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Quotient of Ideals of a Vague Lattice

Research Article

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Abstract: In this Paper we define the Ideal of a Vague Lattice and give some characterization of these Ideals in terms of operations on VS(L). Also we discuss the residual of ideals of a Vague Lattice and prove that the residual of ideals is again a Vague Ideal of the Vague Lattice. Moreover we establish that it is the largest ideal with respect to some property on the operations.

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1. Introduction

Most of our traditional tools for formal modeling, reasoning and computing are crisp and precise. But real life data are not always crisp, and all description can not be always expressed or measured precisely. To deal with such type of real life problems, Zadeh [25] in 1965 proposed a new mathematical model known as Fuzzy set Theory. The genuine necessity of such a new mathematical model stem from the fuzziness of natural phenomenon. Fuzzy sets have been applied in wide variety of fields like Computer Science, Medical Science, Management Science, Social Science, Engineering etc. to list a few only. Let U be a universe of discourse. A fuzzy set A is a class of objects of U along with a membership function μ_A . The grade of membership of u ($u \in U$) in the universe U is 1, but the grade membership of u in a fuzzy subset A (of U) is a real number $[0, 1]$ denoted by $\mu_A(u)$, which signifies that u is a member of the fuzzy set A up to certain extent, the degree of membership could be zero or more and atmost full (i.e., 1). The greater $\mu_A(u)$, the greater is the truth of the statement that " the element u belongs to set A". Different authors ([1, 7, 8, 11, 12, 18]) from time to time have made a number of generalizations Zadeh's fuzzy set theory ([25]). The concept of intuitionistic fuzzy sets was introduced by Atanassov [1, 2] as a generalization of that of fuzzy sets and it is very effective tool to study the case of Vagueness. Furhter many researchers applied this notion in various branches of mathematics especially in algebra and defined intuitionistic fuzzy subgroups, intuitionistic fuzzy subrings, and intuitionistic fuzzy sublattice, and so forth. The concept of ideal of a fuzzy subring was introduced by Mordeson and Malik in [19]. After that N.Ajmal and A.S.Prajapathi introduced the concept of residual of ideals of an L-Ring in [22]. Of these, the notion of Vague set theory introduced by Gau and Buehrer [10] is of interest to us. In most cases of judgements, evaluation is done by human beings (or by an intelligent agent) where there certainly is a limitation of knowledge or intellectual functionaries. Naturallly, every decision -maker hesitates more or less, on every evaluation activity. To judge whether a patient has cancer or not, a doctor (the decision-maker) will hesitate because of

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the fact that a fraction of evaluation he thinks in favour of truthness, another fraction in favour of falseness and rest part remains undecided to him. This is the breaking Philosophy in the notion of vague set theory introduced by Gau and Buehrer [10]. Motivated by this, In this paper we first defined the Vague Ideal of a Vague Lattice and certain characterizations are given. Lastly we defined quotient (residuals) of ideals of a Vague sublattice and studied their Properties.

2. Preliminaries

Definition 2.1 ([10]). *A Vague set (or in Short VS) A in the universe of discourse U is characterized by two membership functions given by: 1) a truth membership function $t_A : U \to [0,1]$, 2) a false membership function $f_A : U \to [0,1]$. Where $t_A(u)$ is a lower bound of the grade of membership of u derived from the evidence for u and $f_A(u)$ is lower bound on the negation of u derived from the evidence against u, and $t_A(u) + f_A(u) \leq 1$. Thus the grade of membership of u in the Vague set A is bounded by a subinterval $[t_A(u), 1 - f_A(u)]$ of $[0,1]$. This indicates that if the actual grade of membership is $\mu(u)$, then $t_A(u) \leq \mu(u) \leq 1 - f_A(u)$. The Vague set A is written as $A = \{< u, [t_A(u), 1 - f_A(u)]/u \in U >\}$ where the interval $[t_A(u), 1 - f_A(u)]$ is called the vague value of u in A and is denoted by $V_A(u)$.*

Definition 2.2 ([10]). *Zero Vague set and Unit Vague set: A vague set A of a set U with $t_A(u) = 0$ and $f_A(u) = 1\ \forall\ u \in U$ is called the zero vague set of U. A vague set A of a set U with $t_A(u) = 1$ and $f_A(u) = 0\ \forall\ u \in U$ is called the unit vague set of U.*

Definition 2.3 ([10]). *A vague set A of a set U with $t_A(u) = \alpha$ and $f_A(u) = 1 - \alpha\ \ \forall\ u \in U$ is called the α-vague set of U, where $\alpha \in [0,1]$.*

Definition 2.4 ([10]). *Let A and B be two vague sets of the universe U. Then*

(1) $A = B$ if $V_A(u) = V_B(u)$,

(2) $A \subset B$ if $V_A(u) \leq V_B(u)$,

(3) $C = A \cup B$ if $V_C(u) = \max\{V_A(u), V_B(u)\}$,

(4) $C = A \cap B$ if $V_C(u) = \min\{V_A(u), V_B(u)\}$.

Definition 2.5 ([25]). *Let $(X, \leq)$ be a Poset, if $\forall\ \ a, b \in S \Rightarrow a \vee b,\ a \wedge b \in X$. Then $(X, \leq)$ or $(X, \vee, \wedge)$ is called a Lattice where $a \vee b = \vee\{a, b\} = \sup\{a, b\}$, $a \wedge b = \wedge\{a, b\} = \inf\{a, b\}$.*

Definition 2.6 ([25]). *Let $(X, \vee, \wedge)$ be a Lattice, if it satisfied following distributivity Laws, then it is called a distributive Lattice*

(i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

(ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$, $\forall\ \ a, b, c \in L$.

Definition 2.7 ([25]). *A Fuzzy subset of L is called a Fuzzy Sublattice of L if*

(i) $\mu(x \vee y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(x \wedge y) \geq \min\{\mu(x), \mu(y)\}\quad \forall\ \ x, y \in L$.

Definition 2.8 ([25]). *A Fuzzy subset of L is called a Fuzzy Sublattice of L if*

(i) $(x \vee y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $(x \wedge y) \geq \max\{\mu(x), \mu(y)\}\quad \forall\ \ x, y \in L$.

3. Ideal of a Vague Lattice

Definition 3.1. *Let A be a Vague Lattice of L and B a Vague set of L with $B \subseteq A$. Then B is called a Vague Ideal (VI) of A if the following conditions are satisfied.*

(i) $V_B(x \vee y) \geq V_B(x) \wedge V_B(y)$.

(ii) $V_A(x \wedge y) \geq [V_A(x) \wedge V_B(y)] \vee [V_B(x) \wedge V_A(y)]\ \ \forall\, x, y \in L$ *where* $V_A = [t_A, 1 - f_A]$.

If B is a Vague Ideal of A, then we write $B \lhd A$.

Example 3.2. *Consider the Lattice* $L = \{1, 2, 5, 10\}$ *under divisibility. Let* $A = \{< x, [t_A(x), 1 - f_A(x)]/x \in L\}$ *be a Vague Lattice of L defined by* $< 1, [.5, .9] >, < 2, [.4, .5] >, < 5, [.4, .7] >, < 10, [.7, .7] >$ *and* $B = \{< x, [t_B(x), 1 - f_B(x)]/x \in L\}$ *be a Vague set of L given by* $< 1, [.5, .7] >, < 2, [.4, .5] >, < 5, [.3, .6] >, < 10, [.3, .6] >$. *Clearly* $B \lhd A$.

Definition 3.3. *Let A be a Vague Lattice and B is also a Vague Lattice with $B \subseteq A$. Then B is called a Vague fuzzy sublattice of A.*

Lemma 3.4. *The intersection of two Vague Ideals of A is again a Vague Ideal of A.*

Proof. Let B,C be Vague Ideals of A. Then we can prove that BC is also a Vague Ideal of A. Since $B \subseteq A$ and $C \subseteq A$, we have $B \cap C \subseteq A$ Also $V_{B\cap C}(x \vee y) = \min\{V_B(x \vee y), V_C(x \vee y)\} \geq \min\{V_B(x) \wedge V_B(y), V_C(x) \wedge V_C(y)\} \geq \min\{V_B(x) \wedge V_C(x), V_B(y) \wedge V_C(y)\} \geq \min\{V_{B\cap C}(x), V_{B\cap C}(y)\} \geq V_{B\cap C}(x) \wedge V_{B\cap C}(y)$. And $V_{B\cap C}(x \wedge y) = \min\{V_B(x \wedge y), V_C(x \wedge y)\} \geq \min\{[V_B(x) \wedge V_A(y) \vee V_B(y) \wedge V_A(x)], [V_C(x) \wedge V_A(y) \vee V_A(x) \wedge V_C(y)]\} \geq \min\{V_B(x), V_C(x), V_B(y)\} \wedge V_A(y) \vee \min\{V_B(y), V_C(y)\} \wedge V_A(x) \geq [V_{B\cap C}(x) \wedge V_A(y)] \vee [V_{B\cap C}(y) \wedge V_A(x)]$. Hence $B \cap C$ is a Vague Ideal of A. □

Theorem 3.5. *Let A be a Vague Lattice and B a Vague set of L with $B \subseteq A$. Then B is a Vague Ideal of A if and only if*

(1). $V_B(x \vee y) \geq V_B(x) \wedge V_B(y)$.

(2). $AB \subseteq B$

Proof. Suppose the Conditions (1), (2) hold. Then we Prove that B is a Vague Ideal of A. We have $V_B(x \wedge y) \geq V_{AB}(x \wedge y) = \sup_{x\wedge y = x_i \wedge y_i} V_A(x_i) \wedge V_B(y_i) \geq V_A(x) \wedge V_B(y)$. Similarly $V_B(x \wedge y) \geq V_B(x) \wedge V_A(y)$. Hence $V_B(x \wedge y) \geq [V_A(x) \wedge V_B(y)] \vee [V_B(x) \wedge V_A(y)]$. Hence B is a Vague Ideal of A. Conversely Suppose that B is a Vague Ideal of A. Then obviously Conditions (1) and (2) holds. Also we have $V_B(x \wedge y) \geq V_A(x) \wedge V_B(y)\ \ \forall\, x, y \in L$. So $\forall\ z \in L$ with $z = x \wedge y$. $V_B(z) \geq \bigvee_{z=x\wedge y}[V_A(x) \wedge V_B(y)] = V_{AB}(z)$. Hence $AB \subseteq B$. □

Theorem 3.6. *Let A be a Vague Lattice of L and B a Vague set with $B \subseteq A$. Then B is a Vague Ideal of A if and only if*

(1). $V_B(x \vee y) \geq V_B(x) \wedge V_B(y)$.

(2). $A.B \subseteq B$.

Proof. Suppose conditions (1), (2) holds. We prove B is a Vague Ideal of A. We have $V_B(x \wedge y) \geq V_{A.B}(x \wedge y) = \sup\limits_{x\wedge y = \bigvee\limits_{i=1}^{n}(x_i \wedge y_i)} [\bigwedge\limits_{i=1}^{n}(V_A(x_i) \wedge V_B(y_i))] \geq V_A(x) \wedge V_B(y)$. Similarly we can prove $V_B(x \wedge y) \geq V_B(x) \wedge V_A(y)$. Hence $V_B(x \wedge y) \geq [V_A(x) \wedge V_B(y)] \vee [V_B(x) \wedge V_A(y)]$. Hence B is a Vague Ideal of A. Conversely suppose that B is a Vague Ideal of A. Then obviously conditions (1) holds. Let $z \in L$ and $z = \bigvee\limits_{i=1}^{n}(x_i \wedge y_i)$, where $x_i \in A$, $y_i \in B$. We have $V_B(z) = V_B[\bigvee\limits_{i=1}^{n}(x_i \wedge y_i)] \geq \bigwedge\limits_{i=1}^{n} V_B(x_i \wedge y_i) \geq \bigwedge\limits_{i=1}^{n}\{V_A(x_i) \wedge V_B(y_i)\} \geq \bigvee[\bigwedge\limits_{i=1}^{n}\{V_A(x_i) \wedge V_B(y_i)\}] = V_{A.B}(z)$. Hence $A \centerdot B \subseteq B$. □

Theorem 3.7. *Let A be a Vague Lattice of L and B, C are Vague Ideals of A. Then $B+C$ is a Vague Ideal of A.*

Proof. We have $V_{B+C}(x \vee y) \geq V_{B+C}(x) \wedge V_{B+C}(y)$. And $A(B+C) \subseteq AB+AC \subseteq B+C$, $(B+C)A = BA+CA \subseteq B+C$. Hence $B+C$ is a Vague Ideal of A. □

4. Quotient of Ideals Over Lattices

Definition 4.1. *Let A be a Vague Lattice of L and B, C are Vague Ideals of A. Then the quotient (residual) of B by C denoted as (B/C) is defined by, $(B/C) = \cup\{D/D \lhd A$ and $DC \subseteq B\}$.*

Theorem 4.2. *Let A be a Vague Lattice of L and B, C are Vague Ideals of A. Then the quotient B/C is a Vague Ideal of A. Also $B \subseteq B/C \subseteq A$.*

Proof. Let $\eta = \{D/D \lhd A$ and $DC \subseteq B\}$. Suppose $D, D' \in \eta$. Then D and D' are Vague Ideals of A such that $DC \subseteq B$ and $D'C \subseteq B$. Then $D + D'$ is a Vague Ideal of A. So $(D + D')C \subseteq DC + D'C \subseteq B + B = B$. Thus $D + D' \in \eta$. Now $V_{B/C}(x) \wedge V_{B/C}(y) = \wedge[\bigvee_{D\in\eta} V_D(x)] \wedge [\bigvee_{D'\in\eta} V_{D'}(y)] = \vee\{V_D(x) \wedge V_{D'}(y)/D, D' \in \eta\} \leq \vee\{\bigvee_{D+D'} (x \vee y)/D, D' \in \eta\} \leq V_{B/C}(x \vee y)$, since $D + D' \in \eta$. That is $V_{B/C}(x \vee y) \geq V_{B/C}(X) \wedge V_{B/C}(y)$. Also $V_{B/C}(x \wedge y) = \bigvee_{D\in\eta} V_D(x \wedge y) \geq \bigvee_{D\in\eta}\{V_D(x) \wedge V_A(y)\}$, since $D \lhd A = [\bigvee_{D\in\eta} V_D(x)] \wedge V_A(y) = V_{B/C}(x) \wedge V_A(y)$. Similarly $V_{B/C}(x \wedge y) \geq V_{B/C}(y) \wedge V_A(x)$. Thus $V_{B/C}(x \wedge y)[V_{B/C}(x) \wedge V_A(y)] \vee [V_{B/C}(y) \wedge V_A(x)]$, B/C is a Vague Ideal of A. Clearly $B/C \subseteq A$. Since B is a Vague Ideal of A, $BA \subseteq B$. Since $C \subseteq A$, $BC \subseteq BA \subseteq B$. Hence $B \in \eta$. So $B \subseteq B/C$. Thus we have $B \subseteq B/C \subseteq A$. □

Theorem 4.3. *Let A be a Vague Lattice and B, C are Vague Ideals of A. Then B/C is the largest Vague Ideal of A with the Property $(B/C)\centerdot C \subseteq B$.*

Proof. Let $\eta = \{D/D \lhd A$ and $DC \subseteq B\}$. We have $B/C = \bigcup_{D\in\eta} D$. Let $x \in L$ such that $x = \bigvee_{i=1}^{n} a_i \wedge b_i$ Then $V_B(a_i \wedge b_i) \geq V_{DC}(a_i \wedge b_i) \geq V_D(a_i) \wedge V_C(b_i), \forall\ D \in \eta$. So $V_B(a_i \wedge b_i) \geq \bigvee_{D\in\eta}[V_D(a_i) \wedge V_C(b_i)] = \bigvee_{D\in\eta}[V_D(a_i) \wedge V_C(b_i)] = V_{B/C}(a_i) \wedge V_C(b_i)$. Hence $V_B(x) \geq \bigwedge\limits_{i=1}^{n} [V_{B/C}(a_i) \wedge V_C(b_i)] \geq \bigwedge\limits_{i=1}^{n} V_B(a_i \wedge b_i) \geq \bigwedge\limits_{i=1}^{n} [V_{B/C}(a_i) \wedge V_C(b_i)]$. Consequently, $V_B(x) \geq \bigvee\left\{\bigwedge\limits_{i=1}^{n} [V_{B/C}(a_i) \wedge V_C(b_i)]/x = \bigvee_{i=1}^{n} ((a_i \wedge b_i))\right\} = V_{B/C}\centerdot C(x)$. Thus $(B/C)\centerdot C \subseteq B$. If D is a Ideal of A such that $D\centerdot C \subseteq B$ then $DC \subseteq D\centerdot C \subseteq B$. So $D \in \eta$. Hence $D \subseteq B/C$. Thus B/C is the largest Vague Ideal of A such that $(B/C)\centerdot C \subseteq B$. □

Theorem 4.4. *Let A be a Vague Lattice and B, C, D are Vague Ideals of A. Then the following holds.*

(1). If $B \subseteq C$ then $B/D \subseteq C/D$ and $D/C \subseteq D/B$.

(2). If $B \subseteq C$ then $C/B = A$

(3). $B/B = A$.

Proof.

(1). Let $B \subseteq C$. Write $\eta = \{E/E \lhd A$ and $ED \subseteq B\}$ and $\xi = \{E/E \lhd A$ and $ED \subseteq C\}$. If $E \in \eta$ then $E \lhd A$ and $ED \subseteq B \subseteq C$. Thus $E \in \xi$ and hence $\eta \subseteq \xi$. So $B/D = \bigcup_{E\in\eta} E \subseteq \bigcup_{E\in\xi} E = C/D$. Similarly, Let $\eta_1 = \{E/E \lhd A$ and $EC \subseteq D\}$ and $\xi_1 = \{E/E \lhd A$ and $EB \subseteq D\}$. If $E \in \eta_1$ then $EC \subseteq D$. But $B \subseteq C$. So $EB \subseteq EC \subseteq D$. Thus $E \in \eta_1$ and hence $\eta_1 \subseteq \xi_1$. So $D/C = \bigcup_{E\in\eta_1} E \subseteq \bigcup_{E\in\xi_1} E = D/B$.

(2). Let $\eta = \{E/E \lhd A$ and $EB \subseteq C\}$. Since $B \lhd A$, we have $AB \subseteq B \subseteq C$, and $A \lhd A$. Thus $A \in \eta$ and hence $A \subseteq \bigcup_{E\in\eta} E = C/B \subseteq A$, since C/B is a Vague ideal of A. Therefore $C/B = A$.

(3). We have $B \subseteq B$. So from (2) $B/B = A$.

$\square$

Corollary 4.5.

(1). $(B/C)/B = A$

(2). $(B/B)/C = A$,

(3). $B/(B \cap C) = A$.

Proof.

(1). Since $B \subseteq B/C$, then $(B/C)/B = A$.

(2). Since $B/B = A$ and $C \subseteq A = B/B$. Therefore $(B/B)C = A$.

(3). Since $B \lhd A$ and $C \lhd A$. So $B \cap C \lhd A$ and $B \cap C \subseteq B$. Hence $B/(B \cap C) = A$.

$\square$

Theorem 4.6. *Let A be a Vague Lattice of L and B_i, $i = 1, 2, \ldots m$, and C be Vague Ideals of A. Then $(\bigcap_{i=1}^{m} B_i)/C = \bigcap_{i=1}^{m} (B_i/C)$.*

Proof. Since $\bigcap_{i=1}^{m} B_i \subseteq B_i$, $(\bigcap_{i=1}^{m} B_i)/C \subseteq B_i/C, \forall\, i$. Hence $(\bigcap_{i=1}^{m} B_i)/C \subseteq \bigcap_{i=1}^{m}(B_i/C)$. Let $\eta_1 = \{E/E \lhd A \text{ and } EC \subseteq B_1\}$, $\eta_2 = \{E/E \lhd A \text{ and } EC \subseteq B_2\}$ and $\eta_3 = \{E/E \lhd A \text{ and } EC \subseteq B_1 \cap B_2\}$. Then $\forall\, x \in L$, $V_{B_1/C \cap B_2/C}(x) = V_{B_1/C}(x) \wedge V_{B_2/C}(x) = (\bigvee_{E \in \eta_1} \mu_E(x)) \wedge (\bigvee_{E' \in \eta_2} \mu_{E'}(x)) = \vee\{[\mu_E(x) \wedge \mu_E(x)/E \in \eta_1,\ E' \in \eta_2]\}$. Now let $E \in \eta_1$ and $E' \in \eta_2$. Then $EC \subseteq B_1$ and $E'C \subseteq B_2$. Also $E \cap E'$ be a Vague Ideal of A. So that $(E \cap E')C \subseteq EC \cap E'C \subseteq B_1 \cap B_2$. Thus $E \cap E' \in \eta_3$. So $\eta_1 \cap \eta_2 \subseteq \eta_3$. Hence $(B_1 \cap B_2)/C = \bigcup_{E \in \eta_3} \supseteq \bigcup_{E \in \eta_1, E' \in \eta_2} (E \cap E')$. So $V_{B_1 \cap B_2/C}(x) \geq \bigvee V_{E \cap E'}(x) = \bigvee [V_E(x) \wedge V_{E'}(x)] = V_{B_1/C \cap B_2/C}(x)$. Hence $(B_1 \cap B_2)/C \supseteq B_1/C \cap B_2/C$. Therefore $(B_1 \cap B_2)/C =_1 /C \cap B_2/C$. Hence Proved. $\square$

Next we denote the set of all Vague Ideals $\{B_i\}$ $i = 1, 2, \ldots, m$ of a Vague Lattice A that satisfies the Property $\mu_{B_i}(0) = \mu_{B_j}(0)$ and $\nu_{B_i}(0) = \nu_{B_j}(0)\ \ \forall\, i, j$ by $VI(A^*)$. Then we have the following results.

Lemma 4.7.

(1). $B \subseteq B + C$ *and* $C \subseteq B + C$.

(2). $B/C = B/B + C$.

(3). $B + C/B = A$ *and* $B + C/B \cap C = A$.

Proof.

(1). We have $V_{B+C}(x) = \bigvee_{x = y \vee z}(V_B(y) \wedge V_C(z)) \geq V_B(x) \wedge V_C(0) = V_B(x) \wedge V_B(0) = V_B(x)$ since $V_B(0) = V_C(0)$, $V_B(0) \geq V_B(x)$. So $B \subseteq B + C$. Similarly we can Prove $C \subseteq B + C$.

(2). We have $B + C \lhd A$ and $C \subseteq B + C$. So $B/B + C \subseteq B/C$. Write $\eta = \{E/E \lhd A \text{ and } EC \subseteq B\}$ and $\xi = \{E/E \lhd A \text{ and } E(B + C) \subseteq B\}$. Let $E \in \eta$, then $E \lhd A$ and $E \subseteq A$. So $EB \subseteq AB$. But $AB \subseteq B$, since $B \lhd A$. Hence $EB \subseteq B$ and also $EC \subseteq B$. So $E(B + C) \subseteq EB + EC \subseteq B + B = B$. Therefore $E \in \xi$. So $\eta \subseteq \xi$. Thus $B/C = \bigcup_{E \in \eta} \subseteq \bigcup_{E \in \xi} E = B/B + C$.

(3). We have $B + C \lhd A$ and $B \subseteq B + C$. So $B + C/B = A$. Also we have $B \cap C \lhd A$ and $B \cap C \subseteq B + C$. Hence $B + C/B \cap C = A$.

$\square$

Theorem 4.8. *Let A be a Vague Lattice of L and $\{B_i\}$, $i = 1, 2, \ldots, m \in VI(A^*)$ and C any Vague Ideal of A. Then* $\sum_{i=1}^{m} B_i = \bigcap_{i=1}^{m} \left(\frac{C}{B_i}\right)$.

Proof. We have $B_1 + B_2 \lhd A$ and $B_1 \subseteq B_1 + B_2$, $B_2 \subseteq B_1 + B_2$. So $C/B_1 + B_2 \subseteq C/B_1$ and $C/B_1 + B_2 \subseteq C/B_2$. Therefore $C/B_1 + B_2 \subseteq C/B_1$ and $C/B_1 + B_2 \subseteq C/B_2$. Therefore $C/B_1 + B_2 \subseteq C/B_1 \cap C/B_2$. Let $\eta_1 = \{E/E \lhd A \quad \text{and} \quad EB_1 \subseteq C\}$, $\eta_2 = \{E/E \lhd A \quad \text{and} \quad EB_2 \subseteq C\}$ and $\eta_3 = \{E/E \lhd A \quad \text{and} \quad E(B_1 + B_2)C\}$. Then $\forall\ x \in L \quad V_{C/B_1 \cap C/B_2}(x) = V_{C/B_1}(x) \wedge V_{C/B_2}(x) = (\bigcup_{E \in \eta_1} V_E(x)) \wedge (\bigcup_{E' \in \eta_2} V_{E'}(x)) = \bigvee\{[V_E(x) \wedge V_{E'}(x)]/E \in \eta_1,\ E' \in \eta_2\}$. Now let $E \in \eta_1$ and $E' \in \eta_2$. Then $EB_1 \subseteq C$ and $E'B_2 \subseteq C$. Also $E \cap E'$ Vague Ideal of A, So that $(E \cap E')(B_1 + B_2) \subseteq (E \cap E')B_1 + (E \cap E')B_2 \subseteq EB_1 + E'B_2 \subseteq C + C = C$. So $E \cap E' \in \eta_3$. Hence $\eta_1 \cap \eta_2 \subseteq \eta_3$. Thus $C/B_1 + B_2 = \bigcup_{E \in \eta_3} E \supseteq \bigcup_{E \in \eta_1, E' \in \eta_2} (E \cap E')$. So $V_{C/B_1 + B_2}(x) \geq \bigvee V_{E \cap E'}(x) = \vee[V_E(x) \wedge V_{E'}(x)] = V_{C/B_1 \cap C/B_2}(x)$. Therefore $C/B_1 + B_2 \supseteq C/B_1 \cap C/B_2$. Hence $C/B_1 + B_2 = C/B_1 \cap C/B_2$. $\square$

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