy Accurate Domination, Fuzzy Accurate Total Domination, Fuzzy Inverse Accurate Domination, Fuzzy Inverse Accurate Total Domination.

1. Introduction

The study of dominating sets in graphs was begun by Orge and Berge, V.R.Kulli wrote on theory of domination in graphs. The concept of fuzzy sets and fuzzy relations was introduced by L.A.Zadeh in 1965 [1] and further studied in [2]. It was Resenfeld [5] who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975.

The domination number is introduced by cockayne and Hedetniemi Resenfeld introduced the notion of fuzzy graph and several fuzzy analogs of graph theoretic concepts such as paths, cycles and connectedness. A.Somasundaram and S.Somasundaram [4] and introduced the domination number for several classes of fuzzy graphs and obtained bounds for the same. NagoorGani and Chandrasekaran discussed domination in fuzzy graph using strong arc.

This concept of Accurate domination number was introduced by V.R.Kulli and M.B.Kattiman. Among the various applications to the theory of domination in fuzzy graphs, here we consider the fault tolerant property in communication network, that is, even if any communication link to a station is failed, still it can communicate the message to that station. We also discuss the fuzzy inverse accurate domination number and fuzzy inverse accurate total domination number of the fuzzy graph.

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2. Preliminaries

A fuzzy subset of a non empty set V is a mapping $\sigma : V \to [0, 1]$. A fuzzy relation on V is a fuzzy subset of $V \times V$. A fuzzy graph $G = (\sigma, \mu)$ is a pair of function $\sigma : V \to [0, 1]$ and $\mu : V \times V \to [0, 1]$, where $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$. The order p and size q of the fuzzy graph $G = (\sigma, \mu)$ are define by $p = \sum_{v \in V} \sigma(v)$ and $q = \sum_{u,v \in E} \mu(u, v)$. The complement of a fuzzy graph $G = (\sigma, \mu)$ is a fuzzy graph $G = (\sigma, \mu)$ where $\sigma = \sigma$ and $\overline{\mu}(u, v) = \sigma(u) \wedge \sigma(v) - \mu(u, v)$ for all u, v in V. The fuzzy cardinality of a fuzzy subset D of V is $|D|_f = \sum_{v \in D} \sigma(v)$. Let $G = (\sigma, \mu)$ be a fuzzy graph on V. Let $x, y \in V$. We say that x dominates y in G if $\mu(uv) = \sigma(u) \wedge \sigma(v)$. The strength of connectedness between two nodes u, v in a fuzzy graph G is $\mu^\infty(u, v) = \sup\{\mu^k(u, v) : k = 1, 2, 3 \ldots ...\}$ where $\mu^k(u, v) = \sup\{\mu(u, u_1) \wedge \mu(u_1, u_2) \wedge \cdots \wedge \mu(u_{k-1}, v)\}$, An arc(u ,v) is said to be a strong arc if $\mu(u, v) \geq \mu^\infty(u, v)$ and the node V is said to be a strong neighbor of u. If $\mu(u, v) = 0$ for every $v \in V$, then u is called isolated node.

A node cover of a graph G is a nodes that covers all the sizes and an size cover of G is a set of sizes that covers all the nodes. The node (size) covering number $(\alpha_0(G), \alpha_1(G))$ of G is minimum cardinality of a node (size) cover. A set S of nodes of G is independent if no two nodes in S are adjacent. The independence number $\beta_0(G)$ of G is the maximum cardinality of an independent set. A β_0 set is a maximum independent set. A set F of nodes of G is independent if no two sizes in F are adjacent. The size independence number $\beta_1(G)$ of G is the maximum cardinality among the independent sets of sizes. Let G be a fuzzy graph and u be a node in G then there exists node v such that (u, v) is a strong arc then u dominates v. A subset S of V is called a dominating set in G if for every $v \notin S$, there exists $u \in S$ such that u dominates v. The minimum fuzzy cardinality of a dominating set in G is called the domination number of G and is denoted by $\gamma(G)$. Let $G = (\sigma, \mu)$ be a fuzzy graph. A subset D of V is said to be fuzzy dominating set of G if for every $v \in V - D$, there exists $u \in D$ such that (u, v) is a Strong arc. A fuzzy dominating set D of a fuzzy graph G is called minimal dominating set of G, if for every node $v \in D$, $D - \{v\}$ is not a dominating set. The domination number $\gamma(G)$ is the minimum cardinalities taken over all minimal dominating sets nodes of G.

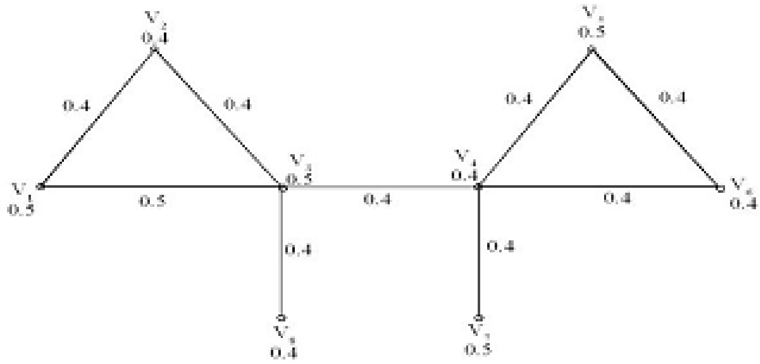
A dominating set D of a graph G is an accurate dominating set, if $V - D$ has no dominating set of cardinality $|D|$. The accurate domination number $\gamma_a(G)$ of G is the minimum cardinality of an accurate dominating set. A total dominating set T of G is a dominating set such that the induced subgraph $\langle T \rangle$ has no isolates. The total domination number $\gamma_t(G)$ of G is the minimum cardinality of a total dominating set. A total dominating set D of G is an accurate total dominating set if $V - D$ has no total dominating set of cardinality $|D|$. The accurate total domination number $\gamma_{at}(G)$ of G is the minimum cardinality of an accurate total dominating set.

Let $G = (V, E)$ be a graph. Let D be a minimum accurate dominating set of G. If $V - D$ contains an accurate dominating set D' of G, then D' is called an inverse accurate dominating set with respect to D. The inverse accurate domination number $\gamma_a^{-'}(G)$ of G is the minimum cardinality of an inverse accurate dominating set of G. Let D be a minimum accurate total dominating set of G, if $V - D$ contains an accurate total dominating set D' of G, then D' is called an inverse accurate total dominating set with respect to D. The inverse accurate total dominating number $\gamma_{at}^{-'}(G)$ of G is the minimum cardinality of an inverse accurate total dominating set of G. Let $G : (\sigma, \mu)$ be a fuzzy graph. The degree of a node u is $d_G(u) = \sum_{u \neq v} \mu(uv)$. Since $\mu(uv) > 0$ for $uv \in E$ and $\mu(uv) = 0$ for $uv \notin E$, This is equivalent to $d_G(u) = \sum_{uv \in E} \mu(uv)$. The minimum degree of G is $\delta(G) = \wedge\{d(v)/v \in V\}$. The maximum degree of G is $\Delta(G) = \vee\{d(v)/v \in V\}$.

3. Inverse Accurate Total Domination in Fuzzy Graphs

Definition 3.1. *Let $G = (V, E)$ be a graph. Let D be a minimum accurate dominating set of G. If $V - D$ contains an accurate dominating set D' of G, then D' is called an fuzzy inverse accurate dominating set with respect to D. The fuzzy inverse accurate domination number $\gamma^{-'}_{fa}(G)$ of G is the minimum cardinality of an fuzzy inverse accurate dominating set of G. Let D be a minimum accurate total dominating set of G, if $V - D$ contains an accurate total dominating set D' of G, then D' is called an fuzzy inverse accurate total dominating set with respect to D. The fuzzy inverse accurate total dominating number $\gamma^{-'}_{fat}(G)$ of G is the minimum cardinality of an fuzzy inverse accurate total dominating set of G.*

Example 3.2. *The graph G in Figure 1 has $\{v_3, v_4\}$ as a γ_{fa}-set and $\{v_3, v_4\}$ as a γ_{fat}-set and $\{v_2, v_5, v_7, v_8\}$ as a $\gamma^{-'}_{fat}$–set. Thus G has $\gamma_{fa}(G) = 0.9$, $\gamma_{fat}(G) = 0.9$, $\gamma^{-'}_{fat}(G) = 1.8$.*



Observation 3.3. *Here we observed the exact values of $\gamma^{-'}_{fa}(G)$ and $\gamma^{-'}_{fat}(G)$ for some standard graphs and proved some standard results.*

Observation 3.4. *For any fuzzy graph G with p nodes then $\gamma^{-'}_{fa}(G) \leq n - \delta(G)$.*

Remark 3.5. *We obtained the relationship between $\gamma^{-'}_{fat}(G)$ and $\gamma^{-'}_{fat}(H)$ where H is any connected spanning subgraph of G. Similar result for $\gamma^{-'}_{fa}(G)$ and $\gamma^{-'}_{fa}(G)$. If H is any connected Spanning subgraph of G then $\gamma^{-'}(G) \leq \gamma^{-'}(H)$.*

Theorem 3.6. *An fuzzy inverse accurate total dominating set D' of G is minimal if and only if for each node $v \in D'$, one of the following conditions is satisfied.*

(a). There exists a node $u \in V - D'$ such that $N(u) \cap D' = \{v\}$

(b). v is not an isolated node in $\langle D' \rangle$

(c). u is not an isolated node in $\langle V - D' \rangle$.

Proof. Suppose D' is a minimal fuzzy inverse accurate total dominating set of G. Suppose the contrary. That is, if there exists a node $v \in D'$ such that v does not satisfy any of the given conditions, then there exists an inverse accurate dominating set $D'' = D' - \{v\}$ such that the induced subgraph $\langle V - D'' \rangle$ is connected. This implies that D'' is an fuzzy inverse accurate total dominating set of G contradicting the minimality of D'. Therefore, the condition is necessary.
Sufficiency from the given conditions. □

Observation 3.7. *If a fuzzy graph G no isolated nodes, then $\gamma_a(G) \leq \gamma^{-'}_{fat}(G)$.*

Observation 3.8. *For any fuzzy graph of G without isolated nodes, then $\gamma_{fat}(G) \leq \gamma_{fat}(G) + \Delta(G) - 1$.*

Theorem 3.9. *For any connected fuzzy graph of G without isolated nodes, then $\gamma_{fa}(G) + \gamma^{-'}_{fat}(G) \leq p$.*

Proof. Suppose $D = \{v_1, v_2, \ldots, v_n\} \subseteq V(G)$ be the γ–set of G, then $D' = \{v_1, v_2, \ldots, v_n\} \subseteq V(G) - D$ forms a minimal fuzzy inverse accurate total dominating set of G. since $|D| \leq p/4$ and $|D'| \leq p/3$, it follows that $|D| \cup |D'| \leq p$. Therefore $\gamma_{fa}(G) + \gamma^{-'}_{fat}(G) \leq p$.
Suppose $V - D$ is not independent, then there exists atleast one node $u \in D'$ such that $N(u) \subseteq V - D$. Clearly $|D'| = |\{V - D\} - \{u\}|$ and hence, $|D| \cup |D'| \leq p$, a contradiction. Conversely, if $V - D$ is independent. Then in this case, $|D'| = |V - D|$ in G. Clearly, it follows that $|D| \cup |D'| = p$. Hence $\gamma_{fidd}(G) + \gamma^{-'}_{fidd}(G) = p$. □

Theorem 3.10. *For any connected fuzzy graph of G then $\gamma^{-'}_{fat}(G) + \gamma_{fc}(G) \leq p + \gamma_{fa}(G)$.*

Proof. For $p \leq 5$, The results follows immediately. Let $p \geq 6$, Suppose $D = \{v_1, v_2, \ldots, v_n\}$, $deg(v_i) \geq 2$, $1 \leq i \leq n$ be a minimal dominating set of G. Now we construct a connected dominating set D_c from D by adding in every step at most two components of D forms a connected component in D. Thus we get a fuzzy connected dominating set D_c after at most $D - 1$ steps. Now in G, suppose $D_1 \subseteq D$ be minimal γ-set of G. Then there exists a vertex set $D' = \{v_1, v_2, \ldots, v_n\} \subseteq V(G) - D_1$, such that dist$(u, v) \geq 2$, for all $u, v \in D'$. Which covers all the nodes in G. Clearly, D' forms a minimal inverse dominating set of G. Hence it follows that $|D'| \cup |D_c| \leq p \cup |D|$. Therefore, $\gamma^{-'}_{fat}(G) + \gamma_{fc}(G) \leq p + \gamma_{fa}(G)$. □

Theorem 3.11. *For any fuzzy graph of G without isolated nodes, then $\lceil \frac{p}{\Delta(G)+1} \rceil \leq \gamma^{-'}_{fat}(G)$.*

Theorem 3.12. *For any fuzzy graph of G without isolated nodes, then $\gamma^{-'}_{fat}(G) \leq \lceil \frac{\Delta(G)p}{\Delta(G)+1} \rceil$.*

Theorem 3.13. *For any fuzzy graph G with $\gamma^{-'}_{fat}$-set, then $\gamma_{fat}(G) + \gamma^{-'}_{fat}(G) \leq q$.*

Theorem 3.14. *Let D be a γ_{fat}-set of G. Then G has an fuzzy inverse accurate total dominating set if and only if the following conditions are satisfied:*

(a). $\langle V - D \rangle$ conditions no isolated nodes.

(b). $|D| \leq |V - D|$.

Observation 3.15. *If a fuzzy graph G has a $\gamma^{-'}_{fat}$-set, then $\gamma_{fat}(G) \leq \gamma^{-'}_{fat}(G)$.*

Theorem 3.16. *For any connected fuzzy graph $\gamma^{-'}_{fat}(G) \leq q - \beta_1(G) + 1$.*

Observation 3.17. *If a fuzzy graph G has a $\gamma^{-'}_{at}$-set , then $\gamma_{ft}(G) \leq \gamma^{-'}_{fat}(G) \leq p$.*

Theorem 3.18. *For any fuzzy graph G without isolated nodes, $\gamma^{-'}_{fat}(G) \leq p - 1$.*

Theorem 3.19. *For any fuzzy graph G has no isolated nodes, then $(2p - q)/3 \leq \gamma^{-'}_{fat}(G)$.*

Proof. Let D be a minimum dominating set and $D' \subseteq V - D$ the corresponding inverse dominating set of G such that $|D| = \gamma_{fat}(G)$ and $|D'| = \gamma^{-'}_{fat}(G)$. If D is a dominating set of G, then there are at least $\gamma^{-'}_{fat}(G)$ edges between D and D'. Suppose $V - D -' = \phi$. Then $\gamma_{fat}(G) + \gamma^{-'}_{fat}(G) = p$. Since $\gamma^{-'}_{fat}(G) \geq \gamma_{fat}(G)$. We have $\gamma^{-'}_{fat}(G) \geq p/2 \geq (2p - q)/3$, since $p \geq 2$.
Suppose $V - D - D' \neq \phi$. If D' is a dominating set of G, then every node in $V - D - D'$ has at least one edge to D and atleast one edge to D'. Then the number of edges from $V - D - D'$ is atleast $2|V - D - D^1|$. Hence $q \geq 2|V - D - D'| + \gamma^{-'}_{fat}(G) = 2(p - \gamma_{fat}(G) - \gamma^{-'}_{fidd}(G)) + \gamma^{-'}_{fat}(G)$. $\gamma^{-'}_{fat}(G) + 2\gamma_{fat}(G) \geq 2p - q$. since $\gamma^{-'}_{fat}(G) \geq \gamma_{fat}(G)$, it follows that $(2p - q)/3 \leq \gamma^{-'}_{fat}(G)$. □

Theorem 3.20. *Let G be any fuzzy graph order ≥ 3. Then*

(a). $\gamma^{-'}_{fat}(G) \leq p-1$ *when G is a K_4-free graph*

(b). $\gamma_{fat}(G) \leq p-2$

Where p is the number of nodes.

Proof.

(a). Let D' be an inverse accurate total dominating set of G. Then, we have D' is an inverse accurate dominating set of G. This implies that, for every node $v \in V - D'$, $N_G(V) \cap D' = \phi$. It means that D' is an inverse accurate dominating set $G - v$. To prove the required result, it is enough to prove that induced subgraph $\langle V - D' \rangle_{G-v}$ is disconnected. By assumption, the induced subgraph $\langle V - D' \rangle$ G is disconnected implies that G is a K_4-free graph and every path in $\langle V - D' \rangle$ contains no node from D'. Also, the removal of the node v from G does not change the above relationship and hence the induced subgraph $\langle V - D' \rangle_{G-v}$ is also disconnected. Thus, $\gamma^{-'}_{fat}(G) \leq p-1$ where p is the number of nodes of G.

(b). Since G is connected, there is a spanning tree T of G with $(p-1)$ nodes. If v is a pendent node of T then $(p-1)$ nodes of T other than v from a minimal inverse accurate total dominating set of G, $\gamma^{-'}_{fat}(G) \leq p-2$.

□

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