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A Note on Matrix Representations of Finite Cyclic Groups

Research Article

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Abstract: In this article we provide a general technique to construct matrix representations of additive cyclic group of any finite order and multiplicative cyclic group of order $p-1$ (p is a positive prime).

Keywords: Group, finite group, matrix group, order of a group, cyclic group.

1. Introduction

In the literature of abstract (modern) algebra ([1–4]) one can find several examples of the group of matrices. However the main aim of this article is to provide matrix representations of finite cyclic groups.

We know that for every positive integer m, $Z_m = \{0, 1, 2, 3...m-1\}$ is an additive cyclic group of order m under the operation of addition modulo m and for every positive prime p, $Z_p = \{1, 2, 3...p-1\}$ is a multiplicative cyclic group of order p under the operation of multiplication modulo p. There are various matrix representations of these two groups. We shall consider only few of them.

2. Matrix Representations of Z_m and Z_p

As mentioned above there are various matrix representations of Z_m and Z_p. We shall consider only few matrix representations of these two groups. Algebraically any two cyclic groups of equal order are same.

Let $M_m^1 = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in Z_m \right\}$. It is easy to verify that this is an additive cyclic group of order m under matrix addition modulo m. If we take $M_m^2 = \left\{ \begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix} : x \in Z_m \right\}$ then we see that M_m^2 is also an additive cyclic group of order m under matrix addition modulo m. Similarly $M_m^3 = \left\{ \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \in Z_m \right\}$ provides another example of an additive cyclic group of order m under matrix addition modulo m. These groups are matrix representations of Z_m. One can find several matrix representations of Z_m however we have considered only three different representations.

We can use all of the above three sets to get matrix representations of Z_p. If the order of additive cyclic group Z_m is same as the order of multiplicative group Z_p, then both groups become algebraically equivalent.

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Let $M_p^1 = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in Z_p, p > 2 \right\}$, $M_p^2 = \left\{ \begin{pmatrix} x & 0 \\ 0 & 0 \end{pmatrix} : x \in Z_p \right\}$ and $M_p^3 = \left\{ \begin{pmatrix} x & 0 \\ 0 & x \end{pmatrix} : x \in Z_p \right\}$. We can see that these are multiplicative cyclic group of order $p-1$ under matrix multiplication modulo p. Therefore these give matrix representations of Z_p.

Using this idea one can construct different examples of an additive cyclic group of any finite order and of any multiplicative cyclic group of order $p-1$. The construction directly follows from the above. For example matrix representations of Z_4 may be given as:

$$A_1 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix} \right\},$$
$$A_2 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix} \right\} \text{ and}$$
$$A_3 = \left\{ \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix} \right\}.$$

A_1, A_2 and A_3 are all additive cyclic group of order four under matrix addition modulo 4.

$$M_1 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \right\},$$
$$M_2 = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix} \right\} \text{ and}$$
$$M_3 = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}, \begin{pmatrix} 4 & 4 \\ 4 & 4 \end{pmatrix} \right\}.$$

M_1, M_2 and M_3 are all multiplicative cyclic group of order four under matrix multiplication modulo 5. Similarly one can construct matrix representations of a finite cyclic group of any finite order.

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