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Another Decomposition of Continuity via $g\mu$-closed Sets

Research Article

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Abstract: There are various types of generalization of continuous functions in the development of topology. Recently some decompositions of continuity are obtained by various authors with the help of generalized continuous functions in topological spaces. In this paper we obtain a decomposition of continuity using a generalized continuity called $g\mu$-continuity in topology.

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1. Introduction

Different types of generalizations of continuous function were introduced and studied by various authors in the recent development of topology. The decomposition of continuity is one of the many problems in general topology. Tong [10] introduced the notions of A-sets and A-continuity and established a decomposition of continuity. Also Tong [11] introduced the notions of B-sets and B-continuity and used them to obtain another decomposition of continuity and Ganster and Reilly [6] improved Tong's decomposition result. Recently, various decompositions of continuity have been established, for example [1–5, 9]. In this paper, we obtain a decomposition of continuity in topological spaces by using $g\mu$-continuity.

2. Preliminaries

Throughout this paper (X, τ) and (Y, σ) (or X and Y) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), cl(A) and int(A) denote the closure of A and the interior of A respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1. *A subset A of a space (X, τ) is called a α-open set [8] if A$\subseteq$ int(cl(int(A))).*

The complements of α-open set is said to be α-closed. The α-closure [8] of a subset A of X, denoted by αcl(A) is defined to be the intersection of all α-closed sets of (X, τ) containing A. It is known that αcl(A) is a α-closed set.

Definition 2.2. *A subset A of a space (X, τ) is called:*

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(1). a $g\alpha$-closed set [7, 12] if $\alpha cl(A) \subseteq int(U)$ whenever $A \subseteq U$ and U is α-open in (X, τ). The complement of $g\alpha$*-closed set is called $g\alpha$*-open.*

(2). a μ-closed set [13] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $g\alpha$-open in (X, τ). The complement of μ-closed set is called μ-open.*

(3). a generalized μ-closed (briefly $g\mu$-closed) set [14] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is μ-open in (X, τ). The complement of $g\mu$-closed set is called $g\mu$-open.

Definition 2.3 ([14]). *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $g\mu$-continuous if for each closed set V of Y, $f^{-1}(V)$ is $g\mu$-closed in X.*

Proposition 2.4 ([14]). *Every closed set is $g\mu$-closed but not conversely.*

Proposition 2.5 ([14]). *Every continuous function is $g\mu$-continuous but not conversely.*

3. Decomposition of Continuity

In this section, we obtain a decomposition of continuity in topological spaces by using $g\mu$-continuity. To obtain a decomposition of continuity, we first recall the notion of $g\mu$lc*-continuous function in topological spaces by using $g\mu$-continuity and prove that a function is continuous if and only if it is both $g\mu$-continuous and $g\mu$lc*-continuous. We introduce the following definition.

Definition 3.1. *A subset A of a space (X, τ) is called $g\mu$lc*-set if $A = M \cap N$, where M is μ-open and N is closed (X, τ).*

Example 3.2. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{c\}, X\}$. Then $\{a\}$ is $g\mu$lc*-set in (X, τ).*

Remark 3.3. *Every closed set is $g\mu$lc*-set but not conversely.*

Example 3.4. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$. Then $\{a, b\}$ is $g\mu$lc*-set but not closed in (X, τ).*

Remark 3.5. *$g\mu$-closed and $g\mu$lc*-sets are independent of each other.*

Example 3.6. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{a, b\}, X\}$. Then $\{a, c\}$ is an $g\mu$-closed set but not $g\mu$lc*-set in (X, τ).*

Example 3.7. *Let $X = \{a, b, c\}$ with $\tau = \{\phi, \{b\}, X\}$. Then $\{a, b\}$ is an $g\mu$lc*-set but not $g\mu$-closed set in (X, τ).*

Proposition 3.8. *Let (X, τ) be a topological space. Then a subset A of (X, τ) is closed if and only if it is both $g\mu$-closed and $g\mu$lc*-set.*

Proof Necessity is trivial. To prove the sufficiency, assume that A is both gμ-closed and gμlc*-set. Then $A = M \cap N$, where M is μ-open and N is closed in (X,τ). Therefore, $A \subseteq M$ and $A \subseteq N$ and so by hypothesis, $cl(A) \subseteq M$ and $cl(A) \subseteq N$. Thus $cl(A) \subseteq M \cap N = A$ and hence $cl(A) = A$ i.e., A is closed in (X, τ).

Definition 3.9. *A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $g\mu$lc*-continuous if for each closed set V of (Y, σ), $f^{-1}(V)$ is $g\mu$lc*-set in (X, τ).*

Example 3.10. *Let $X = Y = \{a, b, c\}$ with $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b, c\}, Y\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Then f is $g\mu$lc*-continuous function.*

Remark 3.11. *Every continuous function is $g\mu$lc*-continuous but not conversely.*

Example 3.12. *Let* $X = Y = \{a, b, c\}$ *with* $\tau = \{\phi, \{b\}, X\}$ *and* $\sigma = \{\phi, \{b\}, \{a, c\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is* $g\mu lc^*$*-continuous function. Since for the closed set* $\{b\}$ *in* (Y, σ), $f^{-1}(\{b\}) = \{b\}$, *which is not closed in* (X, τ), *f is not continuous.*

Remark 3.13. $g\mu$*-continuity and* $g\mu lc^*$*-continuity are independent of each other.*

Example 3.14. *Let* $X = Y = \{a, b, c\}$ *with* $\tau = \{\phi, \{a, b\}, X\}$ *and* $\sigma = \{\phi, \{a\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is* $g\mu$*-continuous but not* $g\mu lc^*$*-continuous.*

Example 3.15. *Let* $X = Y = \{a, b, c\}$ *with* $\tau = \{\phi, \{b\}, X\}$ *and* $\sigma = \{\phi, \{b, c\}, Y\}$. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be the identity function. Then f is* $g\mu lc^*$*-continuous but not* $g\mu$*-continuous.*

We have the following decomposition for continuity.

Theorem 3.16. *A function* $f : (X, \tau) \to (Y, \sigma)$ *is continuous if and only if it is both* $g\mu$*-continuous and* $g\mu lc^*$*-continuous.*

Proof Assume that f is continuous. Then by Proposition 2.5 and Remark 3.11, f is both gμ-continuous and gμlc*-continuous. Conversely, assume that f is both gμ-continuous and gμlc*-continuous. Let V be a closed subset of (Y, σ). Then f^{-1}(V) is both gμ-closed and gμlc*-set. By Proposition 3.8, f^{-1}(V) is a closed set in (X, τ) and so f is continuous.

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