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On Pre-closure Sets in Topological Space

Research Article

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Abstract: The aim of this paper is to introduce the concept of pre-closure set and the pre-closure topological spaces.

Keywords: Closure, pre closure, pre closure space.

1. Introduction

In mathematics the closure of a set A in a topological space (X, τ) is the smallest super set of A which contains all the limit points of A [1]. This paper is devoted to introduce a new type of set-called the Pre-closure-in a topological space and a topology associated with it.

1.1. Preliminaries

Let S be a subset (X, τ). A point x in X is a point of closure of S if every neighbourhood of x must contain at least one element of S [1]. A point x in X is a limit point of S if every neighbourhood of x must contain at least one element of S other than x [2]. The closure Cl(S) of S is the collection of all points of closure of S or in other words Cl(S) is the union of S and its limit point [3]. S called dense (in X) if every point x in X either belongs to S or is a limit point of S [4].

2. Pre-Closure Set

Definition 2.1. *Let (X, τ) be a topological space and $A \subseteq X$. Then we define the Pre-closure set of A to be any set $B \subseteq X$ such that*

(a). $A \subseteq B$.

(b). Every element in B is a point of closure of A.

Example 2.2. *Consider real line with usual topology. Then the Pre-closure sets of $(1, 2)$ are $(1, 2)$, $[1, 2)$, $(1, 2]$, $[1, 2]$.*

Lemma 2.3. *Let (X, τ) be a topological space and $A \subseteq X$. Then the following statements holds*

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(a). The largest Pre-closure set of A is Cl(A).

(b). The smallest Pre-closure of A is A itself.

(c). Arbitrary union of Pre-closures of A is again a Pre-closure of A.

(d). Arbitrary intersection of Pre-closures of A is again Pre-closure of A.

(e). The union of all Pre-closure of A is Cl(A).

(f). The intersection of all Pre-closures of A is A itself.

Proof.

(a). Since any super set of Cl(A) contains at least one element which is not a limit point of A, from the definition of Pre-closure the result follows.

(b). Result directly follows from the definition of Pre-closure.

(c). Let $U = \bigcup_k U_k$ be an arbitrary union of Pre-closures of A. Clearly $A \subseteq U$. If possible U is not a Pre-closure of A. Then there exist at least one $x \in U$ such that x is not a point of closure of A. But then $x \in U_k$ for some k, which is a contradiction.

(d). Let $G = \bigcap_k G_k$be an arbitrary intersection of Pre-closures of A. Clearly $A \subseteq G$. If possible G is not a Pre-closure of A. Then there exist at least one $x \in G$ such that x is not a point of closure A. But then $x \in G_k$ for all k, which is a contradiction.

(e). Result follows from (c) and (a).

(f). Result follows from (d) and (b).

□

Lemma 2.4. *Let (X, τ) be a topological space and $A \subseteq X$. Then the set of all Pre-closures of A together with ϕ forms a topology on Cl(A) denoted by $\tau_{pcl(A)}$ and we call it Pre-closure topology on Cl(A).*

Theorem 2.5. *Let (X, τ) be a topological space and $A \subseteq X$. Then the set of all Pre-closures of A together with ϕ forms a topology on X if and only if A is dense in X.*

Proof. Let the set of all Pre-closures of A together with ϕ forms a topology on X. Then obviously X is a Pre-closure of A. which means every element in X is a point of closure of A. Hence $Cl(A) = X$ and thus A is dense in X.
The converse part is obvious since for a dense set $Cl(A) = X$ and by Lemma 2.4. □

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