International Journal of Mathematics And its Applications
Volume 4, Issue 1–C (2016), 137–142.
ISSN: 2347-1557
Available Online: http://ijmaa.in/

International Journal of Mathematics And its Applications

Regular and Totally Regular Intuitionistic Fuzzy Hypergraph(IFH)

Research Article

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Abstract: Regular and totally regular Intuitionistic fuzzy graph was first introduced by Nagoor gani.A and Radha.K. In this paper, we define regular and totally regular Intuitionistic fuzzy hypergraphs and discusses the size and order along with properties of the regular and totally regular Intuitionistic fuzzy hypergraphs. The work has been extended to completeness of Intuitionistic fuzzy hypergraphs

MSC: 05C65, 05C72.

Keywords: Intuitionistic fuzzy graph, Regular intuitionistic fuzzy graph, Totally Regular intuitionistic fuzzy graph,Degree, Total degree.

1. Introduction

Atanassov introduced the concept of intuitionistic fuzzy relations and intuitionistic fuzzy graphs. Among the various notions of higher-order fuzzy sets, IFS proposed by Atanossov provide a flexible framework to explain uncertainty and vagueness. This domain has recently motivated new research in several directions.Moderson J.N and P.S Nair gave the definition for fuzzy hypergraph.Parvathy.R and M.G.Karunambigai's paper introduced the concepts of IFH and analyzed its components. Nagoor Gani.A and Sajith Begum.S defined degree, order and size in Intuitionistic fuzzy graphs and extend the properties. Nagoor Gani.A and Latha.R introduced irregular fuzzy graphs and discussed some of its properties.

In this paper, we focus on regular and totally regular IFH.Also we proved the necessary and sufficient condition under which regular Intuitionistic fuzzy hypergraphs and totally regular Intuitionistic fuzzy hypergraphs are equivalent. Regular and Totally regular Intuitionistic fuzzy hypergraphs are compared through examples.

2. Preliminaries and Main Results

Definition 2.1. *A hyper graph H is an ordered pair $H = (X, E)$ where*

(a). $X = \{x_1, x_2, \ldots, x_n\}$ a finite set of vertices.

(b). $E = \{E_1, E_2, \ldots, E_m\}$ a family of subsets of V.

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(c). $E_j \neq \phi$, $j = 1, 2, \ldots, m$ *and* $\bigcup_j E_j = X$.

The set X is called the set of vertices and E is the set of edges (or hyper edges).

In a hypergraph two or more vertices $x_1, x_2, \ldots, x_n$ are said to be adjacent if there exist an edge E_j which contains those vertices. In a hypergraph two edges E_i & E_j, $i \neq j$is said to be adjacent if their intersection is not empty. The size of a hyper graph depends on the cardinality of its hyperedges. We define the size of H as the sum of the cardinalities of its hyperedges. A regular hyper graph is one in which every vertex is contained in k edges for some constant k. In a complete hyper graph the edge set consists of the power set $P(X)$, where X is the set of vertices other than singleton set and empty sets. A hyper graph H is said to be k-uniform if the number of vertices in each hyper edge is k.

Definition 2.2. *Let X be a finite set and let ξ be a finite family of non-trivial fuzzy subsets of X such that $X = \bigcup_{\mu \in E} Supp(\mu)$. The pair $H = (X, \xi)$ is called a fuzzy hypergraph (on X) and ξ is called the edge set of H which in sometimes denoted $\xi(H)$. The members of ξ are called the fuzzy edges of H.*

Definition 2.3. *The IFHG H is an ordered pair $H = \{X, E)$ where H satisfies the following conditions*

(a). $X = \{x_1, x_2, \ldots, x_n\}$ *is a finite set of vertices.*

(b). $E = \{E_1, E_2, \ldots, E_m\}$ *is a family on intuitionistic fuzzy subsets of* X.

(c). $E_j = \{(x_i, \mu_j(x_i), \gamma_j(x_i)); \mu_j(x_i),\ \gamma_j(x_i)\ \geq 0\ \&\ \mu_j(x_i) + \gamma_j(x_i) \leq 1\};\ j = 1,\ 2 \ldots m$

(d). $E_j \neq \phi$, $j = 1,\ 2,\ \ldots,\ m$ *and* $\bigcup_j Supp(E_j) = X$, $j = 1,\ 2,\ \ldots\ m$

The edges E_j are an IFSs of vertices, $\mu_j(x_i)$ and $\gamma_j(x_i)$ denote the degree of membership and non-membership of vertex x_i to edge E_j.

Definition 2.4. *The open neighborhood of a vertex x in the Intuitionistic Fuzzy Hypergraph is the set of adjacent vertices of x excluding that vertex and it is denoted by $N(x)$.*

Example 2.5. *Consider the Intuitionistic Fuzzy Hypergraph, Define* $X = \{x_1, x_2, x_3, x_4, x_5\}$ *and* $\xi = \{E_1, E_2, E_3, E_4\}$, *where*

$$E_1 = \{(x_1, 0.1, 0.4)\ (x_2, 0.1, 0.6)\}$$

$$E_2 = \{(x_3, 0.2, 0.4)\ (x_4, 0.8, 0.1)\ (x_5, 0.7, 0.2)\}$$

$$E_3 = \{(x_2, 0.1, 0.2)\ (x_3, 0.6, 0.2)\}$$

$$E_4 = \{(x_1, 0.1, 0.2)(x_4, 0.3, 0.4)\}$$

Here, the Open neighbourhood of vertex x_1 is x_2 and x_4.

Definition 2.6. *The closed neighbourhood of a vertex x in the Intuitionistic Fuzzy Hypergraph is the set of adjacent vertices of x including that vertex and is denoted by $N[x]$.*

Example 2.7. *Consider the Intuitionistic Fuzzy Hypergraph, Define* $X = \{x_1, x_2, x_3, x_4, x_5\}$ *and* $\xi = \{E_1, E_2, E_3, E_4\}$, *where*

$$E_1 = \{(x_1, 0.1, 0.4)\ (x_2, 0.1, 0.6)\}$$

$$E_2 = \{(x_3, 0.2, 0.4)\ (x_4, 0.8, 0.1)\ (x_5, 0.7, 0.2)\}$$

$$E_3 = \{(x_2, 0.1, 0.2)\ (x_3, 0.6, 0.2)\}$$

$$E_4 = \{(x_1, 0.1, 0.2)(x_4, 0.3, 0.4)\}$$

Here, the Closed neighbourhood of vertex x_2 is $x_2,\ x_1,\ x_3$.

Definition 2.8. *In an Intuitionistic Fuzzy Hypergraph H, the open neighborhood degree of a vertex x is denoted by* $\deg(x)$ *and defined by* $\deg(x) = (\deg^{\mu}(x),\ \deg^{\gamma}(x))$*, where*

$$\deg^{\mu}(x) = \sum_{x \varepsilon N(x)} \mu_E(x)$$

$$\deg^{\gamma}(x) = \sum_{x \varepsilon N(x)} \gamma_E(x)$$

Example 2.9. *Consider the Intuitionistic Fuzzy Hypergraph $H = (X, \xi)$. Define $X = \{x_1, x_2, x_3, x_4, x_5\}$ and $\xi = \{E_1, E_2, E_3, E_4\}$, where*

$$E_1 = \{(x_1, 0.1, 0.4)\ (x_2, 0.1, 0.6)\}$$

$$E_2 = \{(x_3, 0.2, 0.4)\ (x_4, 0.8, 0.1)\ (x_5, 0.7, 0.2)\}$$

$$E_3 = \{(x_2, 0.1, 0.2)\ (x_3, 0.6, 0.2)\}$$

$$E_4 = \{(x_1, 0.1, 0.2)(x_4, 0.3, 0.4)\}$$

The open neighborhood degree of a vertex x_1 is (0.4, 1.0).

Definition 2.10. *In an Intuitionistic Fuzzy Hypergraph H,the closed neighbourhood degree of a vertex x is denoted by* $\deg[x]$ *and defined by* $\deg[x] = (\deg^{\mu}[x],\ \deg^{\gamma}[x])$*, where*

$$\deg^{\mu}[x] = \deg^{\mu}(x) + \mu_E(x)$$

$$\deg^{\gamma}[x] = \deg^{\gamma}(x) + \gamma_E(x)$$

Example 2.11. *Consider the Intuitionistic Fuzzy Hypergraph $H = (X, \xi)$. Define $X = \{x_1, x_2, x_3, x_4, x_5\}$ and $\xi = \{E_1, E_2, E_3, E_4\}$, where*

$$E_1 = \{(x_1, 0.1, 0.4)\ (x_2, 0.1, 0.6)\}$$

$$E_2 = \{(x_3, 0.2, 0.4)\ (x_4, 0.8, 0.1)\ (x_5, 0.7, 0.2)\}$$

$$E_3 = \{(x_2, 0.1, 0.2)\ (x_3, 0.6, 0.2)\}$$

$$E_4 = \{(x_1, 0.1, 0.2)(x_4, 0.3, 0.4)\}$$

The closed neighborhood degree of a vertex x_1 is (0.5, 1.4).

Definition 2.12. *Let $H = (X, \xi)$ be a Intuitionistic Fuzzy Hypergraph on crisp hypergraph $H^* = (X, E)$. If all the vertices in X have the same open neighborhood degree n, then H is called n-Regular Intuitionistic Fuzzy Hypergraph.*

Definition 2.13. *Let $H = (X, \xi)$ be a Intuitionistic Fuzzy Hypergraph on a crisp hypergraph $H^* = (X, E)$. If all the vertices in X have the same closed neighborhood degree m, then H is called m-Totally Regular Intuitionistic Fuzzy Hypergraph.*

Example 2.14. *The following example is both n-Regular and m-Totally Regular Intuitionistic Fuzzy Hypergraph. Consider the Intuitionistic Fuzzy Hypergraph $H = (X, \xi)$, Define $X = \{a, b, c, d\}$ and $\xi = \{E_1, E_2 E_3, E_4\}$, where*

$$E_1 = \{(a, 0.7, 0.2)\ (b, 0.7, 0.2)\}$$

$$E_2 = \{(b, 0.7, 0.2)\ (c, 0.7, 0.2)\}$$

$$E_3 = \{(c, 0.7, 0.2)\ (d, 0.7, 0.2)\}$$

$$E_4 = \{(d, 0.7, 0.2)\ (2, 0.7, 0.2)\}$$

Open neighbourhood degree of every vertex is same $\deg(a) = \deg(b) = \deg(c) = \deg(d) = (1.4, 0.4)$*. So regular IFHG.*

Closed neighbourhood degree of every vertex is same $\deg[a] = \deg[b] = \deg[c] = \deg[d] = (2.1, 0.6)$*. So totally regular IFHG.*

Definition 2.15. *Let* $H = (X, \xi)$ *be a Regular Intuitionistic Fuzzy Hypergraph. The order of a Regular Intuitionistic Fuzzy Hypergraph,* H *in* $O(H) = \left(\sum_{x\in X} \mu_{E_i}(x), \sum_{x\in X} \gamma_{E_i}(x)\right)$ *for every* $x \in X$*. The size of Regular Intuitionistic Fuzzy Hypergraph is* $S(H) = \sum_{i=1}^{n} S(E_i)$ *where* $S(E_i) = \left(\sum_{x\in E_i} \mu_{E_i}(x), \sum_{x\in E_i} \gamma_{E_i}(x)\right)$.

Example 2.16. *Consider the Intuitionistic Fuzzy Hypergraph* $H = (X, \xi)$*. Define* $X = \{a, b, c, d\}$ *and* $\xi = \{E_1, E_2E_3, E_4\}$*, where*

$$E_1 = \{(a, 0.7, 0.2)\ (b, 0.7, 0.2)\}$$

$$E_2 = \{(b, 0.7, 0.2)\ (c, 0.7, 0.2)\}$$

$$E_3 = \{(c, 0.7, 0.2)\ (d, 0.7, 0.2)\}$$

$$E_4 = \{(d, 0.7, 0.2)\ (2, 0.7, 0.2)\}$$

Order of the IFHG, $O(H) = (2.8, 0.8)$*, Size of the IFHG,* $S(H) = (5.6, 1.6)$*.*

Remark 2.17.

(a). For an Intuitionistic Fuzzy Hypergraph, $H = (X, \xi)$ *to be both regular & totally regular, the number of vertices in each hyper edge* E_i *must be same.*

(b). And also each vertex lies in exactly same number of hyperedges.

Proposition 2.18. *The order of a n-Regular Intuitionistic Fuzzy Hypergraph* H *is* $nk/2$*,* $|X| = k$*.*

Proposition 2.19. *If* H *is both n-Regular and m-Totally regular Intuitionistic Fuzzy Hypergraph then* $O(H) = K(m - n)$ *where* $|X| = k$*.*

Proposition 2.20. *If* H *is a m-Totally regular Intuitionistic Fuzzy Hypergraph, then* $2S(H) - O(H) = mk$*,* $|X| = k$*.*

Theorem 2.21. *Let* $H = (X, \xi)$ *be a Intuitionistic Fuzzy Hypergraph of a hypergraph* H^**. Then* $\mu_E : X \to [0, 1]$*,* $\gamma_E : X \to [0, 1]$ *is a constant function* $\Leftrightarrow$ *the following are equivalent.*

(i). H *is a Regular Intuitionistic Fuzzy Hypergraph.*

(ii). H *is Totally regular Intuitionistic Fuzzy Hypergraph.*

Proof. Suppose that (μ_E, γ_E) is a constant function. Let $\mu_E(x) = C_1$ and $\gamma_E(x) = C_2$ for all $x \in E_i$

(i) $\Rightarrow$ (ii): Assume that H is n-Regular IFH. Then $\deg^{\mu}(x) = n_1$ and $\deg^{\gamma}(x) = n_2$ for all $x \in E_i$. So

$$\deg^{\mu}[x] = \deg^{\mu}(x) + \mu_E(x) \quad \text{for all} \quad x \in E_i;$$

$$\deg^{\gamma}[x] = \deg^{\gamma}(x) + \gamma_E(x) \quad \text{for all} \quad x \in E_i, \quad i = 1, 2, \ldots, n.$$

Thus, $\deg^{\mu}[x] = n_1 + C_1$; $\deg^{N}[x] = n_2 + C_2$ for all $x \in E_i$. Hence H is Totally regular IFH.

(ii) $\Rightarrow$ (i) Suppose that H is a m-Totally regular IFH. Then $\deg^{\mu}[x] = K_1$, $\deg^{\gamma}[x] = K_2$ for all $x \in E_i$, i = 1, 2, ..., n $\Rightarrow \deg^{\mu}(x) + \mu_E(x) = k_1$ for all $x \in E_i$, $\deg^{\gamma}(x) + \gamma_E(x) = k_2$ for all $x \in E_i$

$$\Rightarrow \quad \deg^{\gamma}(x) + C_1 = k_1 \quad \text{for all} \quad x \in E_i; \deg^{\gamma}(x) + C_2 = k_2 \quad \text{for all} \quad x \in E_i$$

$$\Rightarrow \quad \deg^{\mu}(x) = k_1 - C_1 \quad \text{for all} \quad x \in E_i; \deg^{\gamma}(x) = k_2 - C_2 \quad \text{for all} \quad x \in E_i$$

Thus H is a Regular IFH. Hence (i) and (ii) are equivalent. The converse part is obvious. □

Theorem 2.22. *If a IFHG H is both Regular and Totally regular, then (μ_E, γ_E) is constant function.*

Proof. Let H be a Regular and Totally regular IFH. Then $\deg^{\mu}(x) = n_1$; $\deg^{\gamma}(x) = n_2$ for all $x \in E_i$ and $\deg^{\mu}[x] = k_1$; $\deg^{\mu}[x] = k_2$ for all $x \in E_i$. Now $\deg^{\mu}[x] = k_1$ for all $x \in E_i$

$$\Leftrightarrow \quad \deg^{\mu}(x) + \mu_E(x) = k_1 \quad \text{for all} \quad x \in E_i$$

$$\Leftrightarrow \quad n_1 + \mu_E(x) = k_1 \quad \text{for all} \quad x \in E_i$$

$$\Leftrightarrow \quad \mu_E(x) = k_1 - n_1 \quad \text{for all} \quad x \in E_i$$

$$\Leftrightarrow \quad \deg^{\gamma}[x] = k_2 \quad \text{for all} \quad x \in E_i$$

$$\Leftrightarrow \quad \deg^{\gamma}(x) + \gamma_E(x) = k_2 \quad \text{for all} \quad x \in E_i$$

$$\Leftrightarrow \quad n_2 + \gamma_E(x) = k_2 \quad \text{for all} \quad x \in E_i$$

$$\Leftrightarrow \quad \gamma_E(x) = k_2 - n_2 \quad \text{for all} \quad x \in E_i$$

Hence (μ_E, γ_E) is constant function. □

Remark 2.23. *The converse of the above theorem is not true. Consider $H = (X, \xi)$ define $X = \{a, b, c, d\}$ and $\xi = \{E_1 E_2 E_3 E_4\}$, where*

$$E_1 = \{(a, 0.7, 0.2)\ (b, 0.7, 0.2)\}$$

$$E_2 = \{(b, 0.7, 0.2)\ (d, 0.7, 0.2)\}$$

$$E_3 = \{(c, 0.7, 0.2)\ (d, 0.7, 0.2)\}$$

$$E_4 = \{(a, 0.7, 0.2)(d, 0.7, 0.2)\}$$

Hence (μ_E, γ_E) is constant function but

$$(\deg^{\mu}(a), \deg^{\gamma}(a)) = (1.4, 0.4) \neq (2.1, 0.6) = (\deg^{\mu}(d), \deg^{\gamma}(d)).$$

Also $(\deg^{\mu}[a], \deg^{\gamma}[a]) = (2.1, 0.6) \neq (2.8, 0.8) = (\deg^{\mu}[d], \deg^{\gamma}[a])$. *So H is neither Regular nor Totally regular IFH.*

Proposition 2.24. *The dual of n-Regular and m-Totally regular IFHG is again n-Regular and m-Totally regular.*

Definition 2.25. *An Intuitionistic fuzzy hyper graph is said to be complete if for every $x \in X$, $N(x) = \{x/x \in X - x\}$ that is N(x) contains all the remaining vertices of X except x.*

Example 2.26. *Consider $H = (X, \xi)$ define $X = \{a, b, c, d\}$ and $\xi = \{E_1, E_2, E_3, E_4\}$, where*

$$E_1 = \{(a, 0.7, 0.2)\ (c, 0.7, 0.2)\}$$

$$E_2 = \{(a, 0.7, 0.2)\ (b, 0.7, 0.2)\ (d, 0.7, 0.2)\}$$

$$E_3 = \{(c, 0.7, 0.2)\ (d, 0.7, 0.2)(b, 0.7, 0.2)\}$$

Remark 2.27. *For a complete Intuitionistic fuzzy hyper graph, the cardinality of N(x) is same for every vertex.*

Theorem 2.28. *Every complete Intuitionistic fuzzy hypergraph is both Regular and Totally regular if (μ_E, γ_E) is a constant function.*

Proof. Let $H = (X, \xi)$, be a complete Intuitionistic fuzzy hypergraph. Suppose (μ_E, γ_E) be a constant function in H, So $\mu_E(x) = c_1$ and $\gamma_E(x) = c_2$ for all $x \in E_i$. Since the IFHG is complete. By definition, for every vertex $x \in X$, $N(x) = \{x/x \in X - x\}$. Open neighborhood degree of every vertex is same. That is $\deg^{\mu}(x) = n_1$; $\deg^{\gamma}(x) = n_2$ for all $x \in E_i$. Hence complete IFHG is regular. Also

$$\deg^{\mu}[x] = \deg^{\mu}(x) + \mu_E(x) \text{ for all } x \in E_i$$

$$\deg^{\gamma}[x] = \deg^{\gamma}(x) + \gamma_E(x) \text{ for all } x \in E_i$$

Thus $\deg^{\mu}[x] = n_1 + c_1$, $\deg^{\gamma}[x] = n_2 + c_2$ for all $x \in E_i$. Hence H is Totally regular. □

Remark 2.29. *Every complete IFHG is Totally regular even if (μ_E, γ_E) is not constant in H.*

Definition 2.30. *An IFH is said to be k-uniform if all the hyper edges have the same cardinality.*

Example 2.31. *Consider $H = (X, \xi)$ define $X = \{a, b, c, d\}$ and $\xi = \{E_1 E_2 E_3\}$, where*

$$E_1 = \{(a, 0.1, 0.2)\ (b, 0.1, 0.6)\}$$

$$E_2 = \{(b, 0.1, 0.6)\ (d, 0.7, 0.2)\}$$

$$E_3 = \{(c, 0.4, 0.2)\ (d, 0.7, 0.2)\}$$

Definition 2.32. *An IFH is said to be complete k-uniform IFH, denoted as $K_n^{(k)}$ is a hypergraph on n-vertices where every k-element subset of the vertex set is an edge.*

3. Conclusion

Theoretical concepts of graphs and hypergraphs are highly utilized by computer science applications.The intuitionistic fuzzy hypergraphs are more flexible than fuzzy hypergraphs.The concepts of Intuitionistic fuzzy hypergraphs can be applied in various areas of engineering and computer science.We defined the regular and totally regular intuitionistic fuzzy hypergraphs in this paper.We plan to extend our research work to irregular intuitionistic fuzzy hypergraphs.

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