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Some Bounds On Co-Isolated Locating Domination Number

Research Article

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Abstract: Let $G(V, E)$ be a simple, finite and undirected connected graph. A non-empty set $S \subseteq V$ of a graph G is a dominating set, if every vertex in $V - S$ is adjacent to atleast one vertex in S. A dominating set $S \subseteq V$ is called a locating dominating set, if for any two vertices $v, w \in V - S$, $N(v) \cap S \neq N(w) \cap S$. A locating dominating set $S \subseteq V$ is called a co-isolated locating dominating set (cild - set), if there exists atleast one isolated vertex in $\langle V - S \rangle$. The co-isolated locating domination number γ_{cild} is the minimum cardinality of a co-isolated locating dominating set. In this paper, some bounds on co-isolated locating domination number are obtained. Also minimal cild - sets are characterized. Further the graphs for which γ_{cild} to be $p - 2$ are obtained.

Keywords: Dominating set, locating dominating set, co-isolated locating dominating set, co-isolated locating domination number.

1. Introduction

Let $G = (V, E)$ be a simple graph of order p and size q. For $v \in V(G)$, the neighborhood $N_G(v)$ (or simply $N(v)$) of v is the set of all vertices adjacent to v in G. If a graph and its complement are connected, then the graph is said to be a doubly connected graph. Let v be a vertex of a connected graph G. The eccentricity $e_G(v)$ of v is the distance to a vertex farthest from v. Thus $e_G(v) = \max\{d_G(u, v) : u \in V(G)\}$. The minimum and maximum eccentricities are the radius and diameter of G, denoted $r(G)$ and $diam(G)$ respectively. The length of a shortest cycle of G is called girth of G and is denoted by $g(G)$. A set S of vertices in a graph G is called an independent set if no two vertices in S are adjacent. The independence number $\beta_0(G)$ is the maximum cardinality of an independent set. The concept of domination in graphs was introduced by Ore [10]. A nonempty set $S \subseteq V(G)$ of a graph G is a dominating set, if every vertex in $V(G) - S$ is adjacent to some vertex in S. A special case of dominating set S is called a locating dominating set. It was defined by D. F. Rall and P. J. Slater in [11]. A dominating set S in a graph G is called a locating dominating set in G, if for any two vertices $v, w \in V(G) - S$, $N_G(v) \cap S$, $N_G(w) \cap S$ are distinct. The locating domination number of G is defined as the minimum number of vertices in a locating dominating set in G. A locating dominating set $S \subseteq V(G)$ is called a co-isolated locating dominating set, if $\langle V - S \rangle$ contains atleast one isolated vertex. The minimum cardinality of a co - isolated locating dominating set is called the co - isolated locating domination number $\gamma_{cild}(G)$. A co - isolated locating dominating set of minimum cardinality is called γ_{cild} - set and a γ_{ld} - set is defined likewise. In this paper, some bounds on co - isolated locating domination number are obtained. Also minimal cild - sets are characterized. Further the graphs for which γ_{cild} to be $p - 2$ are found.

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2. Prior Results

The following results are obtained in [2, 6–9]

Theorem 2.1 ([6]). *For any nontrivial simple connected graph G, $1 \leq \gamma_{cild}(G) \leq p-1$.*

Theorem 2.2 ([6]). *For any connected graph G, $\gamma_{cild}(G) = 1$ if and only if $G \cong K_2$.*

Theorem 2.3 ([7]). *For any connected graph G, $\gamma_{cild}(G) = 2$ if and only if G is one of the following graphs*

(a). $P_p(p = 3, 4, 5)$, where P_p is a path on p vertices.

(b). $C_p(p = 3, 5)$, where C_p is a cycle on p vertices.

(c). C_5 with a chord.

(d). G is the graph obtained by attaching a pendant edge at a vertex of C_3 (or) at a vertex of degree 2 in $K_4 - e$.

(e). G is the graph obtained by attaching a path of length 2 at a vertex of C_3.

(f). G is the Bull Graph.

Theorem 2.4 ([6]). *For any connected graph G, $\gamma_{cild}(G) = p-1$ $(p \geq 4)$ if and only if $V(G)$ can be partitioned into two sets X and Y such that one of the sets X and Y say, Y is independent and each vertex in X is adjacent to each in Y and the subgraph $\langle X \rangle$ of G induced by X is one of the following,*

(a). $\langle X \rangle$ is a complete graph

(b). $\langle X \rangle$ is totally disconnected

(c). Any two nonadjacent vertices in $V(\langle X \rangle)$ have common neighbors in $\langle X \rangle$.

Theorem 2.5 ([8]). *Let G be a doubly connected graph of order $p \geq 5$ such that $diam(G) = diam(\bar{G}) = 2$. Then G contains a co - isolated locating dominating set of cardinality $p-3$.*

Observation 2.6 ([8]).

(i). If S is a co-isolated locating dominating set of a connected graph G, then S will not be co - isolated locating dominating set of $\bar{G}$.

(ii). Let S be γ_{cild}- set of G such that $\langle V - S \rangle$ has exactly one isolated vertex, say v. Let there exist a vertex $u \in S$ such that $N(u) \cap S \subset S$ and $N(u) \cap V - S = (V - S) - \{v\}$.

(a). If there exists no vertex $w \in V - S$ such that $S \subseteq N_G(w)$, then $(S - \{u\}) \cup \{v\}$ is a co - isolated locating dominating set of $\bar{G}$. Hence, $\gamma_{cild}(\bar{G}) \leq \gamma_{cild}(G)$.

(b). If there exists a vertex $w \in V - S$ such that $S \subseteq N_G(w)$, then $S \cup \{w\}$ is a co-isolated locating dominating set of $\bar{G}$ and hence $\gamma_{cild}(\bar{G}) \leq \gamma_{cild}(G) + 1$.

Lemma 2.7 ([8]). *If G is a connected graph, then $\delta(G) \leq \gamma_{cild}(G)$, where $\delta(G)$ is the minimum degree of G.*

Theorem 2.8 ([8]). *For any doubly connected graph G of order $p \geq 4$,*

(a). $4 \leq \gamma_{cild}(G) + \gamma_{cild}\bar{G}) \leq 2p - 4$ and

(b). $4 \leq \gamma_{cild}(G)$. $\gamma_{cild}(\bar{G}) \leq (p-2)^2$.

Theorem 2.9 ([8]). *Let G be a doubly connected graph with $p \geq 4$. Then $\gamma_{cild}(G) + \gamma_{cild}(\bar{G}) = 4$ if and only if G is one of the following graphs: P_4, P_5, C_5, C_5 with a chord and the Bull graph, where Bull graph is a graph obtained by attaching exactly one pendant edge at any two vertices of C_3.*

Theorem 2.10 ([8]). *Let $G = (V, E)$ be a connected cubic graph with p vertices ($p \geq 4$). Then $\lfloor \frac{p+1}{3} \rfloor \leq \gamma_{cild}(G) \leq \frac{p}{2}$.*

Theorem 2.11 ([9]). *There exists a connected cubic graph G with $\gamma_{cild}(G) = a$, where a is a positive integer and $a \geq 8$.*

Theorem 2.12 ([2]). *If G is a graph with girth $g(G) \geq 5$, then every maximum independent set S is a minimal locating dominating set. Furthermore, if $\delta(G) \geq 2$, then $V - S$ is a locating dominating set.*

Proposition 2.13 ([2]).

(a). *If G is a bipartite graph, then the independence number $\beta_0 \geq \frac{p+l(G)-s(G)}{2}$, where $l(G)$ and $s(G)$ are number of leaves and that of supports of G respectively.*

(b). *If G is a bipartite graph with $g(G) \geq 6$ and $\delta(G) \geq 2$, then $\gamma_{cild}(G) \leq \frac{p+l(G)-s(G)}{2}$.*

3. Main Results

Observation 3.1. *Since every co-isolated locating dominating set is a dominating set as well as a locating dominating set, $\gamma(G) \leq \gamma_{ld}(G) \leq \gamma_{cild}(G)$. Equality holds if $G \cong P_5$, a path on five vertices.*

Example 3.2. *In the graph G given in Figure 3.1, $\{v_5\}$ is a $\gamma-$ set; $\{v_1, v_2\}$ is a γ_{ld}-set and $\{v_1, v_2, v_3, v_5\}$ is a γ_{cild}-set. Therefore $(G) = 1$, $\gamma_{ld}(G) = 2$ and $\gamma_{cild}(G) = 4$ and hence $\gamma(G) < \gamma_{ld} < \gamma(G) < \gamma_{cild}(G)$.*

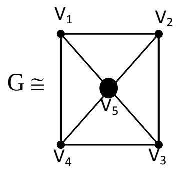

Figure 1.

In the following, the connected graphs for which $\gamma_{cild}(G) = p - 2$ are characterized.

Theorem 3.3. *Let G be a connected graph with p vertices. Then $\gamma_{cild}(G) = p - 2$ if and only if G is one of the following graphs.*

(a). *G is a graph obtained from a complete bipartite graph with bipartition $[A, B]$ by introducing two new nonadjacent vertices u and v such that $N(u) = A$, $N(v) = B$ and $A \cap B = \emptyset$.*

(b). *G is a double star $S_{m,n}$ $(m, n \geq 1)$.*

(c). *G is a graph obtained from a complete graph K_{p-1} by joining a new vertex to atmost $p - 2$ vertices of K_{p-1}.*

(d). *G is a graph obtained from K_n $(n \geq 3)$ and K_2 by joining a vertex of K_2 to a vertex of K_n and the other vertex of K_2 to $n - 2$ vertices of remaining $n - 1$ vertices of K_n, where $n = p - 2$.*

(e). G is a graph obtained from two complete graphs K_n and K_m $(m, n \geq 2)$ by joining a vertex of K_n to $m-1$ vertices of K_m and a vertex of K_m to $n-2$ vertices of K_n.

(f). G is a graph obtained from a complete bipartite graph with bipartition $[A, B]$ by introducing two new nonadjacent vertices u and v such that $N(u) \supset A$, $N(v) \supset B$ and $N(u) \neq A$ and $N(v) \neq B$.

(g). G is a graph obtained from the star $K_{1,p-2}$ by joining a new vertex to s pendant vertices of $K_{1,p-2}$, where $s < p-2$.

(h). G is a graph obtained from a complete bipartite graph $K_{m,n}$ $(m, n \geq 2$ and $m+n = p-1)$ by joining a new vertex to $m+n-1(=p-2)$ vertices of $K_{m,n}$.

(i). G is a graph such that V(G) can be partitioned into two sets X and Y such that $\langle X \rangle$ is complete, $\langle Y \rangle$ is a star and each vertex in Y is adjacent to the same $|V(G)|-1$ vertices of $\langle X \rangle$.

(j). G is a graph such that V(G) can be partitioned into two sets X and Y such that $\langle X \rangle$ is a star and $\langle Y \rangle$ is complete and each vertex in Y is adjacent to all the vertices of the star except the central vertex.

Proof. Assume $\gamma_{cild}(G) = p-2$. Then there exists a γ_{cild} set S of G having $p-2$ vertices and $V-S$ has 2 vertices. Let $V-S = \{u, v\}$, where $u, v \in V(G)$. Since $\langle V-S \rangle$ contains atleast one isolated vertex, $uv \notin E(G)$. Also S is a locating dominating set and hence $N(u) \cap S \neq N(v) \cap S$. Let $N(u) \cap S = A$ and $N(v) \cap S = B$. Therefore $A \neq B$.

Case 1: $A \cap B = \emptyset$

Assume both $\langle A \rangle$ and $\langle B \rangle$ have atleast one edge. Since G is connected, there exists an edge in G joining a vertex of A and a vertex of B. If $\langle A \rangle$ is not complete, then there exists a pair of nonadjacent vertices say a_1, a_2 in $\langle A \rangle$ such that atleast one of a_1, a_2 has degree greater than or equal to 2. Then the set $V(G)-\{u, v, a_1\}$ (or) $V(G)-\{u, v, a_2\}$ is a cild set of G and hence $\gamma_{cild}(G) \leq p-3$. Therefore $\langle A \rangle$ is complete. Similarly, it can be proved that $\langle B \rangle$ is also complete. That is, if both $\langle A \rangle$ and $\langle B \rangle$ have atleast one edge, then $\langle A \rangle$ and $\langle B \rangle$ are complete. Therefore, one of the following cases arises.

(a). Both $\langle A \rangle$ and $\langle B \rangle$ are complete.

(b). Both $\langle A \rangle$ and $\langle B \rangle$ are totally disconnected.

(c). One of $\langle A \rangle$ and $\langle B \rangle$ is complete and the other is totally disconnected.

Let $e = (a, b)$ $(a \in A, b \in B)$ be an edge in G.

Subcase 1.a: Both $\langle A \rangle$ and $\langle B \rangle$ are complete

Assume each vertex in $V(G)-\{u, v\}$ is adjacent to either u (or) v. If $|A| = |B| = 1$, then $G \cong P_4$ and $\gamma_{cild}(G) = 2$. Assume one of A and B has atleast two vertices. Let $|A| = 1$ and $|B| \geq 2$ and $N(u) = \{a_1\}, N(v) = \{b_1, b_2\}$. Then $V(G)-\{u, b_1, b_2\}$ is a cild set of G. Assume $|A| \geq 2$ and $|B| \geq 2$. Consider the set $S_1 = V(G)-\{u, v, a\}$. Then $V-S_1 = \{u, v, a\}$. In $\langle V-S_1 \rangle$, v is an isolated vertex and $N(u) \cap S = A$; $N(v) \cap S = B$ and $A \cap B = \emptyset$. Therefore, S_1 is a cild set of G and hence $\gamma_{cild}(G) \leq p-3$. Similarly, if there exists a vertex in $V(G)-\{u, v\}$ adjacent to neither u nor v, then also there is a cild set of G having $p-3$ vertices.

Subcase 1.b: Both $\langle A \rangle$ and $\langle B \rangle$ are totally disconnected

Let each vertex in $V(G)-\{u, v\}$ be adjacent to either u or v. Assume one of A and B has atleast two vertices. Let A have atleast two vertices. If there exist vertices $a_1 \in A$, $a_2 \in B$ such that $a_1b_1 \notin E(G)$, then the set $S_2 = V(G)-\{a, a_1, b_1\}$ is a cild set of G, since $V(G)-S_2 = \{a, a_1, b_1\}$ is independent, $N(a) \cap S_2 = \{u, b\}$, $N(a_1) \cap S_2 = \{u\}$ and $N(b_1) \cap S_2 = \{v\}$. Therefore $\gamma_{cild}(G) \leq p-3$. Hence each vertex in A is adjacent to each in B. That is, $\langle A \cup B \rangle$ is a complete bipartite graph. Therefore G is a graph obtained from a complete bipartite graph with bipartition $[A, B]$ by introducing two new vertices u and v such that $N(u) = A$, $N(v) = B$ and $A \cap B = \emptyset$. Let there exist a vertex in $V(G) - \{u, v\}$ adjacent to neither u nor v. Assume $|A| \geq 1$ and $|B| \geq 2$. Let $w \in V(G) - \{u, v\}$ be adjacent to neither u nor v. Then w is adjacent to atleast one of the vertices in $N(u) \cup N(v)$. Let $deg_G(w) = 1$ and let w be adjacent to the vertex say a_1 in N(u). Let $b_1, b_2 \in N(v)$. Then the set $S_3 = \{w, a_1, b_1\}$ is a cild set of G and u is isolated in $V(G) - S_3$.

Similarly is the case when $deg_G(w) \geq 2$. Consider $|A| = 1$ and $|B| = 1$. Let a_1, b_1 be adjacent to a vertex of $N(u)$ and $N(v)$ respectively and if a_1 is adjacent to b_1, then there exists a cildset of cardinality $p-3$. Therefore there exist pendant vertices in G adjavent to vertices in $N(u)$ and $N(v)$. If G is a star, then $\gamma_{cild}(G) = p - 1$. Therefore, G is a double star.

Subcase 1.c: $\langle A \rangle$ is complete and $\langle B \rangle$ is totally disconnected.

Here also the set $V(G) - \{u, v, a\}$ is a cild-set of G and hence $\gamma_{cild}(G) \leq p - 3$.

Case 2: $A \cap B \neq \emptyset$

Without loss of generality, the sets A and B $(A \cap B)$ are considered. As in Case(1), one of the following cases arise.

(a). Both $\langle A \rangle$ and $\langle B - (A \cap B) \rangle$ are complete.

(b). Both $\langle A \rangle$ and $\langle B - (A \cap B) \rangle$ are totally disconnected.

(c). $\langle A \rangle$ is complete and $\langle B - (A \cap B) \rangle$ is independent.

(d). $\langle A \rangle$ is independent and $\langle B - (A \cap B) \rangle$ is complete.

Subcase 2.a: Both $\langle A \rangle$ and $\langle B - (A \cap B) \rangle$ are complete.

Assume $B - (A \cap B) \neq \emptyset$. Let $b_1 \in B - (A \cap B)$ be not adjacent to a vertex, say $a_1 \in A \cap B$. Then $V(G) - \{a_1, b_1, u\}$ is a cild-set of G and hence $\gamma_{cild}(G) \leq p - 3$. Therefore each vertex in $B - (A \cap B)$ is adjacent to each in $A \cap B$. Similarly v is adjacent to all the vertices of $A - (A \cap B)$. That is, v is adjacent to all the vertices of A. In this case G is a graph obtained from a complete graph K_{p-1} by joining a new vertex to atmost $p - 2$ vertices of K_{p-1}. Assume $B - (A \cap B) = \emptyset$. Then $B = A \cap B$. That is, u is adjacent to all the vertices of $N(u)(= B)$. Since $A \neq B$, $A \cap B \neq A$. That is v is not adjacent to atleast one vertex of $N(u)(= A)$. Therefore, $|A| \geq 2$, $|B| \geq 1$. Assume $|A| \geq 2$ and $|B| = 1$. Let $|B| = b_1$. Then u is adjacent to b_1 if one of the following conditions holds

(i). v is adjacent to $|A| - 1$ vertices of $A(= N(u))$

(ii). v is adjacent to t vertices of $A(= N(u))$ where $1 \leq t \leq |A| - 2$ and each vertex in A is adjacent to b_1.

If (i) holds, then G is a graph obtained from $K_n (n \geq 2)$ and K_2 by joining a vertex of K_2 to a vertex of K_n and the other vertex of K_2 to $n - 2$ vertices of remaining $n - 1$ vertices of $K_n (n = p - 2)$.

If (ii) holds, then G is a graph obtained from K_{p-1} by joining a new vertex of atmost $p - 2$ vertices of K_{p-1}. Let $|A| \geq 2$, $|B| \geq 2$. As above one of the following holds

(iii). v is adjacent to $|A| - 1$ vertices of A.

(iv). v is adjacent to t vertices of A where $1 \leq t \leq |A| - 2$ and each vertex in A is adjacent to each in B.

If (iii) holds, then G is a graph obtained from complete graphs K_n and K_m $(m, n \geq 2)$ by joining a vertex of K_n to $m - 1$ vertices of K_m and a vertex of K_m to $n - 2$ vertices of K_n.

If (iv) holds, then G is a graph obtained from K_{p-1} by joining a new vertex to atmost $p - 2$ vertices of K_{p-1}.

Subcase 2.b: Both $\langle A \rangle$ and $\langle B(A \cap B) \rangle$ are totally disconnected.

Assume $B-(A\cap B)\neq\emptyset$. Here also each vertex in $B-(A\cap B)$ is adjacent to each in A and v is adjacent to each vertex in A. Therefore G is a graph obtained from a complete bipartite graph with bipartition $[A,B]$ by introducing two new nonadjacent vertices u and v such that $N(u)\supset A$, $N(v)\supset B$ and $N(u)\neq A$, $N(v)\neq B$. Assume $B-(A\cap B)=\emptyset$. Then $B=A\cap B$. That is, u is adjacent to all the vertices of $N(v)$. Then either G is a graph obtained from the star $K_{1,p-2}$ by joining a new vertex to s pendant vertices of $K_{1,p-2}$ where s $< p-2$ and this graph is denoted by $K^{s}_{1,p-2}$ $(s<p-2)$ (or) G is a graph obtained from a complete bipartite graph $K_{m,n}$ $(m,n\geq 2$ and $m+n=p-1)$ by joining a new vertex to $m+n-1(=p-2)$ pendant vertices of $K_{m,n}$.

Subcase 2.c: Either $\langle A\rangle$ is complete and $\langle B-(A\cap B)\rangle$ is totally disconnected (or) $\langle A\rangle$ is totally disconnected and $\langle B-(A\cap B)\rangle$ is complete.

Assume $B-(A\cap B)\neq\emptyset$. In both the cases, each vertex in $B-(A\cap B)$ is adjacent to each in A and v is adjacent to each vertex in A. Then either G is a graph in which $V(G)$ can be partitioned into two sets X and Y such that $\langle X\rangle$ is complete, $\langle Y\rangle$ is a star and each vertex in Y is adjacent to the same $|V(X)|-1$ vertices of $\langle X\rangle$ (or) G is a graph in which $V(G)$ can be partitioned into two sets X and Y such that $\langle X\rangle$ is a star and $\langle Y\rangle$ is complete and each vertex in Y is adjacent to all the vertices of the star except the central vertex. From all the cases, it is concluded that G is one of the graphs given in the theorem. Conversely if G is one of the graphs given in the theorem, then $\gamma_{cild}(G)=p-2$. □

In the following, the minimal cild-sets are characterized.

Theorem 3.4. *A cild-set S of a connected graph G is minimal if and only if each vertex $v\in S$ satisfies one of the following conditions,*

(i). v is an isolated vertex of S.

(ii). There exists a vertex $u\in V-S$ such that $N(u)\cap S=\{v\}$

(iii). v is adjacent to all the isolated vertices in $V-S$.

(iv). there exists a vertex $u\in V-S$ such that both u and v have common neighbor in S.

Proof. Let S be a minimal cild-set of G. Then for every $v\in S, S-\{v\}$ is not a cild-set of G. Then one of the following conditions holds

(a). $S-\{v\}$ is not a dominating set

(b). $V-(S-\{v\})$ does not contain any isolated vertices.

(c). Any two vertices in $V-(S-\{v\})$ have common neighbors in $S-\{v\}$.

(a). implies the conditions (i) and (ii).

(b). implies that, v is adjacent to all the isolated vertices in $V-S$.

(c). implies that there exists a vertex $u\in V-S$ such that u and v have common neighbors in S.

Conversely, let S be a cild-set of G. Assume for each $v\in S$, one of the conditions (i)-(iv) holds. By (i) and (ii), $S-\{v\}$ is not a dominating set of G, since v is not adjacent to any vertex in $S-\{v\}$. By (iii), $V-(S-\{v\})$ has no isolated vertices. By (iv), $S-\{v\}$ is not a locating set of G. Therefore, $S-\{v\}$ is not a cild-set of G, for all $v\in S$. Hence, S is a minimal cild-set of G. □

In the following, an upper bound of $\gamma_{cild}(G)$ in terms of maximum degree $\Delta(G)$ is obtained.

Theorem 3.5. *For any connected graph G on p vertices, $\gamma_{cild}(G) + \Delta(G) \leq 2p - 2$.*

Proof. For any connected graph G, $\gamma_{cild}(G) \leq p-1$ and $\Delta(G) \leq p-1$ and hence $\gamma_{cild}(G) + \Delta(G) \leq 2p-2$. □

In the following, the connected graphs G for which $\gamma_{cild}(G) + \Delta(G) = 2p - 2$ are characterized.

Theorem 3.6. *For any connected graph G on p ($p \geq 4$) vertices, $\gamma_{cild}(G)+\Delta(G) = 2p-2$ if and only if $G \cong K_{1,p-1}$, $p \geq 4$ (or) $V(G)$ can be partitioned into two sets X and Y such that Y is independent and each vertex in Y is adjacent to each in X and either $|Y| = 1$ (or) there exists atleast one vertex in $\langle X\rangle$ of degree $(m-1)$ where $|V(X)| = m$.*

Proof. Let $\gamma_{cild}(G) + \Delta(G) = 2p - 2$. Then $\gamma_{cild}(G) = p - 1$ and $\Delta(G) = p - 1$. But $\gamma_{cild}(G) = p - 1$ if and only if $V(G)$ can be partitioned into two sets X and Y such that one of the sets X and Y, say Y is independent and each vertex in X is adjacent to each vertex in Y and $\langle X\rangle$ is one of the following.

(a). $\langle X\rangle$ is a complete subgraph of G.

(b). $\langle X\rangle$ is totally disconnected.

(c). Any two non - adjacent vertices in V ($\langle X\rangle$) have common neighbors in $\langle X\rangle$.

Case 1: $\langle X\rangle$ is a complete subgraph of G

Since each vertex in X is adjacent to each in Y, the vertices of X have degree $(p-1)$ in G. For this graph, $\Delta(G) = p-1$.

Case 2: $\langle X\rangle$ is totally disconnected

Since $\Delta(G) = p - 1$, X contains exactly one vertex. Therefore, $G \cong K_{1,p-1}$, $p \geq 4$.

Case 3: Any two non-adjacent vertices in V ($\langle X\rangle$) have common neighbors in $\langle X\rangle$

In this case $\langle X\rangle$ is not complete. Let $|V(\langle X\rangle)| = m$, $m < p$. Therefore, $|Y| = p - m$. Let v be vertex in $\langle X\rangle$ of degree t, where $t < m - 1$. Let $u, v \in V$. If $u, v \in X$, then $d_G(u, v) \leq 2$. Therefore, diameter of G is 2. If $diam(G) = rad(G) = 2$, then there exists no vertex of degree $p - 1$ in G. Therefore $\Delta(G) \leq p - 2$. Assume $\gamma(G) = 1$. If Y has exactly one vertex, then that vertex has degree $p - 1$ in G. Otherwise there must exist a vertex in $\langle X\rangle$ of degree $m - 1$ in $\langle X\rangle$. Hence G is a graph with $\gamma(G) = 1$ and $V(G)$ can be partitioned into two sets X and Y such that Y is independent and each vertex in X is adjacent to each vertex in Y and either $|Y| = 1$ (or) $\langle X\rangle$ has a vertex of degree $(m - 1)$, where $|V(X)| = m$. □

In the following, an upper bound of $\gamma_{cild}(G)$ in terms of order and diameter is proved.

Theorem 3.7. *Let G be a connected graph of order p and diameter $d \geq 4$. Then $\gamma_{cild}(G) + \lceil \frac{3d-3}{5} \rceil \leq p$ and the bound is sharp.*

Proof. Let $u, v \in V(G)$ be two diametral vertices and let P be a diametral path joining u and v. Let $V(P) = \{u = 1, 2, 3, \ldots, v = d\}$, where $d = 5h + k$ with $0 \leq k \leq 4$. Then for $k = 0$, the set $A_1 = \{2, 4, \ldots, 5h-3, d-1\}$; for $1 \leq k \leq 2$, the set $A_2 = \{2, 4, \ldots, 5h - 3, 5h - 1, d\}$ and for $3 \leq k \leq 4$, the set $A_3 = \{2, 4, \ldots, 5h - 3, 5h - 1, d - 2, d\}$ are the γ_{cild}-sets of P and these sets have $\lfloor \frac{2d+4}{5} \rfloor$ elements. The set $S = V(G) - V(P) - A_i$ has $p - \lceil \frac{3d-3}{5} \rceil$ elements and it is a co-isolated locating dominating set of G. Hence $\gamma_{cild}(G) \leq p - \lceil \frac{3d-3}{5} \rceil$. That is, $\gamma_{cild}(G) + \lceil \frac{3d-3}{5} \rceil \leq p$. This bound is attained when $G \cong P_{5n+1}$, $n \geq 1$. □

Lemma 3.8. *Let G be a graph of order p and $\gamma_{cild}(G) \geq p - 2$. Then $diam(G) \leq 3$.*

Proof. Assume $\gamma_{cild}(G) \geq p-2$. Suppose that $diam(G) \geq 4$. Let $u, v \in V(G)$ such that $d(u, v) = 4$ and let P be a shortest path joining u and v. Let $P = \{u, x, w, y, v\}$ where $x, w, y \in V(G)$. Let $S = V(G) - \{u, w, v\}$ and $N(u) \cap S$, $N(v) \cap S$, $N(w) \cap S$, are nonempty and distinct. Also the vertices u, w, and v are isolated in $\langle V - S\rangle$. Therefore $\gamma_{cild}(G) \leq p - 3$, which is a contradiction. Hence $diam(G) \leq 3$. □

In the following, an upper bound of $\gamma_{cild}(G)$ in terms of order and independence number is proved.

Theorem 3.9. *If G is a connected graph with* $g(G) \geq 5$ *and* $\delta(G) \geq 2$ *then* $\gamma_{cild}(G) \leq p - \beta_0(G)$.

Proof. By Theorem 2.12, if girth $g(G) \geq 5$ and $\delta(G) \geq 2$ and if S is a maximum independent set, then $V - S$ is also a co-isolated locating dominating set. Hence $\gamma_{cild}(G) \leq |V - S| = p - \beta_0(G)$. Equality holds, if $G \cong C_{2n}$, $n \geq 3$. □

In the following, an upper bound of $\gamma_{cild}(G)$ in terms of order, leaves and supports of G is obtained.

Theorem 3.10. *If G is a bipartite graph with* $g(G) \geq 5$ *and* $\delta(G) \geq 2$ *then* $\gamma_{cild}(G) \leq \frac{p-l(G)+s(G)}{2}$.

Proof. Assume $\delta(G) \geq 2$ and $g(G) \geq 6$. By Proposition 2.13, $\beta_0(G) \geq \frac{p+l(G)-s(G)}{2}$. Therefore, $p - \beta_0(G) \leq p - \frac{p+l(G)-s(G)}{2} = \frac{p-l(G)+s(G)}{2}$. □

4. Conclusion

In this paper, an upper bounds of γ_{cild}(G) in terms of the order, maximum degree, diameter, independence number are obtained. Also the graphs for which $\gamma_{cild}(G) = p - 2$ are characterized. This paper can also be developed by finding the lower bound of $\gamma_{cild}(G)$ in terms of the some other parameters like minimum degree, girth of G. Finding the co-isolated locating domatic number is the future work.

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