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Soft Pre Generalized Closed Sets With Respect to a Soft Ideal in Soft Topological Spaces

Research Article

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Abstract: In this paper, we extend the concept of soft pre generelized closed sets due to J. Subhashini and C. Sekar [15] to soft pre generalized closed sets with respect to a soft ideal and study some of their basic properties.

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1. Introduction

Soft set is collection of approximate descriptions of an object. In 1999, Molodtsov [12] initiated the concept of soft set as a new mathematical tool for dealing with uncertainties. Soft topology was introduced by Muhammad Shabir and Munazza Naz [13] in 2010. Later, Aygunoglu [2], Zorlutuna [17], S.Hussian [3], Aygun [2] continued to study the some of basic concepts and properties of soft topological spaces. The concept of generalized closed set in topology was introduced by Levine [8]. Ideals play an important role in toplogy. Jankovic and Hamlet [5] introduced the notion of I-open sets in topologival spaces. Kuratowski [7] and Vaidyanayhaswamy [16] introduced and studied the concepts of ideals topological spaces. The concept of g-closed sets in soft topology by Kannan [4]. Andrijevic [1] and Mashhour [11] gave many results on pre open sets in general toplogy. Subhashini and Sekar [15] introduced pre generalized closed sets in soft topological spaces. H.I.Mustafa and F.M.Sleim [14] introduced the concept of soft generalized closed sets with respect to a soft ideal, which is the extension of the concept of soft generalized closed sets.

In this paper, we introduce and study the concept of soft I_{pg}-closed sets in soft topological spaces. We also study the relationship between generalized soft I_{pg}-closed sets, soft I_{pg}-open sets, soft I_g-closed sets, soft g-closed sets, soft pg-closed sets and some of their properties.

2. Preliminaries

Let X refers to an initial universe set, E is a set of parameters and $A \subseteq E$. Parameters are attributes, characteristics or properties of the objects in X and $P(X)$ denotes the power set of X.

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Definition 2.1 ([12]). *A pair F_E is called a soft set over X where F is a mapping given by $F : A \to P(X)$. In other words, a soft set over X is a parameterized family of subsets of the universe X.*

Definition 2.2 ([9]). *The union of two soft sets F_A and G_B over U is a soft set H_C, where $C = A \cup B$ and $\forall\ e \in C$,*

$$H(e) = \begin{cases} F(e), & if \quad e \in A - B \\ G(e), & if \quad e \in B - A \\ F(e) \cup G(e), & if \quad e \in A \cap B \end{cases}$$

This relationship is denoted by $F_A \tilde{\cup} G_B = H_C$

Definition 2.3 ([10]). *The intersection of two soft sets F_A and G_B over X is a soft set H_C, where $C = A \cap B$ and $\forall\ e \in C$, $H(e) = F(e) \cap G(e)$. This relationship is denoted by $F_A \tilde{\cap} G_B = H_C$.*

Definition 2.4 ([13]). *Let $\tilde{\tau}$ be the collection of soft sets over X. Then $\tilde{\tau}$ is said to be a soft topology on X if*

(a). $F_\phi, F_A \in \tilde{\tau}$

(b). If $F_E, G_E \in \tilde{\tau}$, then $F_E \tilde{\cap} G_E \in \tilde{\tau}$

(c). If $\{F_{E_i}\}_{i \in I} \in \tilde{\tau}, \forall\ i \in I$, then $\tilde{\bigcup}_{i \in I} F_{E_i} \in \tilde{\tau}$

The pair $(X, \tilde{\tau})$ is called a soft topological space.

Definition 2.5 ([13]). *Let $(X, \tilde{\tau})$ be a soft topological space over X and Y be a non-empty subset of X. Then $\tilde{\tau}_Y = \{\tilde{Y} \tilde{\cap} F_E \mid F_E \in \tilde{\tau}\}$ is said to be the soft relative topology on Y and $(Y, \tilde{\tau}_Y)$ is called a soft subspace of $(X, \tilde{\tau})$. We can easily verify that $\tilde{\tau}_Y$ is a soft topology on Y.*

Definition 2.6 ([12]). *If $(X, \tilde{\tau})$ be a soft topological space. Then,every element of $\tilde{\tau}$ is called a soft open set. The collection of all soft open set is denoted by $G_s(F_E)$. Let $F_c \tilde{\subseteq} F_E$. Then F_c is said to be sofr closed if the soft set F_c^c is soft open in F_E. The collection of all soft closed set is denoted by $F_s(F_E)$.*

Definition 2.7 ([13]). *Let F_E be a soft set over X and Y be a nonempty subset of X. Then the soft subset of F_E over Y denoted by $^Y F_E$, is defined as $^Y F(e) = Y \tilde{\cap} F(e)$, for all $e \in E$. In other words $^Y F_E = \tilde{Y} \cap F_E$.*

Definition 2.8 ([17]). *Let $(X, \tilde{\tau})$ be a soft topological space, a soft set F_A is said to be soft pre-open(soft P-open) if there exists a soft open set F_o such that $F_A \tilde{\subseteq} F_o \tilde{\subseteq} F_A^-$. The set of all soft P-open set of F_E is denoted by $G_{sp}(F_E, \tilde{\tau})$ or $G_{sp}(F_E)$. Then F_A^c is said to be soft pre-closed. The set of all soft P-closed set of F_E is denoted by $F_{sp}(F_E, \tilde{\tau})$ or $F_{sp}(F_E)$.*

Definition 2.9 ([15]). *Let $(X, \tilde{\tau})$ be a soft topological space and $F_A \tilde{\subseteq} F_E$. Then the soft pre-interior(soft P-interior) of F_A denoted by $p(F_A)^o$ is defined as the soft union of all soft P-open subsets of F_A. Note that $p(F_A)^o$is the biggest soft P-open set that contained in F_A.*

Definition 2.10 ([15]). *Let $(X, \tilde{\tau})$ be a soft topological space and $F_A \tilde{\subseteq} F_E$. Then the soft pre closure(soft P-closure)of F_A denoted by $p(F_A)^-$ is defined as the soft intersection of all soft P-closed supersets of F_A. Note that, $p(F_A)^-$ is the soft smallest soft P-closed set containing F_A.*

Definition 2.11 ([6]). *Let $(X, \tilde{\tau})$ be a soft topological space over X. A soft set F_E is called a soft g-closed in X if $F_E^- \tilde{\subseteq} F_o$ whenever $F_E \tilde{\subseteq} F_o$ and F_o is soft open in X.*

Definition 2.12 ([15]). *Let F_A is said to be soft pre generalized closed set (soft Pg-closed set) if $p(F_A)^- \tilde{\subseteq} F_o$ whenever $F_A \tilde{\subseteq} F_o$ and $F_o \in G_{sp}(F_E)$. The collection of all soft Pg-closed sets is denoted by $F_{spg}(F_E)$.*

Definition 2.13 ([14]). *A non empty collections I of soft subsets over X is called a soft ideal on X if the following holds :*

(i) If $F_E \in I$ and $G_E \tilde{\subseteq} F_E$, implies $G_E \in I$ (heredity)

(ii) If F_E and $G_E \in I$, then $F_E \tilde{\cup} G_E \in I$. (additivity)

Definition 2.14 ([14]). *A soft set F_E is called soft generalized closed set with respect to a soft ideal I (soft I_g-closed) in a soft topological space $(X, \tilde{\tau})$ if $F_E^- \setminus G_E \in I$ whenever $F_E \tilde{\subseteq} G_E$ and $G_E \tilde{\in} \tilde{\tau}$.*

Definition 2.15 ([14]). *Two soft sets A_E and B_E are said to be soft separated in a soft topological space $(X, \tilde{\tau})$ if $A_E^c \tilde{\cap} B_E = \phi$ and $A_E \tilde{\cap} B_E^- = \phi$.*

3. Soft Pre Generalized Closed Sets With Respect To Soft Ideal

In this section we introduce the concept of soft I_{pg}-closed sets and study some of its properties.

Definition 3.1. *A soft set F_E is called soft pre generalized closed set with respect to a soft ideal I (soft I_{pg}-closed) in a soft topological space $(X, \tilde{\tau})$ if $p(F_E)^- \setminus G_E \in I$ whenever $F_E \tilde{\subseteq} G_E$ and G_E is soft pre open.*

Example 3.2. *Let $X = \{u_1, u_2, u_3\}$, $E = \{x_1, x_2, x_3\}$,$A = \{x_1, x_2\} \subseteq E$ and $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})\}$. Then $F_{A1} = \{(x_1, \{u_1\})\}$, $F_{A2} = \{(x_1, \{u_2\})\}$, $F_{A3} = \{(x_1, \{u_1, u_2\})\}$, $F_{A4} = \{(x_2, \{u_2\})\}$, $F_{A5} = \{(x_2, \{u_3\})\}$, $F_{A6} = \{(x_2, \{u_2, u_3\})\}$, $F_{A7} = \{(x_1, \{u_1\}), (x_2, \{u_2\})\}$, $F_{A8} = \{(x_1, \{u_1\}), (x_2, \{u_3\})\}$, $F_{A9} = \{(x_1, \{u_1\}), (x_2, \{u_2, u_3\})\}$, $F_{A10} = \{(x_1, \{u_2\}), (x_2, \{u_2\})\}$, $F_{A11} = \{(x_1, \{u_2\}), (x_2, \{u_3\})\}$, $F_{A12} = \{(x_1, \{u_2\}), (x_2, \{u_2, u_3\})\}$, $F_{A13} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2\})\}$, $F_{A14} = \{(x_1, \{u_1, u_2\}), (x_2, \{u_3\})\}$, $F_{A15} = F_A$, $F_{A16} = F_\phi$ are all soft subsets of F_A and $\tau = \{F_\phi, F_A, F_{A1}, F_{A3}\}$ is the soft topology over X. Let $I = \{F_\phi, F_A, F_{A4}\}$ be a soft ideal on X. Since, soft pre open sets=$\{F_\phi, F_A, F_{A1}, F_{A3}, F_{A7}, F_{A8}, F_{A9}, F_{A13}, F_{A14}\}$. Clearly, F_{A8} is soft I_{pg}-closed. In fact, $F_{A8} \subseteq F_{A14}$ and F_{A14} is soft pre open set. But $p(F_{A8})^- = F_A$ and $p(F_{A8})^- \setminus F_{A14} = F_A \setminus F_{A14} = F_{A4} \in I$.*

Proposition 3.3. *Every soft pg-closed set is soft I_{pg}- closed.*

Proof. Let F_E be a soft pg-closed set in soft topological space $(X, \tilde{\tau})$. We show that F_E is soft I_{pg}-closed set. Let $F_E \tilde{\subseteq} G_E$ and G_E is soft pre open. Since F_E is soft pg-closed, then $p(F_E)^- \tilde{\subseteq} G_E$ and hence $p(F_E)^- \setminus G_E = \phi \in I$. Consequently F_E is soft I_{pg} -closed. □

The converse of the above proposition is not true in general as seen from the following example.

Example 3.4. *From Example 3.3, we have F_{A8} is soft I_{pg}-closed, but it is not pg-closed.*

Proposition 3.5. *Every soft g-closed set is soft I_{pg}- closed.*

Proof. Let F_E be a soft g-closed set in soft topological space $(X, \tilde{\tau})$. Let $F_E \tilde{\subseteq} G_E$ and G_E is soft open. Since F_E is soft g-closed, then $F_E^- \tilde{\subseteq} G_E$. Since every soft open is soft pre open, then we have G_E is soft pre open and hence $p(F_E)^- \setminus G_E = \phi \in I$. Consequently F_E is soft I_{pg} -closed. □

The converse of the above proposition is not true in general as seen from the following example.

Example 3.6. *From Example 3.3, F_{A2} is soft I_{pg}-closed, but not soft g-closed.*

Proposition 3.7. *Every soft I_g-closed set is soft I_{pg}- closed.*

Proof. Let F_E be a soft I_g-closed set in soft topological space $(X, \tilde{\tau})$. Let $F_E \tilde{\subseteq} G_E$ and G_E is soft open. Since F_E is soft I_g-closed, then $F_E^- \setminus G_E \in I$. Since every soft open is soft pre open, then we have G_E is soft pre open and hence $p(F_E)^- \setminus G_E = \phi \in I$. Consequently F_E is soft I_{pg} -closed. □

The converse of the above proposition is not true in general as seen from the following example.

Example 3.8. *From Example 3.3, we have F_{A2} is soft I_{pg}-closed. But it is not soft I_g-closed.*

Theorem 3.9. *A soft set A_E is soft I_{pg}-closed in a soft topological space $(X, \tilde{\tau})$ iff $F_E \tilde{\subseteq} p(A_E)^- \setminus A_E$ and F_E is soft pre closed implies $F_E \in I$.*

Proof. Assume that A_E is soft I_{pg}-closed. Let $F_E \tilde{\subseteq} p(A_E)^- \setminus A_E$. Suppose that F_E is soft pre closed. Then $A_E \tilde{\subseteq} F_E^c$. By our assumption, $p(A_E)^- \setminus F_E^c \in I$. But $F_E \tilde{\subseteq} p(A_E)^- \setminus F_E^c$, then $F_E \in I$.
Conversely, assume that $F_E \tilde{\subseteq} p(A_E)^- \setminus A_E$ and F_E is soft pre closed implies $F_E \in I$. Suppose that $A_E \tilde{\subseteq} G_E$ and G_E is soft pre open. Then $p(A_E)^- \setminus G_E^- = p(A_E)^- \tilde{\cap} (G_E^c)^-$ is a soft pre closed set in $(X, \tilde{\tau})$ and $p(A_E)^- \setminus G_E^- \tilde{\supseteq} p(A_E)^- \setminus G_E$. By assumption $p(A_E)^- \setminus G_E \in I$. This implies that A_E is soft I_{pg}-closed. □

Theorem 3.10. *If F_E and G_E are soft I_{pg}-closed sets in a soft topological space $(X, \tilde{\tau})$, then their union $F_E \tilde{\cup} G_E$ is also soft I_{pg}-closed in $(X, \tilde{\tau})$.*

Proof. Suppose that F_E and G_E are soft I_{pg}-closed in $(X, \tilde{\tau})$. If $F_E \tilde{\cup} G_E \tilde{\subseteq} H_E$ and H_E is soft pre open , then $F_E \tilde{\subseteq} H_E$ and $G_E \tilde{\subseteq} H_E$. By assumption $p(F_E)^- \setminus H_E \in I$ and $p(G_E)^- \setminus H_E \in I$ and hence $p(F_E \tilde{\cup} G_E)^- \setminus H_E = (p(F_E)^- \setminus H_E) \tilde{\cup} (p(G_E)^- \setminus H_E) \in I$. That is $F_E \tilde{\cup} G_E$ is soft I_{pg}-closed. □

Theorem 3.11. *If F_E is soft I_{pg}-closed in a soft topological space $(X, \tilde{\tau})$ and $F_E \tilde{\subseteq} G_E \tilde{\subseteq} p(F_E)^-$, then G_E is soft I_{pg}-closed in $(X, \tilde{\tau})$.*

Proof. If F_E is soft I_{pg}-closed and $F_E \tilde{\subseteq} G_E \tilde{\subseteq} p(F_E)^-$ in $(X, \tilde{\tau})$. Suppose that $G_E \tilde{\subseteq} H_E$ and H_E is soft pre open . Then $F_E \tilde{\subseteq} H_E$. Since F_E is soft I_{pg}-closed, then $p(F_E)^- \setminus H_E \in I$. Now, $G_E \tilde{\subseteq} p(F_E)^-$ implies that $p(G_E)^- \tilde{\subseteq} p(F_E)^-$. So $p(G_E)^- \setminus H_E \tilde{\subseteq} p(F_E)^- \setminus H_E$ and thus $p(G_E)^- \setminus H_E \in I$. Consequently G_E is soft I_{pg}-closed in $(X, \tilde{\tau})$. □

Remark 3.12. *The intersection of two soft I_{pg}-closed sets need not be a soft I_{pg}-closed as shown by the following example.*

Example 3.13. *Let $X = \{u_1, u_2, u_3\}$,$A = \{x_1, x_2\}$, $F_A = \{(x_1, \{u_1, u_2\}), (x_2, \{u_2, u_3\})$ and $\tilde{\tau} = \{F_\phi, F_A, F_{A1}, F_{A3}\}$, $\tilde{\tau}^c = \{F_\phi, F_A, F_{A12}, F_{A6}\}$. Let $I = \{F_\phi, (x_2, \{u_2\})\}$ be a soft ideal on X. $G_{sp} = \{F_\phi, F_A, F_{A1}, F_{A3}, F_{A7}, F_{A8}, F_{A9}, F_{A13}, F_{A14}\}$. Clearly, F_{A9} and F_{A13} are soft I_{pg}-closed but their intersection F_{A7} is not I_{pg}-closed.*

Theorem 3.14. *If A_E is soft I_{pg}-closed and F_E is soft pre closed in a soft topological space $(X, \tilde{\tau})$. Then $A_E \tilde{\cap} F_E$ is soft I_{pg}-closed in $(X, \tilde{\tau})$.*

Proof. Assume that $A_E \tilde{\cap} F_E \tilde{\subseteq} G_E$ and G_E is soft pre open. Then $A_E \tilde{\subseteq} G_E \tilde{\cup} F_E^c$. Since A_E is soft I_{pg}-closed, we have $p(A_E)^- \setminus (G_E \tilde{\cup} F_E^c) \in I$. Now, $p(A_E \tilde{\cap} F_E)^- \tilde{\subseteq} p(A_E)^- \tilde{\cap} p(F_E)^- = p(A_E)^- \tilde{\cap} F_E = p(A_E)^- \tilde{\cap} F_E \setminus F_E^c$. Therefore $(A_E \tilde{\cap} F_E) \setminus G_E \tilde{\subseteq} p(A_E)^- \tilde{\cap} F_E) \setminus G_E \tilde{\cap} F_E^c \tilde{\subseteq} p(A_E)^- \setminus (G_E \tilde{\cup} F_E^c) \in I$. Hence $A_E \tilde{\cap} F_E$ is soft I_{pg}-closed. □

Definition 3.15. *Let $(X, \tilde{\tau})$ be a soft topological space and $F_A \tilde{\subseteq} F_E$. Then the soft pre generalized closed set with respect to soft ideal closure(soft I_{pg}-closure)of F_A denoted by $I_{pg}(F_A)^-$ is defined as the soft intersection of all soft I_{pg}-closed supersets of F_A. Note that, $I_{pg}(F_A)^-$ is the soft smallest soft I_{pg}-closed set containing F_A.*

Theorem 3.16. *Let $(X, \tilde{\tau})$ be a soft topological space and let F_A and F_B be a soft sets over X. Then*

(a). $F_A \tilde{\subseteq} I_{pg}(F_A)^-$

(b). F_A *is soft* I_{pg}*-closed iff* $F_A = I_{pg}(F_A)^-$

(c). $F_A \tilde{\subseteq} F_B$*, then* $I_{pg}(F_A)^- \tilde{\subseteq} I_{pg}(F_B)^-$

(d). $I_{pg}(F_\phi)^- = F_\phi$ *and* $I_{pg}(F_E)^- = F_E$

(e). $I_{pg}(F_A \tilde{\cap} F_B)^- = I_{pg}(F_A)^- \tilde{\cap} I_{pg}(F_B)^-$

(f). $I_{pg}(F_A \tilde{\cup} F_B)^- = I_{pg}(F_A)^- \tilde{\cup} I_{pg}(F_B)^-$

(g). $I_{pg}(I_{pg}(F_A)^-)^- = I_{pg}(F_A)^-$.

Theorem 3.17. *Let $Y \tilde{\subseteq} X$ and $F_E \tilde{\subseteq} \tilde{Y} \tilde{\subseteq} \tilde{X}$. Suppose that F_E is soft I_{pg}-closed in $(X, \tilde{\tau})$. Then F_E is soft I_{pg}-closed relative to the soft topolgical subspace $\tilde{\tau}_Y$ of X and with respect to the soft ideal $I_Y = \{H_E \tilde{\subseteq} \tilde{Y} : H_E \in I\}$.*

Proof. Suppose that $F_E \tilde{\subseteq} B_E \tilde{\cap} \tilde{Y}$ and B_E is soft pre open . So $B_E \tilde{\cap} \tilde{Y} \in \tilde{\tau}_Y$ and $F_E \tilde{\subseteq} B_E$. Since F_E is soft I_{pg}-closed in $(X, \tilde{\tau})$, then $p(F_E)^- \setminus B_E \in I$. Now, $(p(F_E)^- \tilde{\cap} \tilde{Y}) \setminus (B_E \tilde{\cap} \tilde{Y}) = (p(F_E)^- \setminus B_E) \tilde{\cap} \tilde{Y} \in I_Y$ whenever F_E is soft I_{pg}-closed relative to the subspace $(Y, \tilde{\tau}_Y)$. □

4. Soft Pre Generalized Open Sets With Respect To Soft Ideal

Definition 4.1. *A soft set F_E is called soft pre generalized open set with respect to a soft ideal I (soft I_{pg}-open) in a soft topological space $(X, \tilde{\tau})$ iff the complement F_E^c is soft I_{pg}-closed in $(X, \tilde{\tau})$.*

Theorem 4.2. *A soft set A_E is soft I_{pg}-open in a soft topological space $(X, \tilde{\tau})$ iff $F_E \setminus B_E \tilde{\subseteq} p(A_E)^o$ for some $B_E \in I$ whenever $F_E \tilde{\subseteq} A_E$ and F_E is soft pre closed in $(X, \tilde{\tau})$.*

Proof. Suppose that A_E is soft I_{pg}-open . Let $F_E \tilde{\subseteq} A_E$ and F_E is soft pre closed . we have $A_E^c \tilde{\subseteq} F_E^c$, A_E^c is soft I_{pg}-closed and F_E^c is soft pre open . By assumption, $p(A_E^c)^- \setminus F_E^c \in I$. Hence $p(A_E^c)^- \tilde{\subseteq} F_E^c \tilde{\cup} B_E$ for some $B_E \in I$. So $(F_E^c \tilde{\cup} B_E)^c \tilde{\subseteq} p((A, E^c)^-)^c = p(A_E)^o$ and therefore $F_E \setminus B_E = F_E \tilde{\cap} B_E^c \tilde{\subseteq} p(A_E)^o$.
Conversely, assume that $F_E \tilde{\subseteq} A_E$ and F_E is soft pre closed. These imply that $F_E \setminus B_E \tilde{\subseteq} p(A_E)^o$ for some $B_E \in I$. Consider G_E is soft pre open such that $A_E^c \tilde{\subseteq} G_E$. Then $G_E^c \tilde{\subseteq} A_E$. By assumption $G_E^c \setminus B_E \tilde{\subseteq} p(A_E)^o = (p(A_E^c)^-)^c$ for some $B_E \in I$. This gives that $(G_E \tilde{\cup} B_E)^c \tilde{\subseteq} (p(A_E^c)^-)^c$. Then $p(A_E^c)^- \tilde{\subseteq} G_E \tilde{\cup} B_E$ for some $B_E \in I$. This shows that $p(A_E^c)^- \setminus G_E \in I$. Hence $A_E^c$is soft I_{pg}-closed and therefore A_E is soft I_{pg}-open. □

Theorem 4.3. *If A_E and B_E are soft separated and soft I_{pg}-open sets in a soft topological space $(X, \tilde{\tau})$ then $A_E \tilde{\cup} B_E$ is soft I_{pg}-open in $(X, \tilde{\tau})$.*

Proof. Suppose that A_E and B_E are soft separated I_{pg}-open sets in $(X, \tilde{\tau})$ and F_E is soft pre closed subset of $A_E \tilde{\cup} B_E$. Then $F_E \tilde{\cap} A_E^- \tilde{\subseteq} A_E$ and $F_E \tilde{\cap} B_E^- \tilde{\subseteq} B_E$. By Theorem 4.2, $(F_E \tilde{\cap} A_E^-) \setminus D_E \tilde{\subseteq} p(A_E)^o$ and $(F_E \tilde{\cap} B_E^-) \setminus C_E \tilde{\subseteq} p(B_E)^o$ for some D_E,$C_E \in I$. This means that $(F_E \tilde{\cap} A_E^-) \setminus p(A_E)^o \in I$ and $(F_E \tilde{\cap} B_E^-) \setminus p(B_E)^o \in I$. Then $((F_E \tilde{\cap} A_E^-) \setminus p(A_E)^o) \tilde{\cup} ((F_E \tilde{\cap} B_E^-) \setminus p(B_E)^o) \in I$. Hence $(F_E \tilde{\cap} (A_E^- \tilde{\cup} B_E^-)) \setminus p(A_E^o \tilde{\cup} p(B_E)^o) \in I$. But $F_E = F_E \tilde{\cap} (A_E \tilde{\cup} B_E) \tilde{\subseteq} F_E \tilde{\cap} (A_E \tilde{\cup} B_E)^-$ and we have $F_E \setminus p(A_E \tilde{\cup} B_E)^o \tilde{\subseteq} (F_E \tilde{\cap} (A_E \tilde{\cup} B_E)^-) \setminus p(A_E^o \tilde{\cup} B_E^o) \in I$. Hence $F_E \setminus H_E \tilde{\subseteq} p(A_E^o \tilde{\cup} B_E^o)$ for some $H_E \in I$. This proves that $A_E \tilde{\cup} B_E$ is soft I_{pg}-open. □

Corollary 4.4. *If A_E and B_E are soft I_{pg}-open set in a soft topological space $(X,\ \tilde{\tau})$, then $A_E \tilde{\cap} B_E$ is soft I_{pg}-open in $(X,\ \tilde{\tau})$.*

Proof. If A_E and B_E are soft I_{pg}-open, then A_E^c and B_E^c are soft I_{pg}-closed. By Theorem 3.10, $(A_E \tilde{\cap} B_E)^c = A_E^c \tilde{\cup} B_E^c$ is soft I_{pg}-closed, which implies $A_E \tilde{\cup} B_E$ is soft I_{pg}-open. □

Remark 4.5. *Let A_E and B_E be soft I_{pg} closed sets and suppose that A_E^c and B_E^c are soft separated in a soft topological space $(X,\ \tilde{\tau})$. Then $A_E \tilde{\cap} B_E$ is soft I_{pg}-closed in $(X,\ \tilde{\tau})$.*

Theorem 4.6. *Let $Y \subseteq X$ and $A_E \tilde{\subseteq} \tilde{Y} \tilde{\subseteq} \tilde{X}$, A_E is soft I_{pg}-open in $(Y,\ \tilde{\tau}_Y)$ and $\tilde{Y}$ is soft I_{pg}-open in $(X,\ \tilde{\tau})$. Then A_E is soft I_{pg}-open in $(X,\ \tilde{\tau})$.*

Proof. Suppose that $A_E \tilde{\subseteq} \tilde{Y} \tilde{\subseteq} \tilde{X}$, A_E is soft I_{pg}-open in $(Y,\ \tilde{\tau}_Y)$ and $\tilde{Y}$ is soft I_{pg}-open in $(X,\ \tilde{\tau})$. We show that A_E is soft I_{pg}-open in $(X,\ \tilde{\tau})$. Suppose that $F_E \tilde{\subseteq} A_E$ and F_E is soft pre closed. Since A_E is soft I_{pg}-open relative to $\tilde{Y}$, by Theorem 4.2 , $F_E \setminus D_E \tilde{\subseteq} p(A_E)^o$ for some $D_E \in I_Y$. This implies that there exists a soft pre open set G_E such that $F_E \setminus D_E \tilde{\subseteq} G_E \tilde{\cap} \tilde{Y} \tilde{\subseteq} A_E$ for some $D_E \in I$. Then $F_E \tilde{\subseteq} \tilde{Y}$ and F_E is soft pre closed. Since $\tilde{Y}$ is soft I_{pg}-open, then $F_E \setminus H_E \tilde{\subseteq} p(\tilde{Y})^o$ for some $H_E \in I$. This implies that there exists a soft pre open set K_E such that $F_E \setminus H_E \tilde{\subseteq} K_E \tilde{\subseteq} \tilde{Y}$ for some $H_E \in I$. Now, $F_E \setminus (D_E \tilde{\cup} H_E) = (F_E \setminus D_E) \tilde{\cap} (F_E \setminus H_E) \tilde{\subseteq} G_E \tilde{\cap} K_E \tilde{\subseteq} G_E \tilde{\cap} \tilde{Y} \tilde{\subseteq} A_E$. This implies that $F_E \setminus (D_E \tilde{\cup} H_E) \tilde{\subseteq} p(A_E)^o$ for some $D_E \tilde{\cup} H_E \in I$ and hence A_E is soft I_{pg}-open in $(X,\ \tilde{\tau})$. □

Theorem 4.7. *If $A_E^o \tilde{\subseteq} B_E \tilde{\subseteq} A_E$ and A_E is soft I_{pg}-open in a soft topological space $(X,\ \tilde{\tau})$, then B_E is soft I_{pg}-open in $(X,\ \tilde{\tau})$.*

Proof. Suppose that $A_E^o \tilde{\subseteq} B_E \tilde{\subseteq} A_E$ and A_E is soft I_{pg}-open. Then $A_E^c \tilde{\subseteq} B_E^c \tilde{\subseteq} (A_E^c)^-$ and A_E^c is soft I_{pg}-closed. By Theorem 3.12, B_E^c is soft I_{pg}-closed and hence B_E is soft I_{pg}-open. □

Definition 4.8. *Let $(X,\ \tilde{\tau})$ be a soft topological space and $F_A \tilde{\subseteq} F_E$. Then the soft pre generalized closed set with respect to soft ideal interior(soft I_{pg}-interior)of F_A denoted by $I_{pg}(F_A)^o$ is defined as the soft union of all soft I_{pg}-open subsets of F_A. Note that, $I_{pg}(F_A)^o$ is the soft biggest soft I_{pg}-open set that contained in F_A.*

Theorem 4.9. *Let $(X,\ \tilde{\tau})$ be a soft topological space and let F_A and F_B be a soft sets over X. Then*

(a). $I_{pg}(F_A)^o \tilde{\subseteq} F_A$

(b). F_A *is soft* I_{pg}*-open iff* $F_A = I_{pg}(F_A)^o$

(c). $F_A \tilde{\subseteq} F_B$, *then* $I_{pg}(F_A)^o \tilde{\subseteq} I_{pg}(F_B)^o$

(d). $I_{pg}(F_\phi)^o = F_\phi$ *and* $I_{pg}(F_E)^o = F_E$

(e). $I_{pg}(F_A \tilde{\cap} F_B)^o = I_{pg}(F_A)^o \tilde{\cap} I_{pg}(F_B)^o$

(f). $I_{pg}(F_A \tilde{\cup} F_B)^o \tilde{\supseteq} I_{pg}(F_A)^o \tilde{\cup} I_{pg}(F_B)^o$

(g). $I_{pg}(I_{pg}(F_A)^o)^o = I_{pg}(F_A)^o$

(h). *(i)*$(I_{pg}(F_A)^-)^c = I_{pg}(F_A^c)^o$ *(ii)*$(I_{pg}(F_A)^o)^c = I_{pg}(F_A^c)^-$

Theorem 4.10. *A soft set A_E is soft I_{pg}-closed in a soft topological space $(X,\ \tilde{\tau})$ iff $p(A_E)^- \setminus A_E$ is soft I_{pg}-open.*

Proof. Suppose that $F_E \tilde{\subseteq} p(A_E)^- \setminus A_E$ and F_E is soft pre closed. Then $F_E \in I$ and this implies that $F_E \setminus D_E = \phi$ for some $D_E \in I$. Clearly, $F_E \setminus D_E \tilde{\subseteq} p(A_E^- \setminus A_E)^o$. By Theorem 4.2, $p(A_E)^- \setminus A_E$ is soft I_{pg}-open.

Conversely, suppose that $A_E \tilde{\subseteq} G_E$ and G_E is soft pre open in $(X, \tilde{\tau})$. Then, $p(A_E)^- \tilde{\cap} G_E^c \tilde{\subseteq} p(A_E)^- \tilde{\cap} A_E^c = p(A_E)^- \setminus A_E$. By hypothesis, $p(A_E)^- \tilde{\cap} G_E^c \setminus D_E \tilde{\subseteq} (p(A_E)^- \setminus A_E)^o = \phi$, for some $D_E \in I$. This implies that $p(A_E)^- \tilde{\cap} G_E^c \tilde{\subseteq} D_E \in I$ and therefore $p(A_E)^- \setminus G_E \in I$. Thus, A_E is soft I_{pg}-closed. □

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