International Journal of Mathematics And its Applications
Volume 4, Issue 1–B (2016), 121–131.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Study of Anomalous Behavior of Blood Flow in Capillary Surrounded by Tissue

Research Article

Rekha Bali[1], Swati Mishra[1]* and P.N.Tandon[1]

1 Department of Mathematics, Harcourt Butler Technological Institute, Kanpur, India.

Abstract: A Mathematical model for the two phase flow of blood in a narrow capillary surrounded by the tissue has been presented in this paper. We have developed the earlier two phase models of blood flow in capillaries into a model of capillary-tissue fluid exchange system by including porous tissue enclosing the capillary. Blood is not a homogeneous fluid, It is a suspension. The suspended blood cells accumulate at the axis in response to flow, resulting in a blunting of the velocity profile near the axis in contrast to the parabolic profile observed in homogeneous fluids. Earlier models have not included the effect of fluid exchange in between the capillary and the tissue while microtubes or capillaries are surrounded by tissue. Therefore, our aim is to include effects of permeability of the tissue with two phase flow of blood in narrow tubes. The model consists of a core region of suspension of all erythrocytes and a peripheral layer of plasma surrounding the core. The governing partial differential equations have been solved by using mathematical and computational techniques. Analytical results, in the proposed model for apparent viscosity, Bluntness and ratio of capillary hematocrit with discharge hematocrit values have been presented and discussed through graphs for various values of parameters and with their axial variations.

Keywords: Anomalous behavior, Apperant viscosity, Bluntness, Capillary-tissue fluid exchange.

1. Introduction

Microcirculation deals with the flow of blood from arterioles to capillaries or sinusoids to venule through a vessel channel called a thoroughfare channel. Capillaries extend from this channel and structures control the flow of blood between the arterioles and capillaries due to vasomotion and dilation. The blood flow through narrow tubes has been of great interest to both physiologist and biomedical engineers due to its relevance to flow in microcirculation, the most important segment in the whole circulatory system. All exchange between the blood and the organs of nutrients, oxygen, and waste products and carbon dioxide takes place through capillary tissue exchange system. The blood fulfills its real functions. One could say that the whole circulatory machine exist for their sake.

Microcirculatory disorders are major contributors to morbidity and mortality. Despite the research and developments efforts of many laboratories, no substitute has yet been developed that can carry out the essential functions that the whole blood performs in the circulatory system particularly in microcirculation. There is need to better understand how the special characteristics of blood and its flow properties make it such an effective means for delivery and exchange in the microcirculation.

Blood is not a homogeneous fluid: it is a suspension. As a result, the viscosity is no longer a well defined material property, but rather must be defined as an observed resistance to flow. Expressed in this way, the apparent viscosity of blood will

* *E-mail: swatimishra1982@yahoo.co.in*

depend upon the confining geometry decreasing with decreasing vessel diameter i.e. it is easier to move blood through a narrow vessel. This is the most famous flow oddity associated with blood and is called the Fahraeus-Lindquist effect. Blood flow in small vessels shear gradients or wall wall interactions lead to some inhomogeneous particle distribution. Therefore geometry of flow is an important consideration. Therefore, we may consider a approach that takes account of the inhomogeneous particle distribution in tube flow by modeling blood as a layered fluid with a viscous core surrounded by rings of less viscous fluid [Scott [20]]

In the study of anomalous flow behavior two types of approach has been considered. (a) The use of a non-linear shear stress-shear rate relationship of some form varying with the flow conditions together with the no slip condition (b) the use of a constant viscosity with slip at the boundaries. The study postulates that the slip may be present blood flow, its magnitude depending on the nature of the wall surface, shear at the wall and relative cell volume immediately adjacent to the wall. As blood possesses a specific, flow independent viscosity, studies proposes to interpret its anomalous flow behavior on the basis of slip.

The rheological properties of blood, including its non-newtonian characteristics have been known for many years. Such studies have yielded valuable information on the properties of blood under certain well defined conditions. For numerous reasons this information is not sufficient to understand the flow behavior of blood in the microcirculation. Firstly, the rheological properties of the blood in a network, such as the microcirculation with its myriad vessel segments of different lengths, diameter and flow rates, cant be adequately predicted from viscometry in much simpler system. Second, in the microcirculation, the luminal surface of the vessel wall is coated with a fibrous material that retards, to varying degrees depending on flow rate, the flow of blood in the immediate vicinity of the wall, the tissue. These factors motivate us to study the flow of blood in a capillary surrounded by tissue and including capillary tissue fluid exchange phenomena.

Pries and coworkers in a series of papers [15–19, 19] derived empirical relationship for the relative apparent viscosity, mean tube diameter and discharge hematocrit in vivo and in vitro. Prahlad and Scultz [14] used a two fluid model of polar fluid to analyze the flow of blood with and without stenotic artery. Sharan and Popel [21] suggested a modification on the models of Haynes [11] and Bugliarello and Sevilla [1] assuming the viscosity in the peripheral layer to be higher than that of plasma due to additional dissipation of energy caused by the red cell motion near the cell free layer. Two fluid model analysis have been carried out by Srivastava [23, 24]. Bassinghwaighte applied the two layered models of Haynes [11] and Sharan and Popel [21] to discuss flow of blood in narrow curved tubes etc. Sheshadri and Jaffrin [22] introduced the outer layer as cell depleted layer. Hematocrit of cell depleted layer is lower than the core hematocrit. Several other authors [Gupta et. al. [10], Haynes [11], Casson [4], Charm and Kurland [3], Eringen [9], Chaturani and Upadhyay [5, 6] have studied the flow behavior of blood subject to physiological aspects observed in small capillaries. Nair et. al. [13] considered a cell rich core surrounded by a cell free plasma layer. In this paper, the hematocrit distribution in radial direction was expressed as power law profile with maximum hematocrit at the centre of the tube. Damiano [7] has presented a semi-empirical model for the blood flow in glycocalyx lined microvessels greater than 20μm in diameter. The model assumes a steady axisymmetric flow of a viscous fluid having a smoothly varying crossectional viscosity throughout the tube.

The above models shows the popularity of layered fluid model in the modeling of blood rheology but in all the above models blood flow has been studied in rigid circular pipe but they have not considered the tube surrounded by tissuea very important aspect of microcirculation. Therefore, in this paper we have taken the capillary surrounded by the tissue.

Two phase continuum models considering a core of Newtonian viscous fluid, representing the concentrated RBC core suspension and an annual concentric layer of a less viscous Newtonian fluid representing the cell depleted layer are in reasonable quantitative agreement with experimental data on the apparent viscosity (Fehraeus-Lindquist effect) of blood flow in tubes $\geq$ 30 μm in diameter.

Blood in the capillary is considered as two phase flow: cell rich core and the peripheral layer of plasma. Under the high shear stress the fluid in both regions are considered as Newtonian fluid of different viscosities. We have studied the effect of tissue permeability and discharge hematocrit on the Fahraeus effect and Fehraeus-Lindquist effect. We have also discussed effects of discharge hematocrit and permeability on apparent viscosity, Bluntness of the velocity profile and ratio of core hematocrit to the discharge hematocrit in capillary.

2. Formulation of The Problem

We have considered a uniform cylindrical capillary of length l and radius R. The capillary is surrounded by a tissue of thickness H. In the capillary region blood is represented as a two fluid model consist of central core region and peripheral layer region. Central core region is assumed to be uniform hematocrit of radius rh and viscosity μ_c and central core region is covered by cell free layer containing plasma with viscosity μ_0. The fluid in both regions is assumed Newtonian. Under the high shear stress the fluid in both regions are considered as Newtonian fluid of different viscosities.

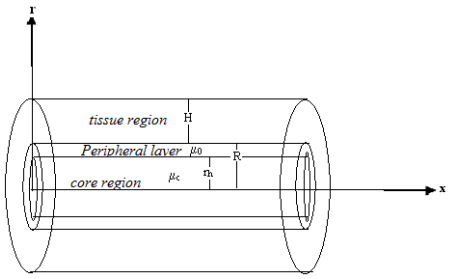

Figure 1. **Schematic diagram of the model**

We restrict ourselves to small enough Reynoldss number so that under the assumptions of slow viscous motion inertia terms are neglected and hence the governing equations for the fluid flow in different regions are represented by the creeping flow equation given below:

(i). For the central core region with red blood cells

$$-\frac{\partial \mathrm{P}'}{\partial x'}+\frac{\mu_c}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial {u_c}'}{\partial r'}\right)=0 \tag{1}$$

$(0 \le r < r_h)$

$$-\frac{\partial {u_c}'}{\partial x'}+\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'{v_c}'\right)=0 \tag{2}$$

where x' and r' are the axial and radial coordinates, P' is the pressure in of the fluid in capillary μ_c is the viscosities of fluid in core and region, μ_c , and v_c are axial and radial component of the velocity in core region, is the radius of central core region.

(ii). For the cell free layer (Plasma layer)

$$-\frac{\partial \mathrm{P}'}{\partial x'}+\frac{\mu_0}{r'}\frac{\partial}{\partial r'}\left(r'\frac{\partial {u_0}'}{\partial r'}\right)=0 \tag{3}$$

$(r_h < r < R)$

$$-\frac{\partial {u_0}'}{\partial x'}+\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'{v_0}'\right)=0 \tag{4}$$

where μ_0 are the viscosity of fluid in peripheral layer, u_0 and v_0 are axial and radial component of the velocity in the cell free layer, R is the radius of capillary region.

(iii). For the Tissue region

$$\bar{u}' = -\frac{K}{\mu_0}\frac{\partial \bar{\mathrm{P}}'}{\partial x'}, \qquad \bar{v}' = -\frac{K}{\mu_0}\frac{\partial \bar{\mathrm{P}}'}{\partial r'} \tag{5}$$

$(R < r' < R + H)$

$$\frac{\partial \bar{u}'}{\partial x'} + \frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\,\bar{v}'\right) = 0 \tag{6}$$

where $\bar{\mathrm{P}}'$ is the pressure in tissue region, $\bar{u}'$ and $\bar{v}'$ are the axial and radial component of velocity of fluid in tissue region, K is the permeability of the fluid at the interface, H is the thickness of the tissue.

The Boundary Conditions for Solving the above Equations are Given as

Due to symmetry, the velocity gradient vanishes along the axis of the tube

$$\frac{\partial {u_c}'}{\partial r'} = 0 \ \text{ at } \ r' = 0$$

The velocity and shear stress are continuous at the interface of plasma and the core

$${u_c}' = {u_0}' \ \text{ at } \ r' = r_h'$$

$$\mu_c \frac{\partial {u_c}'}{\partial r'} = \mu_0 \frac{\partial {u_0}'}{\partial r'} \ \text{ at } \ r' = r_h'$$

Slip velocity is assumed at the porous boundary

$${u_0}' - \bar{u}' = -\sigma \frac{\partial {u_0}'}{\partial r'} \ \text{ at } \ r' = R$$

No flux condition is assumed at the outer surface of tissue

$$\frac{\partial \bar{\mathrm{P}}'}{\partial r'} = 0 \ \text{ at } \ r' = R + H'$$

There is no transfer of fluid through the annular ends of the tissue

$$\frac{\partial \bar{\mathrm{P}}'}{\partial x'} = 0 \ \text{ at } \ x' = 0$$

$$\frac{\partial \bar{\mathrm{P}}'}{\partial x'} = 0 \ \text{ at } \ x' = l'$$

Pressure across the boundary are assumed continuous

$$\mathrm{P}' = \bar{\mathrm{P}}' \ \text{ at } \ r' = R$$

Normal velocity at the central line is assumed zero

$${v_c}' = 0 \ \text{ at } \ r' = 0$$

At the entry of the capillary the fluid pressure is equal to the pressure at the arterial end we assume the fluid pressure equal to the pressure at venous end

$$\mathrm{P}' = {\mathrm{P}_0}' \ \text{ at } \ x' = 0$$

$$\mathrm{P}' = {\mathrm{P}_l}' \ \text{ at } \ x' = l' \tag{7}$$

where σ is the slip parameter and l' is the length of the capillary, P_0' is the pressure at the arterial end and P_1 is the pressure at the venous end.

Solutions

We now introduce the following non-dimensional scheme before finding the solution of the problem.

$$\begin{aligned} &\mathrm{P}=\frac{\mathrm{P}'}{\rho\,{u_{b_0}}^2}, \quad x=\frac{x'}{R}, \quad \mathrm{P}_l=\frac{{\mathrm{P}_l}'}{\rho\,{u_{b_0}}^2}, \quad r=\frac{r'}{R}, \quad u_c=\frac{{u_c}'}{u_{b_0}}, \quad l=\frac{l'}{R}, \quad \bar{\mathrm{P}}=\frac{\bar{\mathrm{P}}'}{\rho\,{u_{b_0}}^2} \\ &u_0=\frac{{u_0}'}{u_{b_0}}, \quad v_c=\frac{{v_c}'}{u_{b_0}}, \quad v_0=\frac{{v_0}'}{u_{b_0}}, \quad r_h=\frac{{r_h}'}{R}, \quad H=\frac{H'}{R}, \quad \mathrm{P}_0=\frac{{\mathrm{P}_0}'}{\rho\,{u_{b_0}}^2}, \quad \mathrm{Re}=\frac{\rho\,u_{b_0}R}{\mu_0} \end{aligned} \tag{8}$$

where U_{b0} is the mean velocity in the capillary, R_e is the Reynolds number.

Solution of the Problem

Substituting and in equation of continuity we get the Laplace equation, which has been solved by method of separation of variables, Thus we get pressure distribution in porous region using boundary and interfaces conditions as:

$$\bar{\mathrm{P}}=E_0+\frac{K\mathrm{Re}}{R^2}\sum_{n=1}^{\infty}E_n\left\{\frac{K_0\left\langle\alpha_n,r\right\rangle I_1\left\langle\alpha_n,\left\langle 1+H\right\rangle\right\rangle+I_0\left\langle\alpha_n,r\right\rangle K_1\left\langle\alpha_n,\left\langle 1+H\right\rangle\right\rangle}{I_1\left\langle\alpha_n,\left\langle 1+H\right\rangle\right\rangle}\right\}\cos\left\langle\alpha_n,x\right\rangle \tag{9}$$

where

$$\begin{aligned} \alpha_n &= \frac{n\pi}{l} \\ E_0 &= 2\left(C_5\frac{l}{2}+C_6\right) \\ E_n &= \frac{2C_5}{\ell}\frac{{\alpha_n}^2\left\{(-1)^n-1\right\}}{\left\{{\alpha_n}^2F_1\left\langle\alpha_n\right\rangle-16DF_3\left\langle\alpha_n\right\rangle\right\}} \\ F_3\left\langle\alpha_n\right\rangle &= \frac{F_2\left\langle\alpha_n\right\rangle+\alpha_nF_1\left\langle\alpha_n\right\rangle}{\mathrm{Re}\left\{{r_h}^2\left(\frac{\mu_0}{\mu_c}\right)-(1+4\sigma)\right\}} \\ F_1\left(\alpha_n\right) &= \left\{\frac{K_0\left\langle\alpha_n\right\rangle I_1\left\langle\alpha_n\left\langle 1+H\right\rangle\right\rangle+I_0\left\langle\alpha_n\right\rangle K_1\left\langle\alpha_n\left\langle 1+H\right\rangle\right\rangle}{I_1\left\langle\alpha_n\left\langle 1+H\right\rangle\right\rangle}\right\} \\ F_2\left(\alpha_n\right) &= \left\{\frac{I_1\left\langle\alpha_n\right\rangle K_1\left\langle\alpha_n\left\langle 1+H\right\rangle\right\rangle-K_1\left\langle\alpha_n\right\rangle I_1\left\langle\alpha_n\left\langle 1+H\right\rangle\right\rangle}{I_1\left\langle\alpha_n\left\langle 1+H\right\rangle\right\rangle}\right\} \end{aligned}$$

and K_0, K_1, I_0 and I_1 are modified Bessel's functions of order zero and one respectively. Solving equation of motion using boundary and interfacial conditions velocity in peripheral layer as given by:

$$u_0=\left[\mathrm{Re}\frac{\partial\mathrm{P}}{\partial x}\left\{\frac{r^2-(1+4\sigma)}{4}\right\}\right]-16\frac{K\mathrm{Re}}{R^2}\sum_{n=1}^{\infty}E_n\alpha_nF_1\left(\alpha_n\right)\sin\left(\alpha_nx\right) \tag{10}$$

and velocity in the core region is obtained as:

$$u_c=\mathrm{Re}\frac{\partial\mathrm{P}}{\partial x}\left[\left\{\frac{\left(r^2-{r_h}^2\right)\mu_0}{4\mu_c}\right\}-\left\{\frac{{r_h}^2-(1+4\sigma)}{4}\right\}\right]-16\frac{K\mathrm{Re}}{R^2}\sum_{n=1}^{\infty}E_n\alpha_nF_1\left(\alpha_n\right)\sin\left(\alpha_nx\right) \tag{11}$$

Using equation of continuity, we obtained the expression for Pressure distribution in capillary region with the help of boundary condition

$$\mathrm{P}=16\frac{K\mathrm{Re}}{R^2}\sum_{n=1}^{\infty}E_n\,F_3\left\langle\alpha_n\right\rangle\cos\left\langle\alpha_nx\right\rangle+C_1x+C_2 \tag{12}$$

where

$$C_1 = \frac{\mathrm{P}_l - \mathrm{P}_0}{\varepsilon + l}$$
$$\varepsilon = 16\,\frac{K\mathrm{Re}}{R^2}\sum_{n=1}^{\infty}\frac{\{(-1)^n - 1\}^2 F_3\,\langle\alpha_n\rangle}{l\alpha_n\left\{{\alpha_n}^2 F_1\,\langle\alpha_n\rangle - 16\frac{K\mathrm{Re}}{R^2}F_3\,\langle\alpha_n\rangle\right\}}$$
$$C_2 = \mathrm{P}_0 - 16\frac{K\mathrm{Re}}{R^2}\sum_{n=1}^{\infty}E_n F_3\,\langle\alpha_n\rangle\,\frac{1}{\alpha_n}$$

Equation (11) can be expressed as

$$u_c = u_{\max}\left(1 - Br^2\right) \tag{13}$$

where

$$u_{\max} = \left\{\mathrm{Re}\frac{\partial \mathrm{P}}{\partial x}\left\{\frac{\left({r_h}^2 - (1+4\sigma)\right)}{4} - \frac{\mu_0}{4\mu_c}{r_h}^2\right\} + \left(\frac{K\mathrm{Re}}{R^2}\right)\sum_{n=1}^{\infty}E_n F_1\,(\alpha_n)\sin\,(\alpha_n x)\right\} \tag{14}$$

and B is defined as bluntness as given by

$$B = \frac{16D\left(\frac{\rho u_{b_0} R}{\mu_c}\right)\sum_{n=0}^{\infty}E_n F_3\,\langle\alpha_n\rangle\cos\langle\alpha_n x\rangle + C_1}{\left\{\mathrm{Re}\frac{\partial \mathrm{P}}{\partial x}\left\{\frac{\left({r_h}^2-(1+4\sigma)\right)}{4} - \frac{\mu_0}{4\mu_c}{r_h}^2\right\} + \left(\frac{16K\mathrm{Re}}{R^2}\right)\sum_{n=1}^{\infty}E_n F_1\,(\alpha_n)\sin\,(\alpha_n x)\right\}}, \tag{15}$$

Volumetric flow rate of the blood is given as

$$Q = 2\pi R^2\int_0^{r_h} u_c(r)\,r\,dr + 2\pi R^2\int_{r_h}^{1} u_0(r)\,r\,dr \tag{16}$$

Overall mass balance of the cells defined in the capillary is

$$QH_d = 2\pi R^2\int_0^1 r\,u(r)\,h(r)\,dr \tag{17}$$

where H_d is the discharge hematocrit

$$h\,(r)\;=\;\left\{\begin{array}{ll} H_c & 0 \le r \le r_h \\ 0 & r_h < r < 1 \end{array}\right\} \tag{18}$$

By solving the equation (16) and (17) and using the velocity from equation (10) and (11) we have

$$Q = 2\pi R^2\left\{\mathrm{Re}\frac{\partial \mathrm{P}}{\partial x}\left\{\frac{{r_h}^4}{16}\left(-\frac{\mu_0}{\mu_c}\right) + \left\{\frac{{r_h}^4}{4} - \frac{(1+4\sigma)}{4}\right\}\right\} + \left(\frac{K\mathrm{Re}}{R^2}\right)\sum_{n=1}^{\infty}E_n\alpha_n F_1\,(\alpha_n)\sin(\alpha_n x)\,\frac{1}{2}\right\} \tag{19}$$

$$QH_d = 2\pi R^2 H_c\left[\mathrm{Re}\frac{\partial \mathrm{P}}{\partial x}\left\{\frac{{r_h}^4}{16}\left(-\frac{\mu_0}{\mu_c}\right) + \left\{\frac{{r_h}^4}{4} - \frac{(1+4\sigma)}{4}\right\}\right\} - \left(\frac{K\mathrm{Re}}{R^2}\right)\sum_{n=1}^{\infty}E_n\alpha_n F_1\,(\alpha_n)\sin(\alpha_n x)\,\frac{{r_h}^2}{2}\right] \tag{20}$$

$$\frac{H_c}{H_d} = \frac{Q}{2\pi R^2\left[\mathrm{Re}\frac{\partial \mathrm{P}}{\partial x}\left\{\frac{{r_h}^4}{16}\left(-\frac{\mu_0}{\mu_c}\right) + \left\{\frac{{r_h}^4}{4} - \frac{(1+4\sigma)}{4}\right\}\right\} - \left(\frac{K\mathrm{Re}}{R^2}\right)\sum_{n=1}^{\infty}E_n\alpha_n F_1\,(\alpha_n)\sin(\alpha_n x)\,\frac{{r_h}^2}{2}\right]} \tag{21}$$

By Poiseuille Law we have

$$Q = \frac{\pi \mathrm{P} R^4}{8\mu_{app}} \tag{22}$$

Comparing (20) and (22) we have obtained the expression for apparent viscosity as given below:

$$\mu_{app} = \frac{16K\mathrm{Re}\sum_{n=0}^{\infty}E_n\frac{1}{\alpha_n}F_3\,\langle\alpha_n\rangle\cos\langle\alpha_n x\rangle + C_1}{16\frac{H_c}{H_d}\left[\mathrm{Re}\frac{\partial \mathrm{P}}{\partial x}\left\{\frac{{r_h}^4}{16}\left(-\frac{\mu_0}{\mu_c}\right) + \left\{\frac{{r_h}^4}{4} - \frac{(1+4\sigma)}{4}\right\}\right\} - \left(\frac{K\mathrm{Re}}{R^2}\right)\sum_{n=1}^{\infty}E_n\alpha_n F_1\,(\alpha_n)\sin(\alpha_n x)\,\frac{{r_h}^2}{2}\right]}$$

It is worth mentioning for the validation of the above model that in the absence of surrounding tissue (i.e. permeability $K = 0$) and when the core fluid changes to fluid as in peripheral region (i.e. when $\mu_c = \mu_0$) Poiseuille's flow is recovered.

3. Results and Discussions

In this paper, we have studied the anomalous behavior of blood in capillaries surrounded by the tissue. When capillary is surrounded by tissue, some fluid exchange also exists between the capillary and tissue. The tissue being porous imbibes the plasma, therefore total flux does not remain constant throughout the capillary length. Therefore, pressure gradient, apparent viscosity, hematocrit and bluntness depend also on axial distance due to the axial variation of fluid flux at the capillary-tissue interface. Therefore, we have analysed the effect of permeability of the tissue on pressure gradient(figures-2 & 3), apparent viscosity of blood(figures-4 & 5), hematocrit (FahraeusLindquist effect (figures-6 & 7) and the bluntness of velocity profile for different discharge hematocrit and permeability of the tissue(figures-8 & 9) with diameter of the tube.

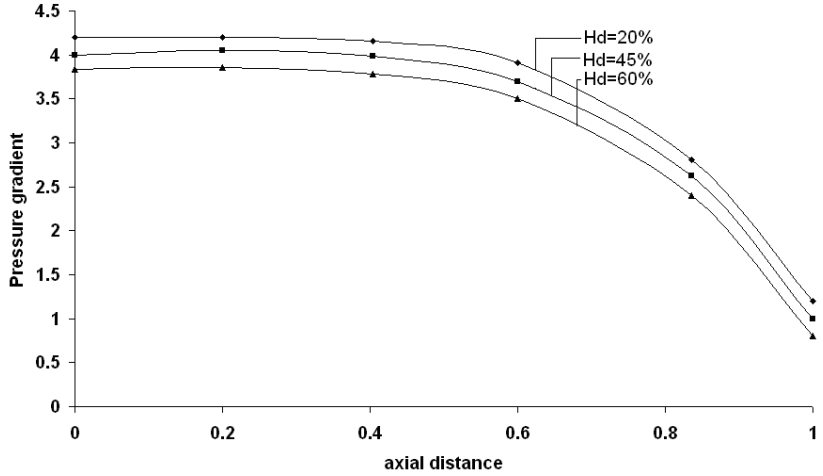

Fig. 2 Variation of dp/dz with axial distance for different values of discharge Hematocrit

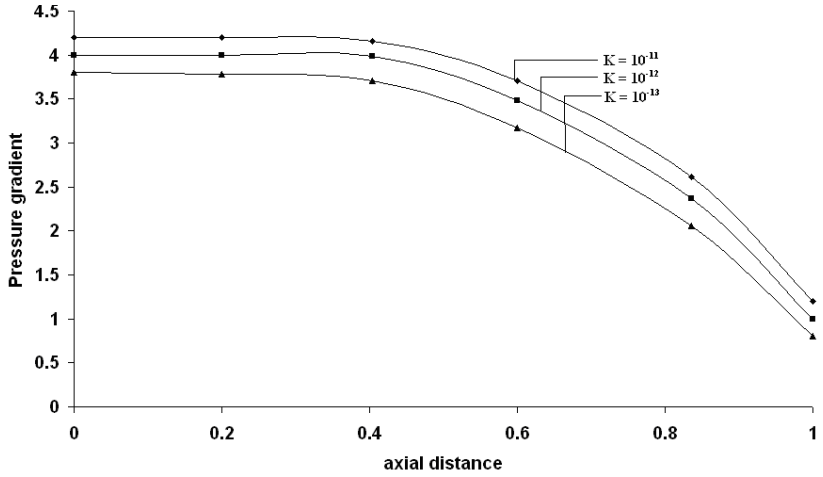

Fig.3 Variation of dp/dz with axial distance for different values of Permeability

Figure 2 and 3 depict that the pressure gradient decreases with axial distance within the capillary and with increasing values of the discharge hematocrit and permeability of the tissue. As the discharge hematocrit decreases core hematocrit increases which intern increases the viscosity of the blood therefore pressure gradient increases. We also know that if the permeability increases, more plasma will enter the tissue and the pressure gradient will increase due to the increase in viscosity of blood in capillary. The same result have been observed through graphs of figure 2 and 3.

Figures 4 and 5 present the variation of apparent viscosity with diameter of the capillary for different values of discharge hematocrit and permeability respectively. Figigure-4 shows the variation of apparent viscosity with the diameter of capillary

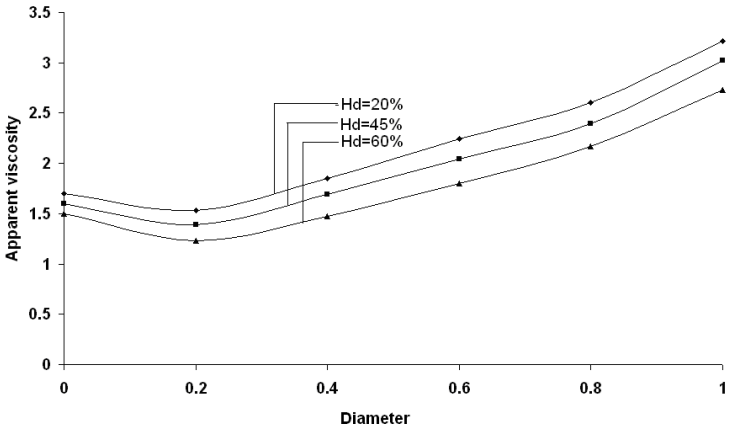

Fig. 4 Variation of Apparent viscosity with axial distance for different values of discharge hematocrit Hd

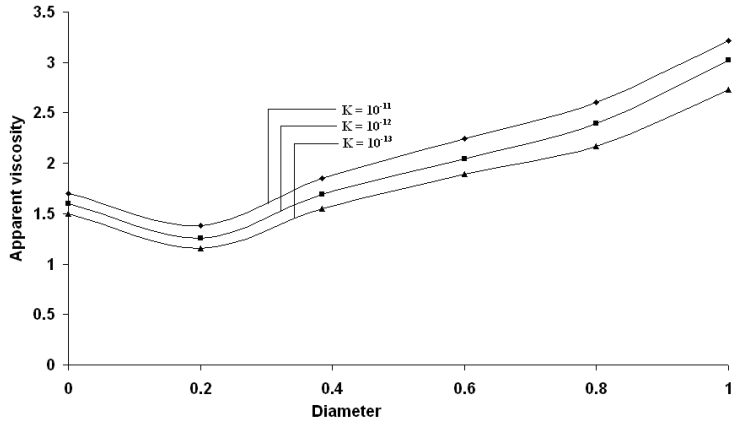

Fig. 5 Variation of Apparent viscosity with axial distance for different values of Permeability

for different discharge hematocrit. When discharge hematocrit increases, ratio of Hc/Hd decreases and apparent viscosity decreases near the arterial end and thereafter it increases with increasing values of the diameter of capillary. This may be due to the capillary tissue fluid exchange phenomena. As the diameter decreases more fluid enters the tissue due to the increase in intra-transmural pressure. Apparent viscosity of the blood in the capillary increases as the discharge hematocrit decreases. It causes an increase in the viscosity. Apparent viscosity decreases as diameter of the vessel decreases. Results are similar to the Fahraeus effect i.e. dependence of tube or vessel hematocrit on tube diameter. This reduction of apparent viscosity may be due to the flexibility of erythrocytes which makes them more prone to radial migration. Figure-4 shows the decrease in apparent viscosity with tube diameter as the permeability increases for the same reason.

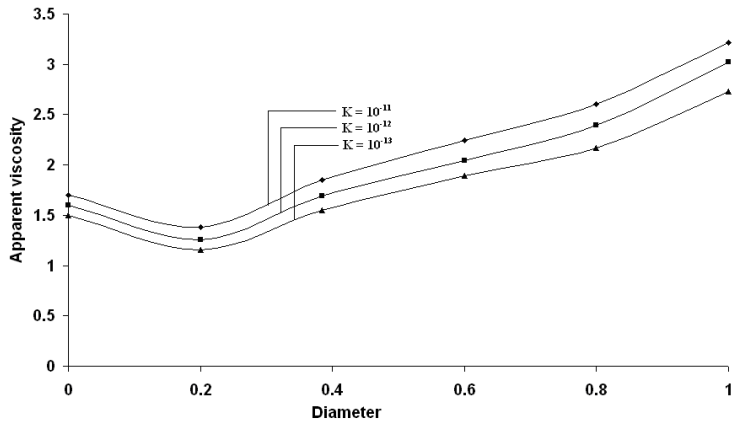

Fig. 5 Variation of Apparent viscosity with axial distance for different values of Permeability

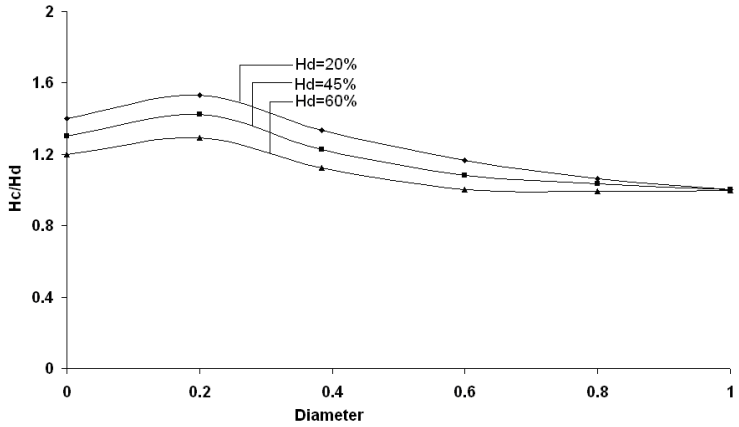

Fig.6 Variation of Hc/Hd with axial distance for different values of discharge Hematocrit

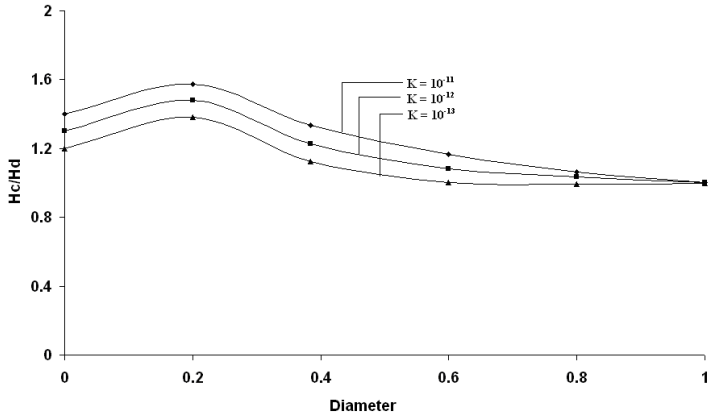

Fig. 7 Variation of Hc/Hd with axial distance for different values of Permeability

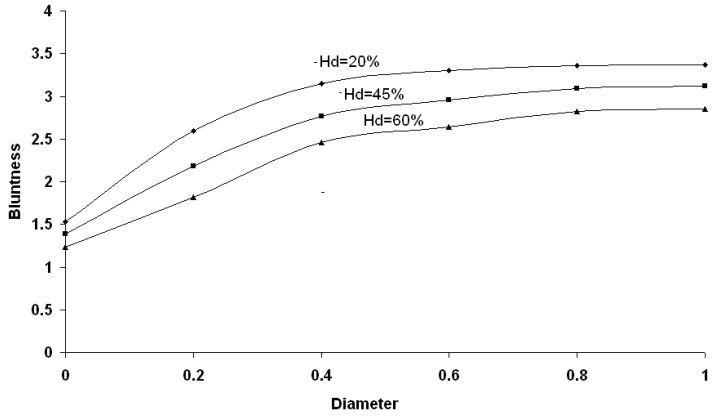

Fig. 8 Variation of Bluntness with axial distance for different values of discharge Hematocrit

Figures 6 & 7 present the variation of H_c/H_d with capillary diameter for different values of discharge hematocrit and tissue permeability respectively. As the discharge hematocrit increases, the ratio (H_c/H_d) decreases and the results of the figure-6 are in order. As permeability increases more fluid (i.e. base fluid or plasma) enters inside tissue which increases concentration of red blood cells in the capillary and the ratio (H_c/H_d) increases near the arterial end and decreases thereafter.

Figures 8 and 9 show that the bluntness increases as the tube diameter increases. B is the bluntness parameter. It represents the deviation from parabolic profile. If $B = 1$, velocity profile is parabolic and when $B = 0$ corresponds to total plug flow, Thus, B represents deviation from parabolic velocity profile. These figures shows that as diameter decreases Bluntness

parameter decreases therefore velocity profile become more blunt. It has already been observed that measurements of velocity in arterioles under $60\mu m$ internal diameter demonstrate blunted velocity profiles with some degree of asymmetry. The degree of bluntness increases in the smaller vessels [Ellsworth & Pittman [8], Nakano et al [12]]. The effects discharge hematocrit and Permeability also depict the anticipated results with increase in tube hematocrit.

4. Conclusion

As we described earlier that blood is not a homogeneous fluid: it is a suspension. As a result the viscosity is no longer a well defined property, but rather must be defined as a observed resistance to flow. Express in this way, the apparent viscosity of blood will depend upon the confining geometry. But in blood vessels like capillaries which are surrounded by porous tissue some fluid exchange occurs between the capillary and tissue. Therefore apparent viscosity total flux, resistance to flow, tube hematocrit does not remain constant throughout the capillary length. The same result has been observed in present model which is evident from the graphical results presented above. The result of the analysis has also some effect on Fahraeus effect and Fahraeus Lindquist effect. The reason behind this is porous boundary of the tube. We have considered a layered fluid approach for modeling blood rheology to account core annular microstructure as at high of red blood cell along the axis. However we may use non Newtonian fluid models as a more general approach rather than a layered fluid model and would be considered in the subsequent communication.

Acknowledge

Authors great fully acknowledge the financial assistance from the CSIR File No. 09/075 (0010)/2010, for this work.

References

[1] Bugliarello and J.Sevilla, *Velocity distribution and other characteristics of steady and pulsatile blood flow in fine glass tubes*, Biorheology, 7(1970), 85-107.

[2] S.E.Charm and G.S.Kurland, *Blood Rheology and Cardiovascular Fluid Dynamics (Ed. D.H. Bergel)*, Academic Press London, (1965).

[3] S.E.Charm and G.S.Kurland, *Blood Flow and Microcirculation*, Wiley, New York, (1974).

[4] N.A.Casson, *Flow equation of pigment oil suspension of printing ink type*, Rheology of Disperse System (Ed. C.C. Mill), (1959), 84-120.

[5] P.Chaturani and V.S.Upadhyay, *On micropolar fluid model for blood flow through narrow tubes*, Biorheology, 16(1979), 419-428.

[6] P.Chaturani and V.S.Upadhyay, *A two fluid model for blood flow through small diameter tubes with non zero couple stress boundary condition at interface*, Biorheology, 18(1981), 245-253.

[7] E.R.Damiano, *Blood flow in microvessels lined with a poroelastic wall layer*, Poromechanics, J. F. Thimus, Y. Abousleiman, A.H.D. Cheng, O. Coussy and E.Detournay, eds, Balkema, (1988), 403-408.

[8] M.L.Ellsworth and R.N.Pittman, *Evaluation of photometric methods for quantifying convective mass transport in microvessels*, Am. J. Physiol., 251(1986), H869-H879.

[9] E.C.Eringen, *Theory of micropolar fluids*, J. Math. Mech., 16(1966), 1-18.

[10] B.B.Gupta, K.M.Nigam and M.Y.Jaffrin, *A three-layer semi-empirical model for flow of blood and other particulate suspensions through narrow tubes*, J. Biomech. Eng., 104(1982), 129-135.

[11] H.R.Haynes, *Physical basis of the dependence of blood viscosity on tube radius*, Am. J. Physiol., 198(6)(1960), 1193-2000.

[12] A.Nakano, Y.Sugii, M.Minamiyama and H.Niimi, *Measurment of red cell velocity in microvessels using particle image velocimetry*, (PIV) Clin. Hemorheol. Microcirc., 29(2003), 445-55

[13] P.K.Nair, J.D.Hellums and J.S.Olson, *Prediction of Oxygen transport rates in blood flowing in large capillaries*, Microvasc. Res., 38(1989), 269-285.

[14] R.N.Pralhad and T.J.Schultz, *Two layered blood flow in stenosed tubes for different diseases*, Biorheology, 25(1988), 715-726.

[15] A.R.Pries, D.Neuhaus and P.Gaehtgens, *Blood viscosity in tube flow: dependence on diameter and hematocrit*, Am. J. Physiol., 263(1992), H1770-H1778

[16] A.R.Pries, T.W.Secomb and P.Gaehtgens, *Biophysical aspects of blood flow in the microvasculature*, Cardiovasc. Res., 32(1996), 654-667.

[17] A.R.Pries, T.W.Secomb, P.Gaehtgens and J.F.Gross, *Blood flow in microvascular networks: experiments and simulation*, Circ. Res. 67(1990), 826-834.

[18] A.R.Pries, T.W.Secomb, T.Gessner, M.B.Sperandio, J.F.Gross and P.Gaehtgens, *Resistance to blood flow in microvessels in vivo*, Circ. Res., 75(1994), 904-915.

[19] A.R.Pries and T.W.Secomb, *Resistance to blood flow in vivo: from Poiseuille to the in vivo viscosity law*, Biorheology, 34(1997), 369-373.

[20] M.Scott, *The modeling of blood rheology in small vessles*, A thesis submitted to University of waterloo Ontario, Canada, (2005).

[21] M.Sharan and A.S.Popel, *A two phase model for flow of blood in narrow tubes with increased effective viscosity near the wall*, Biorheology, 38(2001), 415-428.

[22] V.Sehadri and M.Y.Jaffrin, *Anamolous effects in blood flow through narrow tubes*, Inserm-Euromech, 97(71)(1977), 265-282.

[23] V.P.Srivastava, *Blood flow through stenosed vessels with a peripheral plasma layer and applications*, Automedica, 18(2000), 271-300.

[24] V.P.Srivastava, *Particulate suspension blood flow through stenotic arteries: effect of hematocrit and stenosis shape*, Indian J. Pure and Appl. Math., 33(2002), 1353-1360.

[25] V.P.Srivastava, *A theoretical model for blood flow in small vessels*, Application and Applied Mathematics, 2(1)(2007), 51-65.