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Some Observations of t^* on Mortality Curve at Very Old Age

Research Article

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Abstract: In the absence of age specific mortality data, we give some observations of the point t^* at which starts to decline in a mortality curve by the extended Weibull model (Weon model).

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Keywords: Survival function, Mortality function, Age-dependent stretched exponent (Shape parameter), Characteristic life and Vertex point.

1. Introduction

Faith that aging is yet an unsolved problem in biology is no longer true. Understanding the reasons for the evolution of senescence is of great importance for investigating the aging mechanisms of humans and other organisms [3, 4]. Demography is an essential tool in aging research, in conjunction with evolution, ecology, and medicine, because more information is available on humans than on any other species [5, 7, 10]. Science asks how long can humans live, species are defined by biological criteria, but we suggest a more appropriate question is: How long must humans live?[11].

As humans live longer, the precise modeling of mortality curves in very old age is becoming more important in aging research and public health [8]. A fundamental question in aging research concerns the demographic trajectories at the highest ages, especially for supercentenarians (persons aged 110 or more). These trajectories at the highest ages are a fundamental question for studying aging and longevity, especially for supercentenarians [9]. The demographic analysis of Swedish females over three recent decades revealed an important trend: the maximum human lifespan (existing around 125 years) gradually decreased at a constant rate of ~ 1.6years per decade, while the characteristic life gradually increased at a constant rate of ~ 1.2 years per decade.

The age-dependence of the stretched exponent is the critical difference between the modified stretched exponential (To explain Weon and Je previous papers) and the classical stretched exponential functions. Weon model enables us to predict that the mortality rate will decline after a plateau around ages 110-115 and the maximum longevity emerges around ages 120-130 for the modern demographic data. The maximum human life spans and characteristic lives eventually become closer together over time. This result indicates that the human survival curves became increasingly concentrated at very old age, which is consistent with the definition of population aging [1, 2].

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The estimated maximum lifespan of around ~ 125 years does not immediately indicate the biological limit of human lifespan, and is different from the concept of the 'biological warranty period' [22]. The continual decrease of the estimated maximum lifespan suggests that the estimated maximum lifespan will become closer to the actual biological limit as people live longer [8].

The Weon model by mathematical modeling of the age-dependent shape parameter is more flexible than the Gompertz model to describe dynamical demographic trajectories over the total life span [6]. Weon et.al. described the lifespan limit estimation is obtained from mortality patterns, which are defined as $\int \mu(t) = (t/\alpha)^{\beta(t)}$. For simplification one defines $\delta(t) = \mu(t)/\int \mu(t) = \beta(t)/t + \ln(t/\alpha)d\beta(t)/t$. The point where the mortality curve starts to decline is obtained from $\delta(t)^2 + d\delta(t)/dt$ by solving $d\mu(t)/dt = 0$. In this paper, we delivered that some observations of the point t^* at which starts to decline by providing the mortality curves in very old age for the recent demographic data.

1.1. Mathematical Model: Weon Model

Weon established a model derived from the Weibull model with an age dependent shape parameter [18, 20] to describe the human survival mortality curve [13]. It is simply described by two parameters, the age dependent shape parameter $\beta(t)$ and the charateristic life α,

$$S(t) = e^{(-\frac{t}{\alpha})^{\beta(t)}}, \tag{1}$$

where $S(t)$ denotes the survival probability at any age t. After graphically determining the value α, a mathematical expression determined from(1),

$$\beta(t) = \frac{\ln(-\ln S(t))}{\ln(\frac{t}{\alpha})}. \tag{2}$$

If $\beta(t)$ is not constant with age, this obviously implies that $\beta(t)$ is a function of age t. The calculation of α and the determination of $\beta(t)$ for a survival curve enables the determination of an exact formula for the mortality curve through

$$\mu(t) = \frac{-d\ln S(t)}{dt}. \tag{3}$$

From (1), we get

$$\mu(t) = \frac{d}{dt}\left(\frac{t}{\alpha}\right)^{\beta(t)}$$

or

$$\mu(t) = \left(\frac{t}{\alpha}\right)^{\beta(t)}\left[\frac{\beta(t)}{t} + \ln\left(\frac{t}{\alpha}\right)\frac{d\beta(t)}{dt}\right]. \tag{4}$$

where α denotes the characteristic life ($t = \alpha$) when $S = \exp(-1) \approx 36.78\%$. The trends and causes increased characteristic life may be identical with those of average life (when S = 50%).

The α value can serve as a good alternative to the life expectancy at birth (ϵ) [14, 17]. Empirically, the quadratic formula of

$$\beta(t) = \beta_0 + \beta_1 t + \beta_2 t^2,$$

describes the $\beta(t)$ patterns at very old ages quite well [13]. Particularly, the quadratic patterns in $\beta(t)$ lead to the existence of the maximum human lifespan (t), which can be defined as the specific age of

$$\beta(t) = -t\ln(t/\alpha)\beta'(t) \tag{5}$$

which is taken by the mathematical constraint of $dS(t)/dt \to 0$. The maximum survival tendency is characterized as a "negative" slope of the beta function as $d^2\beta(t)/dt^2 < 0$ for the phase of $t > \alpha$. Supposedly, equation (1) is the survival function of age ($\frac{dS(t)}{dt} < 0$) between $S(0) = 1$ and $S(\omega) = 0$ for a maximum age (ω). For this motive, the slope of the stretched exponent with age can be given by

$$\frac{d\beta(t)}{dt} \begin{cases} < +\epsilon(t), t < \alpha \\ > -\epsilon(t), t > \alpha. \end{cases} \tag{6}$$

where $\epsilon(t) = \left| -\frac{\beta(t)}{t\ln(\frac{t}{\alpha})} \right|$ is the mathematical constraint of $\beta(t)$. Equation (6), shows a mathematical tendency that if change the survival probability is minimized ($\frac{dS(t)}{dt}$) then the slope of stretched exponent becomes lower than the certain positive value at young ages ($t < \alpha$), while it becomes higher than a negative value at old ages ($t > \alpha$). Weon et.al. described the lifespan limit estimatinon is obtained from mortality patterns, which are defined as $\int \mu(t) = (t/\alpha)^{\beta(t)}$. For simplification one defines $\delta(t) = \mu(t)/\int \mu(t) = \beta(t)/t + \ln(t/\alpha)d\beta(t)/t$.

The point where the mortality curve starts to decline is obtained from $\delta(t)^2 + d\delta(t)/dt$ by solving $d\mu(t)/dt = 0$. In our previous work, we described the estimation of ratio of vertex point to the characteristic life and its maximum lifespan; we derive the reduced $'t'$ intercept and its maximum lifespan.

And also, we provide the properties of age-dependent stretched exponent; an estimation of lower and upper bounds for maximum lifespan in the absence of age specific mortality data. In the present work, we address that this model enables the some observations of the point (t^*) at which starts to decline by providing the mortality curves in very old age for the recent demographic data.

2. Some observations of the point t^* at which starts to decline on the mortality curve

A quadratic expression $\beta(t)$ can be expressed in the form [12],

$$\beta(t) = \beta(\nu) - c(\nu - t)^2, \tag{7}$$

where $\beta(\nu)$ and c are unknown constants. In [12], the point where the mortality curve starts to decline is obtained from

$$(\delta(t))^2 + \frac{d\delta(t)}{dt} = 0,$$

by solving $\frac{d\mu(t)}{dt} = 0$. From (4) we get,

$$\left(\ln(t/\alpha)\beta''(t) - \frac{\beta(t)}{t^2} + \frac{2\beta'(t)}{t}\right) + \left(\frac{\beta(t)}{t} + \ln(t/\alpha)\beta'(t)\right)^2 = 0. \tag{8}$$

Or, equivalently

$$\left(\ln(t/\alpha)\beta''(t) - \frac{\beta(t)}{t^2} + \frac{2\beta'(t)}{t}\right) = -\left(\frac{\beta(t)}{t} + \ln(t/\alpha)\beta'(t)\right)^2 \tag{9}$$

We denote the solution of (8) by t^*. To solve above equality (9), we consider the following two cases separately.

Case:(I) When $t = \alpha$

Note that, the L.H.S of above equality is always negative. That is,

$$\left(\ln(t/\alpha)\beta''(t) - \frac{\beta(t)}{t^2} + \frac{2\beta'(t)}{t}\right) < 0.$$

Or, equivalently

$$\left(\frac{\beta(t)}{t^2} - \ln(t/\alpha)\beta''(t) - \frac{2\beta'(t)}{t}\right) > 0. \tag{10}$$

When $t = \alpha$, then (10) implies

$$\frac{\beta(\alpha)}{\alpha^2} - \frac{2\beta'(\alpha)}{\alpha} > 0. \tag{11}$$

Using (2), it can be shown [15] that

$$\beta'(\alpha) = \frac{\beta(\alpha)}{2\alpha}. \tag{12}$$

Now substitution of (12) into (11),

$$\frac{\beta(\alpha)}{\alpha^2} - \frac{2\beta(\alpha)}{2\alpha^2} > 0.$$

Thus, we arrive at a contradiction. Therefore $t \neq \alpha$ is clear.

Case:(II) When $t = \nu$

Rewriting the equation (9) in the form,

$$\sqrt{\left(-\ln(t/\alpha)\beta''(t) + \frac{\beta(t)}{t^2} - \frac{2\beta'(t)}{t}\right)} = \frac{\beta(t)}{t} + \ln(t/\alpha)\beta'(t). \tag{13}$$

Putting $\beta'(t)$ at $t = \nu$, we get

$$\sqrt{\frac{\beta(\nu)}{\nu^2} + 2c\ln(\nu/\alpha)} = \frac{\beta(\nu)}{\nu}.$$

This equality is invalid, since L.H.S is greater than R.H.S. Because, $\beta(\nu)$ is always greater than 1 & c is positive, $\nu > \alpha$. Therefore $t \neq \nu$ is clear. Numerical results show that $t > \nu$. Expanding R.H.S of (9) takes the form,

$$\left(\ln(t/\alpha)\beta''(t) - \frac{\beta(t)}{t^2} + \frac{2\beta'(t)}{t}\right) = -\left(\left(\frac{\beta(t)}{t}\right)^2 + (\ln(t/\alpha)\beta'(t))^2 + \frac{2\beta(t)}{t}\ln(t/\alpha)\beta'(t)\right) \tag{14}$$

Numerical results indicate that $\beta(t)$ is of order 10 and t is of order 10^2 (i.e. $\beta(t) \sim 10$ *and* $t \sim 10^2$). Neglecting the higher order terms $\frac{\beta(t)}{t^2}$ and $(\ln(t/\alpha)\beta'(t))^2$, equation (14) reduces to,

$$\ln(t/\alpha)\beta''(t) + \frac{2\beta'(t)}{t} = -\left(\frac{\beta(t)}{t}\right)^2 - \frac{2\beta(t)}{t}\ln(t/\alpha)\beta'(t). \tag{15}$$

Or, equivalently

$$\ln(t/\alpha)\beta''(t) + \frac{2\beta'(t)}{t} + \left(\frac{\beta(t)}{t}\right)^2 = -\frac{2\beta(t)}{t}\ln(t/\alpha)\beta'(t). \tag{16}$$

Using Mathematica version 6.0, the graphical representation shows that, the solution t^* of (14) is also a solution t^* of (16). Its follows from (15) that

$$\left(\frac{\beta(t)}{t}\right)^2 > -\frac{2\beta(t)}{t}\ln(t/\alpha)\beta'(t), \tag{17}$$

since R.H.S is negative. Further, its follows from (16) that

$$\left(\frac{\beta(t)}{t}\right)^2 > \ln(t/\alpha)\beta''(t) + \frac{2\beta'(t)}{t}, \tag{18}$$

since L.H.S is positive. In table (1), using Mathematica version 6.0. we list the numerical values of (17) and (18), to choose maximum value for a given ν and α.

$\alpha(years)$	$\nu(years)$	t^*	u	v	w
88.57223	96.34	111.160	0.00403039	0.00530165	0.00935457
88.52680	94.96	112.580	0.00359697	0.00511099	0.00861772
87.83450	94.73	111.383	0.00371288	0.00516082	0.00888749
87.81336	95.93	112.665	0.00343064	0.00474252	0.00819737
87.66873	95.65	112.594	0.00338436	0.00468819	0.00810823
87.34920	94.04	113.940	0.00301382	0.00448949	0.00750347
86.91140	94.57	112.362	0.00298234	0.00421136	0.00785261

Table 1. **(reprinted from [12])**

Here

$$u = \ln(t/\alpha)\beta''(t) + \frac{2\beta'(t)}{t} \quad \& \quad v = -2\ln(t/\alpha)\beta'(t)\frac{\beta(t)}{t} \quad \& \quad w = \left(\frac{\beta(t)}{t}\right)^2$$

Hence we choose maximum of (u, v),

$$\left(\frac{\beta(t)}{t}\right)^2 > -\frac{2\beta(t)}{t}\ln(t/\alpha)\beta'(t), \tag{19}$$

The inequality (18) is valid for $t < 116$, which is of order 10^2.

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