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Two Methods to Approach Collatz Conjecture

Research Article

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Abstract: Two methods are explained in this article which may lead to a positive solution to Collatz conjecture, if these methods stop at finite stage.

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Keywords: Asymptotic density, stopping time.

1. Introduction

The Collatz conjecture is a well known open problem. This is also quoted in the literature as the $3x+1$ problem, Ulams conjecture and Hasse's algorithm. The conjecture is described by a function $T : N \to N$ defined by

$$T(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even;} \\ \frac{3n+1}{2}, & \text{if } n \text{ is odd.} \end{cases}$$

The Collatz conjecture asserts that repeated iteration at $T(n)$, starting from any positive integer n, eventually reaches the value 1. The stopping time of n is the least positive integer k, such that $T^k(n) < n$. The total stopping time is the least positive integer k such that$T^k(n) = 1$. Using this term Collatz conjecture says " Every integer $n \geq 2$ has a finite total stopping time", and the Collatz conjecture would be false, if $T^k(n)$ is either periodic (or) $\lim_{k\to\infty} T^k(n) = \infty$, for some natural number n. That is the folowing result is true.

Result 1.1. *The Collatz conjecture is true if and only if* $\{x \in N - \{1\}; T^k(x) < x, \text{ for some } k \in N\} = N - \{1\}$. *The proof easily follows by induction.*

This result gives a motivation to find integers x satisfying $T^k(x) < x$, for some k, which may be considered as favourable integers which are favourable to solve the conjecture positively. It is assumed hereafter in this article that the function $T : N \to N$ refers to the following function:

$$T(x) = \begin{cases} \frac{x}{2}, & \text{if } x \text{ is even;} \\ 3x+1, & \text{if } x \text{ is odd.} \end{cases}$$

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Let x be a positive even integer and $x = 2n$, for some $n \in N$. Then $T(x) = 2n/2 = n < x$. Hence $T(x) < x$ is true for all even positive integers. If $T^k(x) < x$, for some $k(x) = k$, for all positive integers $x \geq 3$, then the Collatz conjecture is true for all the elements in N. Consider the iterations performed by T on an integer $x \geq 2$. By an iteration, let us mean applying T once. So, successive iterations performed by T on x are considered. Let p denote smallest number of iterations performed on odd numbers and q denote smallest number of iterations performed on even numbers, until $\frac{3^p}{2^q} < 1$. Note that $\frac{3^p}{2^q} < 1$ if and only if $\frac{p}{q} < \frac{log2}{log3} \approxeq 0.6309.....$ Hence if T performs p number of iterations performed on odd integers and q number of iterations performed on even integers until $\frac{3^p}{2^q} < 1$, then there is a chance to have the relation $T^{p+q}(x) < x$.

So if $\frac{p}{q} < \frac{log2}{log3}$ and $T^{p+q}(x) < x$, then Collatz iterations may be stopped with the assumption that the integer is a number favorable to Collatz conjecture. So one may find p and q corresponding to each x so that $\frac{p}{q} < \frac{log2}{log3}$ and $\frac{p}{q-1} \geq \frac{log2}{log3}$. This selection is a favourable one for number x, because $T^{p+q}(x) < x$ is satisfied which is verified (not proved) for many cases. Thus it is proposed to find p and q for different cases of numbers. The method of finding p and q such that $\frac{p}{q} < \frac{log2}{log3} \leq \frac{p}{q-1}$ is discussed in the next section. However, finding i and j satisfying $T^{i+j}(x) < x$ after performing i number of iterations on odd integers and j number of iterations on even integers are being helpful to understand $T^{i+j}(2^j n + x) < (2^j n + x)$. This is explained in section 3 of this article.

These computational procedures are followed with an expectation of getting a chance of solving Collatz conjecture, eventhough there are favourable theoretical results in literature (for example [2, 3, 5, 6, 9, 10]), which are not sufficient to solve Collatz conjecture. There are articles(see for example [1, 4, 7, 8]) which discusses particular cases for Collatz conjecture, and the present article is also of this type.

2. Finding p and q

Let $m(x)$ denote a positive integer multiple of x.

(1). Let $A_1 = \{x : x = m(2)\}$. Let $x \in A_1$ be arbitrary, then x is an even integer. Then $T(x) = \frac{x}{2} < x$. So $T(x) < x$, for all x in A_1. Here number of iterations performed on odd integers is $p = 0$, and number of iterations performed on even integers is $q = 1$, and $\frac{p}{q} = 0 < \frac{log2}{log3}$.

(2). Let $A_2 = \{x : x = 1 + m(2^2)\}$. Let $x \in A_2$ be arbitrary. Then $x = 1 + m(2^2)$, and x is an odd integer. Then $T(x) = 4 + m(2^2)$. $T^2(x) = 2 + m(2)$. $T^3(x) = 1 + m(1)$. Now number of iterations performed on odd integers is p =1, number of iterations performed on even integers is $q = 2$, and $\frac{p}{q} = \frac{1}{2} < \frac{log2}{log3}$ where as $\frac{p}{q-1} = 1 > \frac{log2}{log3}$. So the iteration procedure may be stopped. Moreover if $x \in A_2$, then x is an odd Integer. $T(x) = 3x + 1$. $T^2(x) = \frac{3x+1}{2}$ and $T^3(x) = \frac{3x+1}{4}$. For this x, $T^3(x) < x \Leftrightarrow \frac{3x+1}{4} < x \Leftrightarrow 3x + 1 < 4x \Leftrightarrow x > 1 \Leftrightarrow (x - 1) > 0$. Here $(x - 1) > 0$, for all $x \in A_2$. So $T^3(x) < x$, for all $x \in A_2$.

(3). Let $A_3 = \{x : x = 1 + 2 + 2^2 + 2^4 + m(2^5)\}$. Let $x \in A_3$ be arbitrary. Then $x = 1 + 2 + 2^2 + 2^4 + m(2^5)$, and x is an odd integer, $T(x) = 4 + 3.2 + 3.2^2 + 3.2^4 + m(3.2^5)$, $T^2(x) = 2 + 3 + 3.2 + 3.2^3 + m(3.2^4)$= $1 + 2^2 + 3.2 + 3.2^3 + m(3.2^4)$, $T^3(x) = 4 + 3.2^2 + 3^2.2 + 3^2.2^3 + m(3^2.2^4)$, $T^4(x) = 2 + 3.2 + 3^2 + 3^2.2^2 + m(3^2.2^3) = 1 + 2^4 + 3^2.2^2 + m(3^2.2^3)$, $T^5(x) = 4 + 3.2^4 + 3^3.2^2 + m(3^3.2^3)$, $T^6(x) = 2 + 3.2^3 + 3^3.2 + m(3^3.2^2)$, $T^7(x) = 1 + 3.2^2 + 3^3 + m(3^3.2) = 40 + m(3^3.2)$, and $T^8(x) = 20 + m(3^3.2)$.

Now number of iterations performed on odd integers is $p = 3$, number of iterations performed on even integers is $q = 5$.

Here $\frac{p}{q} = \frac{3}{5} < \frac{\log 2}{\log 3}$ where as $\frac{p}{q-1} = \frac{3}{4} > \frac{\log 2}{\log 3}$. So the iteration procedure may be stopped. Moreover, if $x \in A_3$, then x is an odd integer. $T(x) = 3x+1$. $T^2(x) = \frac{3x+1}{2}$. $T^3(x) = \frac{9x+5}{2}$. $T^4(x) = \frac{9x+5}{4}$. $T^5(x) = \frac{27x+19}{4}$. $T^6(x) = \frac{27x+19}{8}$. $T^7(x) = \frac{27x+19}{16}$ and $T^8(x) = \frac{27x+19}{32}$. If $T^8(x) < x \Leftrightarrow \frac{27x+19}{32} < x \Leftrightarrow 27x+19 < 32x \Leftrightarrow (5x-19) > 0 \Leftrightarrow 5x > 19 \Leftrightarrow x > \frac{19}{5}$. Here $x > \frac{19}{5}$, for all $x \in A_3$. So $T^8(x) < x$, for all $x \in A_3$.

(4). Let $A_4 = \{x : x = 1+2+2^2+m(2^7)\}$. Let $x \in A_4$ be arbitrary. $x = 1+2+2^2+m(2^7)$, x is an odd integer. $T(x) = 4+3.2+3.2^2+m(3.2^7)$. $T^2(x) = 1+2+2^3+m(3.2^6)$. $T^3(x) = 4+3.2+3.2^3+m(3^2.2^6)$. $T^4(x) = 1+2^4+m(3^2.2^5)$. $T^5(x) = 4+3.2^4+m(3^3.2^5)$. $T^6(x) = 2+3.2^3+m(3^3.2^3)$. $T^8(x) = 4+3^2.2+m(3^4.2^3)$. $T^9(x) = 2+3^2.2+m(3^4.2^2)$. $T^{10}(x) = 10+m(3^4.2)$. $T^{11}(x) = 5+m(3^4)$.

Now number of iterations performed on odd integers is $p = 4$. Number of iterations performed on even integers is $q = 7$. Here $\frac{p}{q} = \frac{4}{7} = 0.578 < \frac{\log 2}{\log 3}$, where as $\frac{p}{q-1} = \frac{4}{6} = \frac{2}{3} > \frac{\log 2}{log3}$. So the iteration procedure may be stopped. Moreover, if $x \in A_5$, then x is an odd integer. $T(x) = 3x+1$. $T^2(x) = \frac{3x+1}{2}$. $T^3(x) = 3[\frac{3x+1}{2}]+1 = \frac{9x+5}{2}$. $T^4(x) = \frac{9x+5}{4}$. $T^5(x) = \frac{27x+19}{4}$. $T^6(x) = \frac{27x+19}{8}$. $T^7(x) = \frac{27x+19}{16}$. $T^8(x) = \frac{81x+73}{16}$. $T^9(x) = \frac{81x+73}{32}$. $T^{10}(x) = \frac{81x+73}{64}$. $T^{11}(x) = \frac{81x+73}{128}$. For this x, $T^{11}(x) = \frac{81x+73}{128} < x \Leftrightarrow 81x+73 < 128x \Leftrightarrow 47x-73 > 0 \Leftrightarrow 47x > 73 \Leftrightarrow x > \frac{73}{47}$. Here $x > \frac{73}{47}$ for all $x \in A_4$. So $T^{11}(x) < x$, for all $x \in A_4$.

(5). Let $A_5 = \{x : x = 1+2+2^3+m(2^6)\}$. Let $x \in A_5$ be arbitrary. $x = 1+2+2^3+m(2^6)$ and x is an odd number. $T(x) = 4+3.2+3.2^3+m(3.2^6)$. $T^2(x) = 1+2^4+m(3.2^5)$. $T^3(x) = 4+3.2^4+m(3^2.2^5)$. $T^4(x) = 2+3.2^3+m(3^2.2^4)$. $T^5(x) = 1+3.2^2+m(3^2.2^3)$. $T^6(x) = 4+3^2.2^2+m(3^3.2^3)$. $T^7(x) = 2+3^2.2+m(3^3.2^2)$. $T^8(x) = 1+3^2+m(3^3.2)$.

Now number of iterations performed on odd integers is $p = 3$. Number of iterations performed on even integers is $q = 5$. Here $\frac{p}{q} = \frac{3}{5} < \frac{\log 2}{\log 3}$, where as $\frac{p}{q-1} = \frac{3}{4} > \frac{\log 2}{log3}$. So the iteration procedure may be stopped. Moreover, if $x \in A_6$, then x is an odd Integer. $T(x) = 3x+1$. $T^2(x) = \frac{3x+1}{2}$. $T^3(x) = 3[\frac{3x+1}{2}]+1 = \frac{9x+5}{2}$. $T^4(x) = \frac{9x+5}{4}$. $T^5(x) = \frac{9x+5}{8}$. $T^6(x) = \frac{27x+23}{8}$. $T^7(x) = \frac{27x+23}{16}$. $T^8(x) = \frac{27x+23}{32}$. For this x, $T^8(x) = \frac{27x+23}{32} < x \Leftrightarrow 27x+23 < 32x \Leftrightarrow (5x-23) > 0 \Leftrightarrow x > 5$. Here $x \geq 5$ for all $x \in A_5$. So $T^8(x) < x$, for all $x \in A_5$.

(6). Let $A_6 = \{x : x = 1+2+m(2^4)\}$. Let $x \in A_6$ be arbitrary. $x = 1+2+m(2^4)$, x is an odd integer. $T(x) = 4+3.2+m(3.2^4)$. $T^2(x) = 1+2^2+m(3.2^3)$. $T^3(x) = 4+3.2^2+m(3^2.2^4)$. $T^4(x) = 2+3.2+m(3^2.2^3)$. $T^5(x) = 1+3+m(3^2.2^2) = 2^2+m(3^2.2^2)$. $T^6(x) = 2+m(3^2.2)$.

Now number of iterations performed on odd integers is $p = 2$. Number of iterations performed on even integers is $q = 4$. Here $\frac{p}{q} = \frac{2}{4} < \frac{\log 2}{\log 3}$, where as $\frac{p}{q-1} = \frac{2}{3} > \frac{\log 2}{\log 3}$. So the iteration procedure may be stopped. Moreover, if $x \in A_7$ then x is an odd integer. $T(x) = 3x+1$. $T^2(x) = \frac{3x+1}{2}$. $T^3(x) = \frac{9x+5}{2}$. $T^4(x) = \frac{9x+5}{4}$. $T^5(x) = \frac{9x+5}{8}$ and $T^6(x) = \frac{9x+5}{16}$. For this $T^6(x) = \frac{9x+5}{16} < x \Leftrightarrow 9x+5 < 16x \Leftrightarrow 7x-5 > 0 \Leftrightarrow x > \frac{5}{7}$. Here $x > \frac{5}{7}$ for all $x \in A_6$. So $T^6(x) < x$, for all $x \in A_6$.

(7). Let $A_7 = \{x : x = 1+2+2^2+2^3+m(2^7)\}$. Let $x \in A_7$ be arbitrary, then x is an odd integer. $x = 1+2+2^2+2^3+m(2^7)$. $T(x) = 4+3.2+3.2^2+3.2^3+m(3.2^7)$. $T^2(x) = 1+2+2^2+2^4+m(3.2^6)$. $T^3(x) = 4+3.2+3.2^2+3.2^4+m(3^2.2^6)$. $T^4(x) = 1+2+2^3+3.2^3+m(3^2.2^5)$. $T^5(x) = 4+3.2+3.2^3+3^2.2^3+m(3^3.2^5)$. $T^6(x) = 1+13.2^2+m(3^3.2^4)$. $T^7(x) = 4+3.13.2^2+m(3^4.2^4)$. $T^8(x) = 2+3.13.2+m(3^4.2^3)$. $T^9(x) = 1+3.13+m(3^4.2^2) = 2^5+2^3+m(3^4.2^2)$. $T^{10}(x) = 2^4+2^2+m(3^4.2)$. $T^{11}(x) = 2^3+2+m(3^4)$.

Now number of iterations performed on odd integers is $p = 4$. Number of iterations performed on even integers is $q = 7$. Now $\frac{p}{q} = \frac{4}{7} < \frac{\log 2}{\log 3}$, whereas $\frac{p}{q-1} = \frac{4}{6} > \frac{\log 2}{\log 3}$. So the iteration procedure maybe stopped. Moreover,if $x \in A_8$, then x is an odd integer. $T(x) = 3x + 1$. $T^2(x) = \frac{3x+1}{2}$. $T^3(x) = \frac{9x+5}{2}$. $T^4(x) = \frac{9x+5}{4}$. $T^5(x) = \frac{27x+19}{4}$. $T^6(x) = \frac{27x+19}{8}$. $T^7(x) = \frac{81x+65}{8}$. $T^8(x) = \frac{81x+65}{16}$. $T^9(x) = \frac{81x+65}{32}$. $T^{10}(x) = \frac{81x+65}{64}$ and $T^{11}(x) = \frac{81x+65}{128}$. For this $T^{11}(x) = \frac{81x+65}{128} < x \Leftrightarrow 81x + 65 < 128x \Leftrightarrow 47x > 65 \Leftrightarrow x > \frac{65}{47}$. Here $x > \frac{65}{47}$ for all $x \in A_7$. So $T^{11}(x) < x$, for all $x \in A_7$.

Verification for Disjointness of sets

(a) A_1 contains all even positive integers, and each A_i, $i = 2, 3, 4, 5, 6, 7$ contains odd positive integers. Then $A_1 \cap A_i = \emptyset$ for all $i = 2, 3, 4, 5, 6, 7$.

(b) If $A_2 \cap A_i \neq \emptyset, i = 3, 4, 5, 6, 7$, then there are some $n, m \in N$, such that, for respective cases, one has:

   i. $4n + 1 = 32m + 23$ or $32m - 4n = -22$ or $8m - n = \frac{-11}{2}$. This is not possible, since $8m - n$ is an integer. Hence $A_2 \cap A_3 = \emptyset$.

   ii. $4n + 1 = 128m + 74$ or $128m - 4n = -6$ or $32m - n = \frac{-3}{2}$. This is not possible, since $32m - n$ is an integer. Hence$A_2 \cap A_4 = \emptyset$.

   iii. $4n + 1 = 64m + 11$ or $64m - 4n = -10$ or $16m - n = \frac{-5}{2}$. This is not possible, since $16m - n$ is an integer. Hence $A_2 \cap A_5 = \emptyset$.

   iv. $4n + 1 = 16m + 3$ or $16m - 4n = -2$ or $4m - n = \frac{-1}{2}$. This is not possible, since $4m - n$ is an integer. Hence$A_2 \cap A_6 = \emptyset$.

   v. $4n + 1 = 128m + 15$ or $128m - 4n = -14$ or $32m - n = \frac{-7}{2}$. This is not possible, since $32m - n$ is an integer. Hence $A_2 \cap A_7 = \emptyset$.

(c) If $A_3 \cap A_i \neq \emptyset, i = 4, 5, 6, 7$, then there are some $m, n \in N$, such that, for respective cases, one has:

   i. $32n + 23 = 128m + 7$ or $128m - 32n = 16$ or $8m - 2n = 1$ or $4m - n = \frac{1}{2}$. This is not possible, since $4m - n$ is an integer. So $A_3 \cap A_4 = \emptyset$.

   ii. $32n + 23 = 64m + 11$ or $64m - 32n = 12$ or $2m - n = \frac{3}{8}$. This is not possible, since $2m - n$ is an integer. So $A_3 \cap A_5 = \emptyset$.

   iii. $32n + 23 = 16m + 3$ or $32n - 16m = -20$ or $2n - m = \frac{-5}{4}$. This is not possible, since $2n - m$ is an integer. So $A_3 \cap A_6 = \emptyset$.

   iv. $32n + 23 = 128m + 15$ or $128m - 32n = -8$ or $4m - n = \frac{-1}{4}$. This is not possible, since $4m - n$ is an integer. So $A_3 \cap A_7 = \emptyset$.

(d) If $A_4 \cap A_i \neq \emptyset, i = 5, 6, 7$, then there are some $m, n \in N$ such that, for respective cases, one has:

   i. $128n + 7 = 64m + 11$ or $128n - 64m = 4$ or $2n - m = \frac{1}{18}$. This is not possible, since $2n - m$ is an integer. So $A_4 \cap A_5 = \emptyset$.

   ii. $128n + 7 = 16m + 13$ or $128n - 16m = 6$ or $8n - m = \frac{3}{8}$. This is not possible, since $8n - m$ is an integer. So $A_4 \cap A_6 = \emptyset$.

   iii. $128n + 7 = 128m + 15$ or $128n - 128m = 8$ or $n - m = \frac{1}{16}$. This is not possible, since $n - m$ is an integer. So $A_4 \cap A_7 = \emptyset$.

(e) i. If $A_5 \cap A_6 \neq \emptyset$, then there are some m, n such that $64n + 11 = 16m + 3$ or $64n - 16m = -8$ or $4n - m = \frac{-1}{2}$. This is not possible, since $4n - m$ is an integer. So $A_5 \cap A_6 = \emptyset$

	A concluding table				
S. No	Sets	Number of Even iterations (p)	Number of odd iterations (q)	p/q (app.value)	p/(q-1) (app.value)
1	$A_1 = \{x : x = 2n\}$ $= \{2, 4, 7, \}$ General term $x = 2n$ Asymptotic density=$\frac{1}{2}$	0	1	0	-
2	$A_2 = \{x : x = 1 + m(2^2)\}$ {1,5,9,......} General term $x = 4n + 1$ Asymptotic density=$\frac{1}{4}$	1	2	0.5	1
3	$A_3 = \{x : x = 1 + 2 + 22 + 24 + m(2^5)\}$ =$\{23, 55, 87, ...\}$ General term $x = 32n + 23$ Asymptotic density=$\frac{1}{32}$	3	5	0.6	0.75
4	$A_4 = \{x : x = 1 + 2 + 2^2 + m(2^7)\}$ {7,135,263,....} General term $x = 128n + 1$ Asymptotic density=$\frac{1}{128}$	4	7	0.58	0.67
5	$A_5 = \{x : x = 1 + 2 + 2^3 + m(2^6)\}$ =$\{11, 75, 139, ...\}$ General term $x = 64n + 11$ Asymptotic density=$\frac{1}{64}$	3	5	0.6	0.75
6	$A_6 = \{x : x = 1 + 2 + m(2^4)\}$ $= \{3, 19, 35, 51, ...\}$ General term $x = 16n + 3$ Asymptotic density=$\frac{1}{16}$	2	4	0.5	0.67
7	$A_7 = \{x : x = 1 + 2 + 2^2 + 2^3 + m(2^7)\}$ = {15,143,271,....} General term $x = 128n + 15$ Asymptotic density=$\frac{1}{128}$	4	7	0.58	0.67

ii. If $A_5 \cap A_7 = \emptyset$, then there are some m, n such that $64n+11 = 128m+15$ or $128m-64n = -4$ or $2m-n = \frac{-1}{16}$. This is not possible, since $2m - n$ is an integer. So $A_5 \cap A_7 = \emptyset$.

(f) If $A_6 \cap A_7 \neq \emptyset$, then there are some m, n such that $16n + 3 = 128m + 15$ or $128m = 16n = -12$ or $8m - n = \frac{-3}{4}$. This is not possible, since $8m - n$ is an integer. So $A_6 \cap A_7 = \emptyset$.

Asymptotic density of a union of pairwise disjoint sets is the sum of the corresponding asymptotic densities. So, if this sum reduces to the value 1 at a finite stage, then the conjecture would be settled out positively. So, one may try to find a way to extend the following table, until the sum value reaches the value 1 at a finite stage. So, the following table is generated.

3. To find i and j

This section also provides a method to generate a table of values satisfying $T^k(x) < x$, for some k. If the previous iteration procedure is adopted for $x = 3$, then $T(3) = 10$, $T^2(3) = 5$, $T^3(3) = 16$, $T^4(3) = 8$, $T^5(3) = 4$, $T^6(3) = 2 < 3$. Hence $T^6(x) < x$. Let i denote smallest number of iterations on odd numbers and j denote smallest number of iterations on even numbers until $T^{i+j}(x) < x$. In this case it happens that $\frac{2^j}{3^i} < 1$.

Here for $x = 3$, the values of i and j are $i = 2, j = 4$. In this procedure integers are divided by 2 four times. When $2^j n$ is divided by 2^j, it leads to n, an integer. So $T^{i+j}(2^j n + 3) < 2^j n + 3$, because $\frac{2^j}{3^i}(2^j n) < 2^j n$, (with $i = 2$ and $j = 4$) and the one is added with the triples of odd integer which affects the odd integer corresponding to 3. So $T^{i+j}(16n+3) < 16n+3$, for all $n = 1, 2, 3, \ldots$. Here $16 = 2^4 = 2^j$. Now the next integer is $x = 5$ is of the form $4n+1$ and the value of $i = 1, j = 2$. Since any number in the form $4n+1 \in A_2$, it has the values $i = 1, j = 2$, and $T^3(4n+1) < 4n+1$. So hereafter the number of the form $x = 4n+1$ may be excluded; when numbers x satisfying $T^k(x) < x$, for some k, are searched.

The next odd integer is $x = 7$. For this 7, $T(7) = 22$, $T^2(7) = 11$, $T^3(7) = 34$, $T^4(7) = 17$, $T^5(7) = 52$, $T^6(7) = 26$, $T^7(7) = 13$, $T^8(7) = 40$, $T^9(7) = 20$, $T^{10}(7) = 10$, $T^{11}(7) = 5 < 7$. So $T^{11}(7) < 7$, with $i = 4$, $j = 7$. Hence $T^{i+j}(2^jn+7)$ $= T^{4+7}(2^7n+7) = T^{11}(2^7n+7) < 2^7n+7$. The next odd integer is $x = 9$, which is of the form $4n+1$.(so that $T^3(9) < 9$). The next odd integer is $x = 11$, and $T(11) = 34$, $T^2(11) = 17$, $T^3(11) = 52$, $T^4(11) = 26$, $T^5(11) = 13$, $T^6(11) = 40$, $T^7(11) = 20$, $T^8(11) = 10 < 11$, and $T^8(x) < x$ with $i = 3, j = 5$.

Hence $T^{i+j}(2^jn+11) = T^{5+3}(2^5n+11) = T^8(2^5n+11) < 2^5n+11$. It is not a difficult work to find asymptotic density of the sets mentioned in the following table generated by following procedure mentioned above. But the difficulty lies in deciding disjointness of these sets.

	Another concluding table		
S.No	**General Form**	i	j
1	$2n$	0	1
2	$4n+1$	1	2
3	$16n+3$	4	2
4	$128n+7$	7	4
5	$64n+11$	5	3
6	$256n+15$	7	4
7	$16n+19$	4	2
8	$32n+23$	5	3
9	$2^{59}n+27$	59	37
10	$2^{56}n+31$	56	35
11	$16n+35$	4	2
12	$256n+39$	8	5
13	$32n+43$	5	3
14	$2^{54}n+47$	54	34
15	$32n+55$	5	3
17	$128n+59$	7	4
18	$2^{54}n+63$	54	34
19	2^4n+67	4	2
20	$2^{51}n+71$	51	30
21	2^5n+75	5	3
22	2^8n+79	8	5
23	2^4n+83	4	2
24	2^5n+87	5	3
25	$2^{45}n+91$	45	28
26	2^8n+95	8	5
27	2^4n+99	4	2
28	$2^{42}n+103$	42	26
29	2^5n+107	5	3
30	$2^{31}n+111$	31	19
31	2^4n+115	4	2
32	2^5n+119	5	3
33	2^8n+123	8	5
34	$2^{15}n+127$	15	9
35	2^4n+131	4	2

S.No	**General Form**	i	j
36	$128n+135$	7	4
37	$32n+139$	5	3
38	$128n+143$	7	4
39	$16n+147$	4	2
40	$32n+151$	5	3
41	$2^{40}n+155$	40	25
42	$2^{21}n+159$	21	13
43	2^4n+163	4	2
44	$2^{29}n+167$	29	18
45	2^5n+171	5	3
46	$256n+175$	5	3
47	$16n+179$	4	2
48	$32n+183$	5	3
49	$128n+187$	7	4
50	$2^{13}n+191$	13	8
51	$16n+195$	4	2
52	$256n+199$	8	5
53	$32n+203$	5	3
54	$2^{13}n+207$	13	8
55	$16n+211$	4	2
56	$32n+215$	5	3
57	$256n+219$	8	5
58	$231n+223$	31	19
59	$16n+227$	4	2
60	$2^{12}n+231$	12	7
61	$32n+235$	5	3
62	$2^{20}n+239$	20	12
63	$16n+243$	4	2
64	$32n+247$	5	3
65	$2^{27}n+257$	27	17
66	$2^{13}n+255$	13	8
67	$16n+259$	4	2

Among all these sets, some pairwise disjoint sets and their asymptotic densities are given below.

S.No	Sets	Asymptotic density
1)	$B_1 = \{2n : n \in N\}$	$\frac{1}{2}$
2)	$B_2 = \{4n + 1 : n \in N\}$	$\frac{1}{4}$
3)	$B_3 = \{16n + 3 : n \in N\}$	$\frac{1}{16}$
4)	$B_4 = \{32n + 23 : n \in N\}$	$\frac{1}{32}$
5)	$B_5 = \{32n + 43 : n \in N\}$	$\frac{1}{32}$
6)	$B_6 = \{128n + 7 : n \in N\}$	$\frac{1}{128}$
7)	$B_7 = \{128n + 59 : n \in N\}$	$\frac{1}{128}$
8)	$B_8 = \{128n + 143 : n \in N\}$	$\frac{1}{128}$
9)	$B_9 = \{256n + 79 : n \in N\}$	$\frac{1}{256}$
10)	$B_{10} = \{256n + 79 : n \in N\}$	$\frac{1}{256}$
11)	$B_{11} = \{256n + 95 : n \in N\}$	$\frac{1}{256}$
12)	$B_{12} = \{256n + 123 : n \in N\}$	$\frac{1}{256}$
13)	$B_{13} = \{256n + 175 : n \in N\}$	$\frac{1}{256}$
14)	$B_{14} = \{256n + 199 : n \in N\}$	$\frac{1}{256}$
15)	$B_{15} = \{256n + 219 : n \in N\}$	$\frac{1}{256}$

Again, the following should be noted regarding this work. If it is possible to find the sum of asymptotic densities of a finite number of pairwise disjoint sets as 1, then it would settle out the Collatz conjecture positively.

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