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Obscuring Instability in C Language Using Euler's Modified Method

Research Article

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Abstract: The solutions of ordinary Differential equations having initial value problems are analyzed numerically. Computationally effective numerical methods, Euler's method is given meticulously while Euler's Modified method presented here gives a better exactness. No higher order derivatives are required to be tabulated. Drawbacks and advantages of the methods are discussed here and the results obtained by them have been compared. The idea behind this paper is to illustrate the facts of implementing a small number of steps of Euler's modified method, in addition, how to apply built-in functions available in C^{++} [1]. Firstly, we use Euler methods to bring in the fundamental thoughts linked with initial value problems (IVP) and then later on, we apply the Euler's Modified method associated to the built-in C^{++} . The purpose of this paper is to go from textbook formula on ODE to production software.

Keywords: Euler's Method, Euler's Modified Method, C^{++}, ODE.

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1. Introduction

The system which changes can be illustrated with the help of Differential equations. The nature of these equations have been understood and calculated by many scientists and many complex problems can relatively be described accurately with numerical terminology. The general solution of an ordinary differential equation of the n^{th} order contains n constant. To obtain its particular solution, the conditions are described at one point only or at two or more points. Based on this the problem of solving ordinary differential equations is categorized into initial value and boundary value problems. In this paper, we shall describe methods for solving initial value problems in ordinary differential equations such as Euler's method and modified Euler's method respectively. Many exceptional texts on this subject that may be studied, such as [1, 6, 9, 10, 12]. It is significant to specify that the center of attention of this paper here is on the practicality of numerical methods with the purpose to solve some distinctive problems and not to present any regular theoretical background, such as [1–4, 8, 11, 12].

2. Euler's Method

Euler's method is an elementary and purely numerical method for solving initial value problem in first order ordinary differential equation. Consider the first order ordinary differential equation

$$\frac{dy}{dx} = f(x, y) \tag{1}$$

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Let the solution of (1) be

$$y = F(x) \tag{2}$$

Let different points (x_0, y_0), (x_1, y_1), (x_2, y_2),..., (x_n, y_n) be on the curve (2) and all the points x_0, x_1, x_2,..., x_n are equi spaced having equal interval h. If (x_{n+1}, y_{n+1}) lies on (2) then $y_{n+1} = F(x_{n+1})$. As $x_{n+1} = x_n + h$, therefore $y_{n+1} = F(x_n + h)$. Using Taylor's series

$$y_{n+1} = F(x_n) + hF'(x_n) + \frac{1}{2}\, h^2 F''(x_n) + \dots \tag{3}$$

As h is very small $y_{n+1} = F(x_n) + hF'(x_n)$

$$y_{n+1} = F(x_n) + hf(x_n, y_n) \tag{4}$$

This formula (4) can be used to find y_{n+1} where y_n is known. Equation (4) is called Euler's method.

3. Drawbacks in Euler's Method

The input information is rarely exact since it comes from some measurement or the other and the method also introduces further error [3] and [7]. Rounding errors arise from the process of rounding off the numbers during the computation and Truncation errors are caused by using approximate results or on replacing an infinite process by a finite one. Since we have neglected $\frac{1}{2}h^2F''(x_n)$ and higher powers of h from formula (3) there will be a larger truncation error in y_{n+1}. Therefore Euler's method is not used in practical problems.

3.1. Geometrical Meaning

Let PQ represents the curve of solution $y = F(x)$. The ordinate of P i.e. y_n is known. Then Ordinate Q i.e,

$$\begin{aligned} y_{n+1} = MQ = MR + RQ = PL + RT + TQ \ \ &(\text{TQ Error}) \\ = y_n + h(\tan\ x_n)& \\ = y_n + h(\frac{dy}{dx}) = y_n + hf(x_n, y_n)& \end{aligned}$$

4. Modified Euler's Method

In the Euler's method, the curve of the solution in the interval PQ is approximated by the tangent at P such that at Q, we have

$$y_1 = y_0 + hf(x_0, y_0).$$

Then the slope of the curve of the solution through Q is computed and a tangent at Q is drawn meeting the ordinate through $x_0 + 2h$ in $(x_0 + 2h, y_2)$. Now we find a better approximation $y_1^{(1)}$ of $y(x_0 + h)$ by taking the slope of the curve as the mean of the slopes of the tangents at P and Q, i.e,

$$y_1^{(1)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_0 + h, y_1)] \tag{5}$$

As y_1 is not known so we take y_1 as found in usual Euler's method and insert it in (5) to obtain the first modified value $y_1^{(1)}$. Therefore modified Euler's method is given by $y_1 = y_0 \frac{1}{2}(k_1 + k_2)$, where $k_1 = hf(x_0, y_0)$ and $k_2 = hf(x_0 + h, y_0 + k_1)$. Similarly we proceed to calculate y_2 and y_3 and so on. This is the modified Euler's method which gives great improvement in accuracy over the original method.

5. Numerical Experiments

In order to verify the correctness of the methods for solution of initial value problems in ordinary differential equations, it was computerized in Fortran Programming language and implemented on a macro-computer adopting double precision arithmetic. On the basis of their accurateness and effectiveness the presentation of the methods was compared . The number of iterations decides the effectiveness whereas the size of interval decides the accuracy

5.1. The Problem Discussed Here is the Linear First Order Initial Value Problem, With Equal Interval

The problem is $F(x, y) = x^* x + y$. The results obtained shown in Table 1 and Table 2, the comparison of the methods to the exact solution and the error incurred respectively.

C language code:

```
#include<stdio.h>
#include <math.h>
#include<conio.h>
#define F(x,y) (x)*(x)+(y)
void main()
{
double y0,x0,y1,x1,y1_0,a,n,h,f,f1;
int j,count,flag;
clrscr();
printf(”\nEnter the value of x0: ”);
scanf(”%lf’,&x0);
printf(”\nEnter the value of y0: ”);
scanf(”%lf’,&y0);
printf(”\nEnter the value of h: ”);
scanf(”%lf’,&h);
printf(”\nEnter the value of last point: ”);
scanf(”%lf’,&n);
for(x1=x0+h,j=1; x1<=n+h; x1=x1+h,j++)
{
count=0;
flag=0;
f=F(x0,y0);
y1_0 = y0 + (h * f);
printf(”\n\n * * y%d_0 = %.3lf * *”,j,y1_0);
do
{
count++;
f=F(x0,y0);
f1=F(x1,y1_0);
y1 = y0 + h/2 * ( f + f1);
printf(”\n\n * * x = %.3lf => y%d_%d = %.3lf * *”,x1,j,count,y1);
if(fabs(y1-y1_0)<0.00001)
{
printf(”\n\n\n\n * * * * y%d = %.3lf * * * *\n\n”,j,y1);
flag=1;
}
else
y1_0 = y1;
}while(flag!=1);
y0 = y1;
}
getch();
}
```

6. Table of Results

The results are calculated by manually and by c programming also, show in Table 1:

S. No	Step Size	By Euler's Method	By Euler's Modified Method	Error Incurred
1	0.1	1.1	1.1105	0.0105
2	0.2	1.21	1.2432	0.0332
3	0.3	1.351	1.4004	0.0494
4	0.4	1.5161	1.5846	0.0685
5	0.5	1.7077	1.7988	0.0911
6	0.6	1.9385	2.0460	0.1075
7	0.7	2.2023	2.3298	0.1275
8	0.8	2.5025	2.6534	0.1509
9	0.9	2.8428	3.0227	0.1799
10	1.0	3.2271	3.4409	0.2138

Table 1.

7. Conclusion

Normally, on using Modified Euler's Method, it has its own advantages and disadvantages .Hence it is most excellent to set aside this method for those derivatives which can be shown with some terms only .Consequently. one can observe that this particular method gives a better approximate result with small step size. One can effortlessly get used to these C++ codes as required for an unlike problem. By means of numerical method, such as Euler's Modified method, one should intellectually check the question whether the consequences are precise enough to be useful. Also, it is widely used in solving initial value problems in ordinary differential equations.

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