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Nano Generalized Closed Sets in Nano Bitopological Space

Research Article

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Abstract: The basic objective of this paper is to introduce and investigate the properties of Nano (1,2)* generalized closed sets in Nano Bitopological Spaces which is the extension work of Nano closed sets introduced by K. Bhuvaneswari et.al [1] also further investigated the properties of Nano generalized closed set in Nano bi topological space.

Keywords: Nano (1,2)* closed sets , Nano (1,2)* Interior, Nano (1,2)* closure, Nano(1,2)* generalized closed sets.

1. Introduction

In 1970 Levine [6] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. The notion of nano topology was introduced by Lellis Thivagar [5]. K.Bhuvaneswari et.al [1] introduced and investigated the concept of nano generalized closed set in nano topological space. Kelly [4] introduced the concept of bitopological space in 1963. T.Fukutake [3] introduced the generalized closed sets in bitoplogical space in 1985. K.Bhuvaneswari et.al [2] have introduced the Nano bitopological space. In this paper some properties of Nano (1,2)* generalized closed sets in Nano bitopological spaces are studied.

2. Preliminaries

Definition 2.1 ([6]). *A subset A of a topological space (X,τ) is called a generalized closed set (briefly g-closed set) if $cl(A) \subseteq U$ Whenever $A \subseteq U$ and U is open in (X,τ).*

Definition 2.2 ([5]). *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$*

(1). The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and its is denoted by $L_R(X)$. That is, $L_R(x) = U_{X\in U}\{R(x) : R(x) \subseteq X\}$ where $R(X)$ denotes the equivalence class determined by X.

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(2). The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = U_{X\in U}\{R(x) : R(x) \cap X \neq \phi\}$.

(3). The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X which respect to R and it is denoted by $B_R(x)$. That is, $B_R(x) = U_R(X) - L_R(X)$.

Property 2.3 ([7]). *If (U, R) is an approximation space and $X, Y \subseteq U$, Then*

(1). $L_R(X) \subseteq X \subseteq U_R(X)$

(2). $L_R(\phi) = U_R(\phi)$ *and* $L_R(U) = U_R(U) = U$

(3). $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$

(4). $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$

(5). $L_R(X \cup Y) \subseteq L_R(X) \cup L_R(Y)$

(6). $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$

(7). $L_R(x) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$

(8). $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$

(9). $U_R U_R(X) = L_R U_R(X) = U_R(X)$

(10). $L_R L_R(X) = U_R L_R(X) = L_R(X)$

Definition 2.4 ([5]). *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(x)\}$ where $X \subseteq U$. Then by Property 2.3 $\tau_R(X)$ satisfies the following axioms:*

1. *U and ϕ belongs to R(X)*
2. *The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$*
3. *The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.*

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano-open sets.

Remark 2.5 ([5]). *If $\tau_R(X)$ is the Nano topology on U with respect to X, then the set $B = \{U, L_R(X), U_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.6 ([5]). *If $(U, \tau_R(X))$ is a Nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, Then the Nano interior of A is defined as the union of all Nano-open subsets of A and it is denoted by NInt(A). That is, NInt(A) is the largest Nano-open subset of A.*
The Nano closure of A is defined as the intersection of all Nano closed sets containing A and it is denoted by Ncl(A). That is, Ncl(A) is the smallest Nano closed set containing A.

Definition 2.7 ([1]). *Let $(U, \tau_R(X))$ is a Nano topological space. A subset A of $(U, \tau_R(X))$ is called Nano generalized closed set (briefly Ng-closed) if $Ncl(A) \subseteq V$ where $A \subseteq V$ and V is Nano open.*

Example 2.8. *Let $U = \{x, y, z\}$, $U/R = \{\{x\}, \{y, z\}\}$ and $X = \{x, z\}$. Then the Nano topology, $\tau_R(X) = \{U, \phi, \{x\}, \{y, z\}\}$. Nano closed sets are $\{\phi, U, \{y, z\}, \{x\}\}$. Let $V = \{y, z\}$ and $A = \{y\}$. Then $Ncl(A) = \{y, z\} \subseteq V$. That is A is said to be Nano generalized closed in $(U, \tau_R(X))$.*

Definition 2.9 ([3])**.** *A subset A of a bitopological space $(X, \tau_{1,2})$ is called (1,2)* generalized closed set if $\tau_{1,2}cl(A) \subseteq U$ whenever $A \subseteq U$ and U is (1,2)* open.*

Definition 2.10 ([2])**.** *Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = \bigcup\{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where $X \subseteq U$. Then it is satisfies the following axioms:*

(1). U and $\Phi \in \tau_{R_{1,2}}(X)$

(2). The union of the elements of any sub-collection of $\tau_{R_{1,2}}(X)$ is in $\tau_{R_{1,2}}(X)$

(3). The intersection of the elements of any finite sub collection of $\tau_{R_{1,2}}(X)$ is in $\tau_{R_{1,2}}(X)$

Then $\tau_{R_{1,2}}(X)$ is a topology on U called the Nano bitopology on U with respect to X. $(U, \tau_{R_{1,2}}(X))$ is called the Nano bitopological space. Elements of the Nano bitopology are known as nano (1,2) open sets in U. Elements of $[\tau_{R_{1,2}}(X)]^C$ are called Nano(1,2)* closed sets in $\tau_{R_{1,2}}(X)$.*

Definition 2.11 ([2])**.** *If $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to X where $X \subseteq U$ and if $A \subseteq U$, then*

(1). The nano (1,2) closure of A is defined as the intersection of all Nano (1,2)* closed sets containing A and it is denoted by $N\tau_{1,2}cl(A)$. $N\tau_{1,2}cl(A)$ is the smallest Nano (1,2)* closed set containing A.*

(2). The nano (1,2) interior of A is defined as the union of all Nano (1,2)* open subsets of A contained in A and it is denoted by $N\tau_{1,2}Int(A)$. $N\tau_{1,2}Int(A)$ is the largest Nano (1,2)* open subset of A.*

3. Nano (1,2)* Generalized Closed Set

Throughout this paper $(U, \tau_{R_{1,2}}(X))$ is a Nano bitopological space with respect to X where $X \subseteq U$, R is an equivalence Relation on U, U/R denotes the family of equivalence classes of U by R.

Definition 3.1. *Let $(U, \tau_{R_{1,2}}(X))$ be a Nano bitopological space. A subset A of $(U, \tau_{R_{1,2}}(X))$ is called Nano (1,2)* generalized closed set if $N\ \tau_{1,2}cl(A) \subseteq V$ where $A \subseteq V$ and V is Nano (1,2)* open*

Example 3.2. *Let $U = \{a, b, c, d\}$ with $U/R_1(X) = \{\{b\}, \{d\}, \{a, c\}\}$, $X_1 = \{b, d\}$, $\tau_{R_1} = \{U, \phi, \{b, d\}\}$, $U/R_2(X) = \{\{b\}, \{d\}, \{a, c\}\}$, $X_2 = \{a, c\}$, $\tau_{R_2} = \{U, \phi, \{a, c\}\}$. Then $\tau_{R_{1,2}}(X) = \{U, \phi, \{b, d\}, \{a, c\}\}$. Nano (1,2)* closed sets are $\{\phi, U, \{a, c\}, \{b, d\}\}$. Then Nano generalized open sets are, $\{U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, c\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}\}$. Nano generalized closed sets are $\{\phi, U, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{a, d\}, \{a, b\}, \{b, c\}, \{a, c\}, \{b, d\}, \{d\}, \{a\}, \{c\}, \{b\}\}$. Let $V = \{a, c\}$ and $A = \{a\}$. Then $N\tau_{R_{1,2}}cl(A) = \{a, c\} \subseteq V$. That is A is said to be Nano (1,2)* generalized closed in $(U, \tau_{R_{1,2}}(X))$.*

Theorem 3.3. *A subset A of $(U, \tau_{R_{1,2}}(X))$ is Nano (1,2)* generalized closed if $N\tau_{1,2}cl(A)$-A contains no nonempty Nano (1,2)* generalized closed set.*

Proof. Suppose if A is Nano (1,2)* generalized closed. Then $N\tau_{1,2}cl(A) \subseteq A$ where $A \subseteq V$ and V is Nano (1,2)* open. Let Y be a Nano (1,2)* closed subset of $N\tau_{1,2}cl(A) - A$. Then $A \subseteq Y^c$ and Y^c is Nano (1,2)* open. since A is Nano (1,2)* generalized closed, $N\tau_{1,2}cl(A) \subseteq Y^c$ (or) $Y \subseteq [N\tau_{1,2}cl(A)]^c$ That is, $Y \subseteq [N\tau_{1,2}cl(A)]$ and $Y \subseteq [N\tau_{1,2}cl(A)]^c$ implies that $Y \subseteq \phi$. So Y is empty. □

Theorem 3.4. *If A and B are Nano (1,2)*generalized closed , then A U B is Nano (1,2)* generalized closed.*

Proof. Let A and B are Nano (1,2)* generalized closed sets. Then $N\tau_{1,2}cl(A) \subseteq V$ where $A \subseteq V$ and V is Nano(1,2)* open and, $N\tau_{1,2}cl(A) \subseteq V$ where $B \subseteq V$ and V is Nano (1,2)* open. since A and B are subsets of V, $(A \cup B)$ is a subset of V and V is Nano (1,2)*open. Then $N\tau_{1,2}cl(A \cup B) = N\tau_{1,2}cl(A) \cup N\tau_{1,2}cl(B) \subseteq V$. which implies that $(A \cup B)$ is Nano (1,2)* generalized closed. □

Remark 3.5. *The Intersection of two Nano (1,2)* generalized closed sets is again an Nano (1,2)* generalized closed set which is shown in the following example.*

Example 3.6. *Let* $U = \{a, b, c, d\}$ *with* $U/R_1(X) = \{\{b\}, \{d\}, \{a, c\}\}$, $X_1 = \{b, d\}$, $\tau_{R_1} = \{U, \phi, \{b, d\}\}$, $U/R_2(X) = \{\{b\}, \{d\}, \{a, c\}\}$, $X_2 = \{a, c\}$, $\tau_{R_2} = \{U, \phi, \{a, c\}\}$. *Then* $\tau_{R_{1,2}}(X) = \{U, \phi, \{b, d\}, \{a, c\}\}$. *Nano (1,2)* closed sets are* $\{\phi, U, \{a, c\}, \{b, d\}\}$. *Then Nano generalized sets are,* $\{U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, d\}, \{b, d\}, \{a, c\}, \{a, b, c\}, \{b, c, d\}, \{a, b, d\}, \{a, c, d\}\}$. *Nano generalized closed sets are* $\{\phi, U, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{a, b, c\}, \{c, d\}, \{a, d\}, \{a, b\}, \{b, c\}, \{a, c\}, \{b, d\}, \{d\}, \{a\}, \{c\}, \{b\}\}$, *let* $A = \{a\}$, $B = \{a, c\}$ *and* $A \cup B = \{a\}$. *Here* $N\tau_{1,2}cl(A \cap B) \subseteq V$. *When* $(A \cap B) \subseteq V$ *and V is Nano (1,2)*open.*

Theorem 3.7. *If A is Nano (1,2)* generalized closed and* $A \subseteq B \subseteq N\tau_{1,2}cl(A)$, *Then B is Nano (1,2)* generalized closed.*

Proof. Let $B \subseteq V$ where V is Nano (1,2)* open in $\tau_{R_{1,2}}(X)$. Then $A \subseteq B$ implies $A \subseteq V$. Since A is Nano (1,2)* generalized Closed. $N\tau_{1,2}cl(A) \subseteq V$. Also $B \subseteq N\tau_{1,2}cl(A)$ implies $N\tau_{1,2}cl(B) \subseteq N\tau_{1,2}cl(A)$. Thus $N\tau_{1,2}cl(A) \subseteq V$ and so B is Nano (1,2)* generalized closed. □

Definition 3.8. *Let A be Nano (1,2)* closed set. Then A is Nano (1,2)* generalized closed set.*

Proof. Let $A \subseteq V$ and V is Nano (1,2)* open in $\tau_{R_{1,2}}(X)$. since A is Nano (1,2)* closed $N\tau_{1,2}cl(A) \subseteq A$. That is $N\tau_{1,2}cl(A) \subseteq A \subseteq V$. Hence A is Nano (1,2)* generalized closed set. □

Theorem 3.9. *An Nano (1,2)* generalized closed set A is Nano (1,2)* closed if and only if* $N\tau_{1,2}cl(A) - A$ *is Nano (1,2)* closed.*

Proof. **Necessity:** Let A be Nano (1,2)* closed. Then $N\tau_{1,2}cl(A) = A$ and so, $N\tau_{1,2}cl(A) - A = \phi$ which is Nano (1,2)* closed.

Sufficiency: Suppose, $N\tau_{1,2}cl(A) - A$ is Nano (1,2)* closed. Then $N\tau_{1,2}cl(A) - A = \phi$. Since A is Nano (1,2)* closed. That is, $N\tau_{1,2}cl(A) = A$ (or) A is Nano (1,2)* closed. □

Theorem 3.10. *Suppose that* $B \subseteq A \subseteq U$, *B is an Nano (1,2)* generalized closed set relative to A and that A is an Nano (1,2)* generalized closed subset of U. Then B is Nano (1,2)* generalized closed relative to U.*

Proof. Let $B \subseteq V$ and suppose that V is Nano (1,2)* open in U. Then $B \subseteq A \cap V$. Therefore $N\tau_{1,2}clA(B) \subseteq A \cap V$. It follows that, $A \cap N\tau_{1,2}cl(B) \subseteq A \cap V$ and $A \subseteq V \cup N\tau_{1,2}cl(B)$. since A is Nano (1,2)* generalized closed in U. we have $N\tau_{1,2}cl(A) \subseteq V \cup N\tau_{1,2}cl(B)$. Therefore $N\tau_{1,2}cl(B) \subseteq N\tau_{1,2}cl(A) \subseteq V \cup N\tau_{1,2}cl(B)$ and so, $N\tau_{1,2}cl(B) \subseteq V$. Then B is N(1,2)* generalized closed relative to V. □

Corollary 3.11. *Let A be a Nano (1,2)* generalized closed set and suppose that F is a Nano (1,2)* closed set. Then* $A \cap F$ *is an Nano (1,2)* generalized closed set which is given in the following example.*

Example 3.12. *Let* $U = \{a,b,c,d\}$ *with* $U/R_1(X) = \{\{b\},\{d\},\{a,c\}\}$, $X_1 = \{b,d\}$, $\tau_{R_1} = \{U,\phi,\{b,d\}\}$, $U/R_2(X) = \{\{b\},\{d\},\{a,c\}\}$, $X_2 = \{a,c\}$, $\tau_{R_2} = \{U,\phi,\{a,c\}\}$. *Then* $\tau_{R_{1,2}}(X) = \{U,\phi,\{b,d\},\{a,c\}\}$. *Nano (1,2)* closed sets are* $\{\phi,U,\{a,c\},\{b,d\}\}$. *Then Nano generalized open sets are,* $\{U,\phi,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{b,c\},\{c,d\},\{a,d\},\{b,d\},\{a,c\},\{a,b,c\},\{b,c,d\},\{a,b,d\},\{a,c,d\}\}$, *Nano generalized closed sets are,* $\{\phi,U,\{b,c,d\},\{a,c,d\},\{a,b,d\},\{a,b,c\},\{c,d\},\{a,d\},\{a,b\},\{b,c\},\{a,c\},\{b,d\},\{d\},\{a\},\{c\},\{b\}\}$, *let* $A = \{a,b,d\}$ *and* $F = \{a,c,d\}$. *Then* $A \cap F = \{a,d\}$ *is an Nano (1,2)* generalized closed set.*

Theorem 3.13. *For each* $a \in U$, *either* $\{a\}$ *is Nano(1,2)* closed (or)* $\{a\}^c$ *is Nano (1,2)* generalized closed in* $\tau_{R_{1,2}}(x)$.

Proof. Suppose $\{a\}$ is not Nano (1,2)* closed in U. Then $\{a\}^c$ is not Nano (1,2)* open and the only Nano (1,2)* open set containing $\{a\}^c$ is $V \subseteq U$. That is $\{a\}^c \subseteq U$. Therefore $N\tau_{1,2}cl\{a\}^c \subseteq U$ which implies $\{a\}^c$ is Nano (1,2)* generalized closed set in $\tau_{R_{1,2}}(X)$. □

References

[1] K. Bhuvaneswari and K.Mythili Gnanapriya, *Nano Generalized Closed sets in Nano topological spaces*, International Journal of Scientific and Research Publications, 4(5)(2014).

[2] K.Bhuvaneswari and J.Sheeba priyadharshini, *On Nano semi generalized closed sets in Nano bitoplogical space.*

[3] T.Fukutake, *On generalized closed sets in bitoplogical space*, Bulletin of Fukuoka university of Education, 35(1985), 19-28.

[4] J.C.Kelly, *Bitopological spaces*, Proc. London Math. Soc., 3(13)(1963).

[5] M.Lellis Thivagar and Carmel Richard, *On Nano forms of Weakly open sets*, International Journal of Mathematics and Statistics Invention, 1(1)(2013), 31-37.

[6] N.Levine, *Generalized Closed sets in Topology*, Rend. Cire. Math. Palermo, 19(2)(1963), 89-96.

[7] I.L.Reilly and Vamanamurthy, *On α-sets in Topological spaces*, Tamkang J.Math, 16(1985), 7-11.

[8] M.K.R.S.VeeraKumar, *Between Closed sets and g-closed Sets*, Mem.Fac.Sci. Kochin University (Math)., 21(2000), 1-19.