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Construction of Two Special Integer Triples

Research Article

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Abstract: This paper concerns with the study of constructing a special non zero integer triple (a, b, c) such that the product of any two elements of the set added with the other is a perfect square. Also, the product of any two elements of the set added with the square of the other is a perfect square.

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1. Introduction

Number theory is that branch of Mathematics which deals with properties of the natural numbers 1, 2, 3,. . . . also called the positive numbers. These numbers together with the negative numbers an zero form the set of integers. Properties of these integers have been studied since antiquity. Number theory is an out enjoyable and pleasing to everybody. It has fascinated and inspired both armatures and mathematicians alike. Diophantine problems have fewer equations than unknown variables and involve finding integers that work correctly for all equations. Certain Diophantine problems come from physical problems or from immediate mathematical generalizations and others come from geometry in a variety of ways. Certain Diophantine problems are neither trivial nor difficult to analyze [1–9].

In this context one may refer [10–13]. This paper consists of two sections A and B. In section A, we search for a special non zero integer triple (a, b, c) such that the product of any two elements of the set added with the other is a perfect square.

In section B, we search for a non zero integer triple (a, b, c) such that, the product of any two elements of the set added with the square of the other is a perfect square.

2. Method of Analysis

Section A:

Let

$$a = x, \quad b = x + 4k + 4, \quad c = (2k + 2)^2, \quad (x, k \in Z - \{0\}) \tag{1}$$

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be such that

$$ab + c = (x + 2k + 2)^2 \tag{2}$$

$$ac + b = [(2k + 2)^2 + 1]x + 4k + 4 = \alpha^2 \tag{3}$$

$$bc + a = [(2k + 2)^2 + 1]x + (4k + 4)(2k + 2)^2 = \beta^2 \tag{4}$$

Subtracting (3) from (4), we get

$$\beta^2 - \alpha^2 = (4k + 4)[(2k + 2)^2 - 1]$$

which is satisfied by

$$\alpha = 4k^3 + 12k^2 + 11k + 2, \beta = 4k^3 + 12k^2 + 11k + 4$$

Substituting the values of $\alpha$and β either in (3) or (4), we have

$$x = 4k^4 + 16k^3 + 21k^2 + 8k \tag{5}$$

In view of (1), the non zero distinct integral values of a, b, c are given by

$$a = 4k^4 + 16k^3 + 21k^2 + 8k$$

$$b = 4k^4 + 16k^3 + 21k^2 + 12k + 4$$

$$c = 4k^2 + 8k + 2$$

Note that the above values of a, b, c satisfying (2) to (3). A few numerical examples are given below. A few interesting

k	a	b	C	ab+c	bc+a	ca+b
1	49	57	16	2809	961	841
2	292	304	36	88804	11236	10816
3	969	985	64	954529	64009	63001
6	9444	9472	196	89453764	1865956	1860496
7	16177	16209	256	262273249	4165681	4157527

properties between a, b, c are presented below.

(1). $a + c =$ sum of two squares

(2). $b - a + c + 1$ is a square.

(3). $5b - 5a + c + 5k^2 + 2k + 1$ is a perfect square.

(4). $b - c + 3k^2 + 4k + 1$ is a perfect square.

(5). $6(b - a + k^2)$ is a nasty number.

(6). $\frac{c+b-a-8(k+1)}{4}$ is a triangular number of rank k.

(7). $c + b - a \equiv 0(mod4)$

Section B:

Let $a = x$, $b = 4x + 4k$, $c = k$, $(x, k \in Z - \{0\})$ be any three non zero distinct integers. It is observed that each of the expressions $ab + c^2$, $bc + a^2$ is a perfect square. Now $ac + b^2 = 16x^2 - 33xk + 16k^2$. Assume $ac + b^2 = (4x - N)^2$. From the above two equations, we obtain

$$x = \frac{N^2 - 16k^2}{33k + 8N}$$

Substituting the values of x in (1) and performing a simple algebra, the integer values of a, b, c satisfy the required criteria are represented by

$$a = N^2 - 16k^2$$
$$b = 4(N^2 - 16k^2) + 4k(33k + 8N)$$
$$c = k(33k + 8N)$$

A few numerical examples are given below.

N	k	a	b	c	$ab + c^2$	$bc + a^2$	$ca + b^2$
5	1	9	328	73	8281	24025	108241
5	2	-39	692	212	17956	148225	470596
11	2	57	1460	308	178084	452929	2149456
20	3	256	4132	777	1661521	3276100	17272336
25	6	49	9748	2388	6180196	23280625	95140516

Remark 2.1. *It is worth mentioning here that, apart from the values of k and N presented in the above table, for which the value of a is a perfect square the other choices of k and N for a to be a perfect square are as follows:*

(1). $k = rs, N = 4r^2 + s^2$.

(2). $k = 2(r^2 - s^2)$, $N = 4(r^2 - s^2)$.

3. Conclusion

In this paper, we have illustrated methods of obtaining three non-zero distinct integers a, b, c such that A) each of the expressions $ab + c$, $ac + b$, $bc + a$ is a perfect square and B) each of the expressions $ab + c^2$, $ac + b^2$, $bc + a^2$ is a perfect square. As Diophantine problems are rich in variety, one may attempt to construct triples whose elements satisfy other characterization.

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