International Journal of Mathematics And its Applications
Volume 4, Issue 1–C (2016), 65–68.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Balanced Cordial Labeling and its Application to Produce new Cordial Families

Research Article

V.J.Kaneria[1], Kalpesh M.Patadiya[2]* and Jaydev R.Teraiya[3]

1 Department of Mathematics, Saurashtra University, Rajkot, India.
2 Department of Science and Humanities, School of Engineering, RK University, Rajkot, India.
3 Department of Mathematics, Marwadi Engineering College, Rajkot, India.

Abstract: In this paper we have introduced a balanced cordial labeling for a graph G, which is a cordial labeling f with condition $e_f(0) = e_f(1), v_f(0) = v_f(1)$. We proved that $P_n \times C_{4t}, C_n \times C_{4t}$ (n is even) are balanced cordial graphs. We also proved that the corona graph $G_1 \odot G_2$ is cordial, when G_1 a cordial graph and G_2 is a balanced cordial graph.

MSC: 05C78.

Keywords: Binary vertex labeling, balanced cordial graph, corona graph.

1. Introduction

Labeled graph has many diversified applications. Gallian [3] survey provides vast amount of literature with comprehensive bibiliography of papers on different types of graph labeling. The cordial labeling introduced by Cahit [1] is a weaker version of graceful and harmonious labeling. Also he proved that the complete graph K_n is cordial if and only if $n \leq 3$. After this, many researchers have studied cordial graph and similar type graph labeling. Kaneria and Vaidya [5] discussed cordiality of graphs in different context. They introduced the index of cordiality for a graph G and proved that it is precisely 2 for K_{t^2}, $\forall\, t \in \mathbb{N}$. Kaneria, Makadia and Meera [4] proved that $C(n \cdot K_n)$ and K_n^* are cordial graphs, $\forall n \in \mathbb{N}$. All graphs in this paper are finite, simple and undirected. For a graph G, we take $p = |V(G)|$, number of vertices and $q = |E(G)|$, number of edges in G.

We follow Harary [2] for the basic notation and terminology of graph theory. A function $f : V(G) \to \{0, 1\}$ is called a binary vertex labeling of a graph G. $f(v)$ is called label of the vertex v of G under f . For any edge $e = (u, v) \in E(G)$, the edge induced function $f^* : E(G) \to \{0, 1\}$ defined as $f^*(e) = |f(u) - f(v)|$. Let $v_f(0)$, $v_f(1)$ be the number of vertices of G having 0 and 1 vertex label respectively under f and $e_f(0)$, $e_f(1)$ be the number of edges of G having 0 and 1 edge labels respectively under f^*. A binary vertex labeling function f of a graph G is called cordial labeling if $|e_f(0) - e_f(1)| \leq 1$ and $|v_f(0) - v_f(1)| \leq 1$. A graph G is called a cordial graph if it admits a cordial labeling. A cordial graph G with a cordial labeling f is called a balance cordial graph if $|e_f(0) - e_f(1)| = |v_f(0) - v_f(1)| = 0$. It is said to be edge balanced cordial graph

* E-mail: kalpesh.patadiya@rku.ac.in

if $|e_f(0) - e_f(1)| = 0$ and $|v_f(0) - v_f(1)| = 1$. Similarly it is said to be vertex balanced cordial graph if $|e_f(0) - e_f(1)| = 1$ and $|v_f(0) - v_f(1)| = 0$. A cordial graph G is said to be unbalanced cordial graph if $|v_f(0) - v_f(1)| = |e_f(0) - e_f(1)| = 1$.

For any cordial graph G, if f is a cordial labeling of G and it has one of above four categories, then $1 - f$ is also a cordial labeling function for G and it has same one of above four categories. The corona graph $G_1 \odot G_2$ is obtained from two graphs G_1 and G_2, by taking one copy of G_1 and $|V(G_1)|$ copies of G_2 and join the i^{th} vertex of G_1 with all the vertices in the i^{th} copy $G_2^{(i)}$ of G_2 by an edge. The corona graph $C_5 \odot P_3$ is shown in following Figure 1.

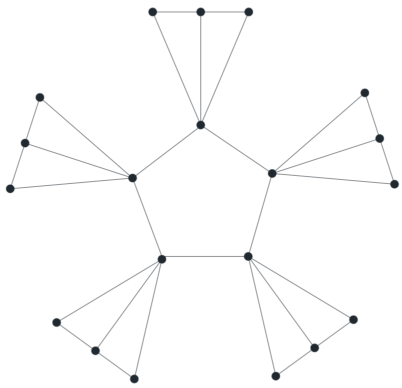

Figure 1. **Corona graph $C_5 \odot P_3$ with $|V(C_5 \odot P_3)| = 20$ and $|E(C_5 \odot P_3)| = 30$**

2. Main Results

Theorem 2.1. *$P_n \times C_{4t}$ is a balanced cordial graph, where $t, n \in \mathbb{N}$.*

Proof. Let $v_{ij} (1 \le j \le 4t)$ be the vertices of i^{th} copy $C_{4t}^{(i)}$ in $P_n \times C_{4t}$, $\forall\ i = 1, 2, ..., n$. It is obvious that the vertex labeling function $f_1 : V(C_{4t}^{(1)}) \to \{0, 1\}$ defined by

$$\begin{aligned} f_1(u_{1,j}) &= 0;\ \text{if}\ j \equiv 0, 1 (mod\ 4), \\ &= 1;\ \text{if}\ j \equiv 2, 3 (mod\ 4),\ \forall\ j = 1, 2, ..., 4t; \end{aligned}$$

is a cordial labeling for $C_{4t}^{(1)}$ and it is a balanced cordial graph. For each $i = 1, 2, ..., n-1$ we join $u_{i,j}$ with $u_{i+1,j}, \forall\ j = 1, 2, ..., 4t$ by an edge to form the graph $P_n \times C_{4t}$. Define $f_2 : V(C_{4t}^{(2)}) \to \{0, 1\}$ as follows.

$$\begin{aligned} f_2(u_{2,j}) &= 0;\ \text{if}\ j \equiv 1, 2 (mod\ 4), \\ &= 1;\ \text{if}\ j \equiv 0, 3 (mod\ 4),\ \forall\ j = 1, 2, ..., 4t; \end{aligned}$$

It is obvious that above defined labeling function f_2 on $C_{4t}^{(2)}$ is also cordial labeling and $C_{4t}^{(2)}$ is also a balanced cordial graph. Now define $f : V(P_n \times C_{4t}) \to \{0, 1\}$ as follows.

$$\begin{aligned} f(u_{i,j}) &= f_1(u_{1,j})\ \ \text{if i is odd} \\ &= f_2(u_{2,j})\ \ \text{if i is even}\ \ \forall\ i = 1, 2, ..., n,\ \ \forall\ j = 1, 2, ..., 4t. \end{aligned}$$

For each i = 1,2,...,n, it is observe that

$$\begin{aligned} f^*((u_{i,j}, u_{i+1,j})) &= |f(u_{i,j}) - f(u_{i+1,j})| \\ &= (\frac{1}{2})(-1)^j + \frac{1}{2}, \qquad \forall\ j = 1, 2, ..., 4t. \end{aligned}$$

Thus, $|e_f(0)-e_f(1)|=0$ in $P_n \times C_{4t}$. Since $V(P_n \times C_{4t}) = \bigcup_{i=1}^{n} V(C_{4t}^{(i)})$, it is observed that $|v_f(0)-v_f(1)|=0$ in $P_n \times C_{4t}$. Hence, $P_n \times C_{4t}$ is a balanced cordial graph. □

Illustration 2.2. *$P_5 \times C_4$ and its balanced cordial labeling are shown in Figure 2.*

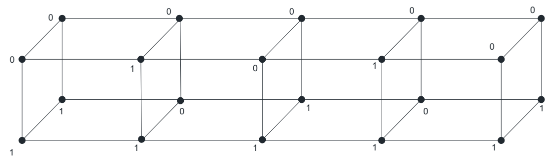

Figure 2. $P_5 \times C_4$ **with its cordial labeling** f **and** $v_f(0)=v_f(1)=10$ **as well as** $e_f(0)=e_f(1)=18$**.**

Theorem 2.3. *$C_n \times C_{4t}$ is a balanced cordial graph, where $t \in \mathbb{N}$, n is even and $n \geq 4$.*

Proof. Let $u_{i,j}(1 \leq j \leq 4t)$ be the vertices of i^{th} copy $C_{4t}^{(i)}$ in $C_n \times C_{4t}$, $\forall\ i = 1,2,\ldots,n$. It is obvious that the vertex labeling function $f_1 : V(C_{4t}^{(1)}) \to \{0,1\}$ defined by

$$\begin{aligned} f_1(u_{i,j}) &= 0;\ \text{if}\ j \equiv 0,1(mod\ \ 4), \\ &= 1;\ \text{if}\ j \equiv 2,3(mod\ \ 4),\ \ \forall\ j = 1,2,...,4t; \end{aligned}$$

is a cordial labeling function for $C_{4t}^{(1)}$ and it is a balanced cordial graph. For each $i = 1,2,...,n$, we join $u_{i,j}$ with $u_{i+1,j}$, $\forall\ j = 1,2,...,4t$ and $u_{n,j}$ with $u_{1,j}$, $\forall\ j = 1,2,...,4t$ by an edge to form the graph $C_n \times C_{4t}$. Define $f_2 : V(C_{4t}^{(2)}) \to \{0,1\}$ as follows.

$$\begin{aligned} f_2(u_{2,j}) &= 0;\ \text{if}\ j \equiv 1,2(mod\ \ 4), \\ &= 1;\ \text{if}\ j \equiv 0,3(mod\ \ 4),\ \ \forall\ j = 1,2,...,4t; \end{aligned}$$

It is obvious that above defined labeling function f_2 on $C_{4t}^{(2)}$ is also a labeling and $C_{4t}^{(2)}$ is a balanced cordial graph. Now defined $f : V(C_n \times C_{4t}) \to \{0,1\}$ as follows.

$$\begin{aligned} f(u_{i,j}) &= f_1(u_{1,j})\ \text{ if i is odd}, \\ &= f_2(u_{2,j})\ \text{ if i is even},\ \ \forall\ i = 1,2,...,n. \end{aligned}$$

For each i = 1,2,...,n , it is observe that

$$\begin{aligned} f^*((u_{i,j},u_{i+1,j})) &= |f(u_{i,j}) - f(u_{i+1,j})| \\ &= (\frac{1}{2})(-1)^j + \frac{1}{2}, \qquad \forall\ j = 1,2,...,4t; \end{aligned}$$

and

$$\begin{aligned} f^*((u_{n,j},u_{1,j})) &= |f(u_{n,j}) - f(u_{1,j})| \\ &= (\frac{1}{2})(-1)^j + \frac{1}{2}, \qquad \forall\ j = 1,2,...,4t; \end{aligned}$$

Thus, $|e_f(0)-e_f(1)|=0$ in $C_n \times C_{4t}$. Since $V(C_n \times C_{4t}) = \bigcup_{i=1}^{n} V(C_{4t}^{(i)})$, it is observed that $|v_f(0)-v_f(1)|=0$ hold in $C_n \times C_{4t}$. Therefore, it is a balanced cordial graph. □

Theorem 2.4. *The corona graph $G_1 \odot G_2$ is cordial, when G_1 is cordial graph and G_2 is a balanced cordial graph.*

Proof. Let $G_1 \odot G_2$, a corona graph obtained by two cordial graphs G_1 and G_2, among G_2 is a balanced cordial graph. Let $|V(G_1)| = p_1, |V(G_2)| = p_2, f_1$ be a cordial labeling function for G_1 and f_2 be a balanced cordial labeling function for G_2. It is obvious that $|e_{f_1}(0) - e_{f_1}(1)| \leq 1, |v_{f_1}(0) - v_{f_1}(1)| \leq 1$ in G_1 and $|e_{f_2}(0) - e_{f_2}(1)| = |v_{f_2}(0) - v_{f_2}(1)| = 0$ in G_2. Also $|V(G)| = p_1(p_2 + 1)$ and $|E(G)| = (p_2 + |E(G_2)|) + |E(G_1)|$. Now define $f : V(G) \to \{0, 1\}$ as follows.

$$\begin{aligned} f(x) &= f_1(x) \text{ if } x \in V(G_1) \\ &= f_2(x) \text{ if } x \in V(G_2^{(i)}), \ \forall\ i = 1, 2, ..., p_1. \end{aligned}$$

Let $V(G_1) = \{u_1, u_2, ..., u_{p_1}\}$ be the vertex set for G_1. For each $i = 1, 2, ..., p_1$ join u_i with all the vertices of $G_2^{(i)}$ by an edge to form corona graph $G_1 \odot G_2$ and each u_i, i^{th} copy $G_2^{(i)}$ produce p_2 edges, among half edges got 0 edge label and rest got 1 edge label, as $v_{f_2}(0) = v_{f_2}(1)$. Therefore,

$$\begin{aligned} e_f(0) &= p_1 e_{f_2}(0) + e_{f_1}(0) + p_1 v_{f_2}(0), \\ e_f(1) &= p_1 e_{f_2}(1) + e_{f_1}(1) + p_1 v_{f_2}(1), \\ v_f(0) &= v_{f_1}(0) + p_1 v_{f_2}(0) \text{ and} \\ v_f(1) &= v_{f_1}(1) + p_1 v_{f_2}(1). \end{aligned}$$

Thus, $|e_f(0) - e_f(1)| = |e_{f_1}(0) - e_{f_1}(1)|$ and $|v_f(0) - v_f(1)| = |v_{f_1}(0) - v_{f_1}(1)|$. So, above defined labeling pattern give rise cordial labeling to the corona graph $G_1 \odot G_2$, as G_1 is a cordial graph. Hence $G_1 \odot G_2$ is cordial. □

Corollary 2.5. *Corona graphs $C_{4n+1} \odot C_{4t}$ and $C_{4n-1} \odot C_{4t}$ are unbalanced cordial graphs,* $\forall\ t, n \in \mathbb{N}$.

Corollary 2.6. *Corona graph $C_{4n} \odot C_{4t}$ is a balanced cordial graph,* $\forall\ t, n \in \mathbb{N}$.

References

[1] I.Cahit, *Cordial graphs: A weaker version of graceful and harmonious graphs*, Ars Combin., 23(1987), 201-207.

[2] F.Harary, *Graph Theory*, Addition Wesley, Massachusetts, (1972).

[3] J.A.Gallian, *A dynamic Survey of graph labeling*, The electronics J. Combin., 17(2014), ♯ DS6.

[4] V.J.Kaneria, Hardik Makadia and Meera Meghpara, *Cordiality of star of the complete graph and a cycle graph $C(n \cdot K_n)$*, J. of Math. Research, 6(4)(2014), 18-28.

[5] V.J.Kaneria and S.K.Vaidya, *Index of cordiality for complete graphs and cycle*, IJAMC, 2(4)(2010), 38-46.